EXPERIMENTS ON THE ELLIPTIC INSTABILITY IN VORTEX PAIRS WITH AXIAL CORE FLOW

Clément Roy*, Thomas Leweke*, Mark C. Thompson** & Kerry Hourigan**
*Institut de Recherche sur les Phénomènes Hors Equilibre (IRPHE), CNRS/Universités Aix-Marseille
49 rue Frédéric Joliot-Curie, BP 146, F-13384 Marseille Cedex 20, France
**Fluids Laboratory for Aeronautical and Industrial Research (FLAIR),
Department of Mechanical Engineering, Monash University, Clayton, 3800, Australia

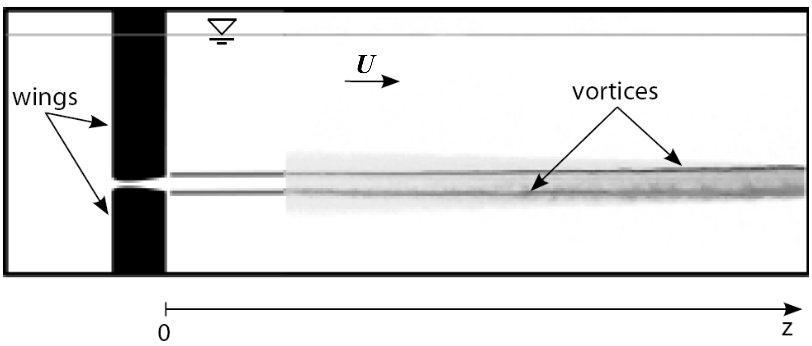
Summary We show experimental results concerning the interaction of two parallel counter-rotating vortices presenting a jet-like axial velocity profile in their cores. The vortex pair is generated by two wings in a water channel; it is visualized by dye injection, and characterized quantitatively by stereoscopic Particle Image Velocimetry. Two types of three-dimensional instability are observed: the long-wavelength Crow instability and a short-wavelength core instability caused by the mutually induced elliptic deformation of the vortex streamlines. We here focus on the short-wave (elliptic) instability, for which we deduce the perturbation mode structure, which is different from the one observed without axial flow, the axial wavelength and the growth rate from the experimental data. Comparisons are made with the results from a numerical linear stability analysis of the measured flow, and good overall agreement is found.

INTRODUCTION

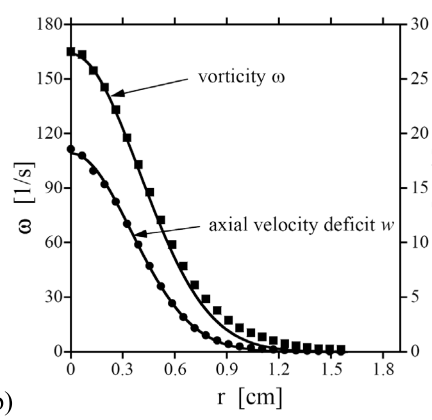
Pairs of parallel vortices are simple configurations for the study of elementary vortex interactions. The case of two counter-rotating vortices of equal and opposite circulation has been studied extensively, due to its relevance for the problem of aircraft wake turbulence. Two types of three-dimensional instability have previously been identified in this system. The long-wavelength Crow [1] instability involves symmetric wavy displacements of the vortex cores, whereas the short-wave elliptic instability (Kerswell [2]) results in more complicated internal deformations of the cores. Both mechanisms and their interaction lead to the eventual break-up and decay of the initial pair. The short-wave instability of uniform vortex pairs without axial flow has been studied experimentally by Leweke & Williamson [3] and theoretically/numerically by Le Dizès & Laporte [4]. The present work considers the additional feature of axial core flow, which more realistically represents the situation in aircraft trailing vortices. Theoretical and numerical studies of this case have been carried out recently by Lacaze *et al.* [5] and Roy *et al.* [6]. Here we present the first detailed experimental results.

