INSTABILITY OF STEADY AND PULSATILE FLOWS IN STENOTIC GEOMETRIES

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Summary A numerical and experimental investigation of steady and pulsatile flows through a tube partially obstructed by an axisymmetric blockage is presented. The geometry serves as an idealised model of an arterial stenosis. The pulsatile flow consists of a steady downstream flow with a single-harmonic sinusoidally varying oscillation. Comparisons are made between the numerical and experimental results. Linear absolute instabilities are found numerically, although convective instability is found to be important for the experimental flows.

The cardiovascular arterial system can present a range of biological responses to fluid mechanical properties. Arterial thickening is often the precursor to the most common form of arterial disease, atherosclerosis, which involves the long-term deposition of cholesterol-related particles in the arterial wall [1]. Such deposits eventually form a blockage, or stenosis, which can have several effects, ranging from reduced blood fbw to the development of thrombi, or blood clots. Fluid mechanics researchers have sought to characterise the fundamental fbw behaviour of stenotic fbws, by investigating idealised models of stenosis using numerical fluid fbw solvers [2, 3]. This study follows in a similar vein, also investigating an idealised geometry experimentally.

METHOD

The geometry under investigation consists of a long, straight tube with an axisymmetric stenosis, semi-circular in cross-section. The stenosis degree, defined by $S = 1 - (d/D)^2$, where d is the diameter at the centre of the blockage and D is the diameter of the tube, is varied from 0.20 to 0.90, along with the Reynolds number, defined by $Re = \overline{U}D/\nu$, where $\overline{U}$ is the temporal and cross-sectional average of the fluid velocity and ν is the kinematic viscosity. For pulsatile fbw, the sectionally-averaged velocity oscillates sinusoidally around the temporally-averaged fbw velocity $\overline{U}$, at a period T (non-dimensionalised by $D/\overline{U}$) and an amplitude A.
Numerical two-dimensional axisymmetric fbw field simulations were obtained using a spectral-element method. Experimental results were obtained from a rig consisting of a transparent perspex tube of 20 mm diameter, with inlet and outlet lengths of 2000 mm, or $100D$. Vibrations to the system were isolated and the alignment of the tube verified. A piston located at the inlet allowed the creation of pulsatile fbws and steady fbws subjected to a high-frequency low-amplitude forcing.

RESULTS

Steady Flow

Figure $1(a)$ depicts the generic fbw structure under investigation, for $b = 0.75$ and $Re = 194$. For this Reynolds number, the fbw is laminar and steady. Experimentally, dye becomes trapped in the axisymmetric recirculation zone which forms downstream of the stenosis. The recirculation zone extends further downstream with stenosis degree and Reynolds number. Numerically, critical Reynolds numbers for linear absolute instability, (i.e., instability modes which break the axisymmetry of the fbw) are predicted. However, experimentally, the fbw becomes unstable at much lower Reynolds