EXPERIMENTAL DETAILS

Vortex pairs with axial core flow were generated in a free-surface recirculating water channel with a 37 cm × 50 cm test section of length 150 cm, using two rectangular NACA 0012 airfoils of chord c = 10 cm, positioned vertically in the test section and facing each other tip to tip (Fig. 1a). When placed at the same angle of attack with respect to the free stream (magnitude U = 57.5 cm/s), two counter-rotating vortices, of circulation $\pm\Gamma$ and separated by a distance b, are generated at the wing tips. They present a longitudinal (z) velocity deficit near their axes, which in the frame of reference moving with the external flow represents a jet flow (directed upstream) in the vortex cores. Stereoscopic Particle-Image Velocimetry (PIV) was used to determine the three-dimensional velocity field of the vortices. An example of the radial profiles of axial vorticity ω and axial velocity defect $w = (U-u_z)$, azimuthally averaged around the given vortex, are shown in Fig. 1(b). Both are closely approximated by Gaussian distributions, $\omega = (\Gamma/\pi a^2) \cdot \exp(-r^2/a^2)$ and $w = w_o \cdot \exp(-r^2/a_W^2)$, with characteristic radial scales a and a_W and amplitudes $\Gamma/\pi a^2$ and w_o, respectively. For the present observations, the initial unperturbed vortex pair was characterized by the following non-dimensional parameters: Reynolds number $Re = \Gamma/\nu$ = 16800 (ν: kinematic viscosity), core size a/b = 0.15, velocity defect (inverse swirl number) $W = 2\pi a w_o/\Gamma$ = 0.44, and axial velocity radius a_W/a = 0.86.



(a)



(b)

Figure 1. (a) Schematic of the experimental set-up (side view). (b) Azimuthally averaged radial profiles of axial vorticity (■) and axial velocity (●) for the upper vortex, measured at z/c = 5.6. Lines represent Gaussian fits.

RESULTS AND DISCUSSION

For the above parameters, both long- and short-wavelength instabilities have been observed in the experiments. Focusing on the latter, Fig. 2 shows a close-up side view of the short-wave core perturbation observed on one of the vortices at a downstream position $z/c = 10$, visualized by fluorescent dye injected at the airfoil tips and illuminated in volume by laser light. The initially cylindrical dye volume presents periodic edge deformations whose axial wavelength scales on the core diameter. The symmetry of the pattern, and the dye streaks linking a given crest with the adjacent opposite one, indicate that this perturbation is helical, with an azimuthal wave number $m = 2$. (For vortex pairs without axial flow, perturbation modes with $m = 1$ are generally observed.) The wavelength λ of the unstable mode can be measured directly from instantaneous side views as in Fig. 2. The experimental determination of the instability growth rate is much more difficult. Instantaneous PIV velocity measurements are too noisy, and the vortices are subject to time-dependent overall lateral displacements, caused by the growth of the Crow instability mode, but also in a random way by the fluctuations generated in the wake of the airfoils at this high *Re*, which excludes meaningful time-averaged measurements. Therefore, perturbation amplitudes A were also measured from the dye visualizations, as shown in Fig. 2, at different downstream locations z, which were converted to the non-dimensional life-time t^* of the vortex pair using $t^* = (z/U)/(2\pi b^2/\Gamma)$. The result of these measurements is plotted in Fig. 3(a). The growth appears indeed to be exponential, and a least-squares fit allows determination of the non-dimensional growth rate σ^*. The curve in Fig. 3(b) gives the growth rate of the most unstable short-wave perturbation mode, as function of the (non-dimensionalised) axial wave number $k = 2\pi/\lambda$, as determined by a numerical stability analysis (details are given in [6]) of the experimentally measured vortex pair velocity profile. The distribution of the corresponding axial vorticity perturbation (see inset) corresponds indeed to an azimuthal wave number $m = 2$. The overall agreement between the theoretical/numerical prediction and the experimental measurement is good, considering the uncertainties of the latter, and in particular the unknown initial noise spectrum, whose characteristics could possibly explain why the observed wavelength is shorter than the one with the highest growth rate in the numerical study.

More details on these observations, also for the case of co-rotating vortex pairs, will be presented at the Congress.

This work is supported by the European Commission under project "FAR-Wake" (www.FAR-Wake.org).

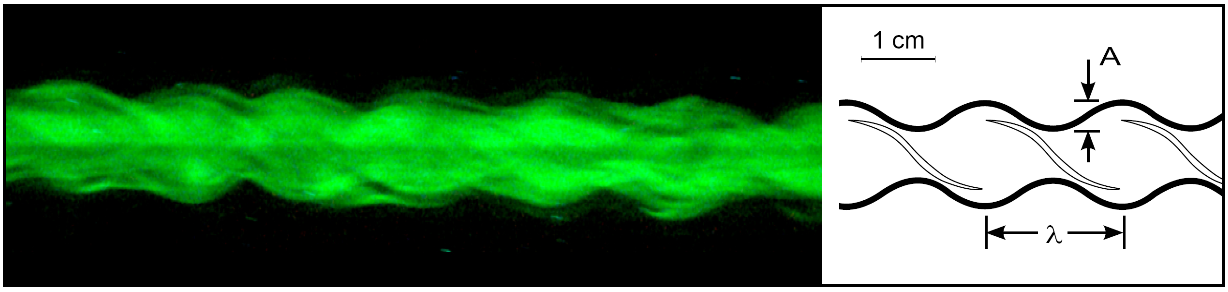


Figure 2. Dye visualisation of the short-wave core perturbation, and schematic explaining the quantitative measurements made from such images.

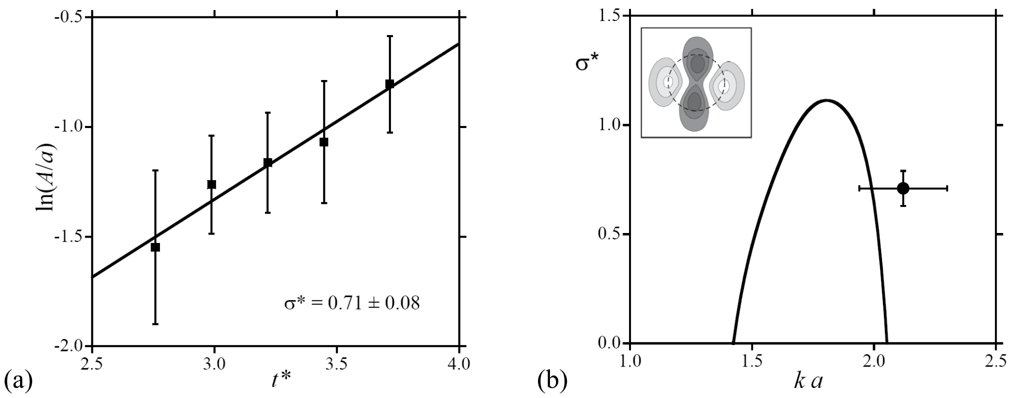


Figure 3. (a) Growth of the perturbation amplitude, as determined from dye visualisations at different downstream positions. (b) Growth rate as function axial wavelength: experiment (●) and numerical stability analysis of the experimental flow (——). Inset: axial vorticity perturbation of the unstable mode.

References

[1] Crow S. C.: Stability theory for a pair of trailing vortices. *AIAA J.* **8**:2172-2179, 1970.
[2] Kerswell R. R.: Elliptical instability. *Annu. Rev. Fluid Mech.* **34**:83-113, 2002.
[3] Leweke T., Williamson C. H. K.: Cooperative elliptic instability of a vortex pair. *J. Fluid Mech.* **360**:85-119, 1998.
[4] Le Dizès S., Laporte F.: Theoretical predictions for the elliptical instability in a two-vortex flow. *J. Fluid Mech.* **471**:169-201, 2002.
[5] Lacaze L., Ryan K., Le Dizès S.: Elliptic instability in a strained Batchelor vortex. *J. Fluid Mech.* **577**:341-361, 2007.
[6] Roy C., Schaeffer N., Le Dizès S., Thompson M. C.: Stability of a pair of co-rotating vortices with axial flow, *Phys. Fluids* (submitted), 2008.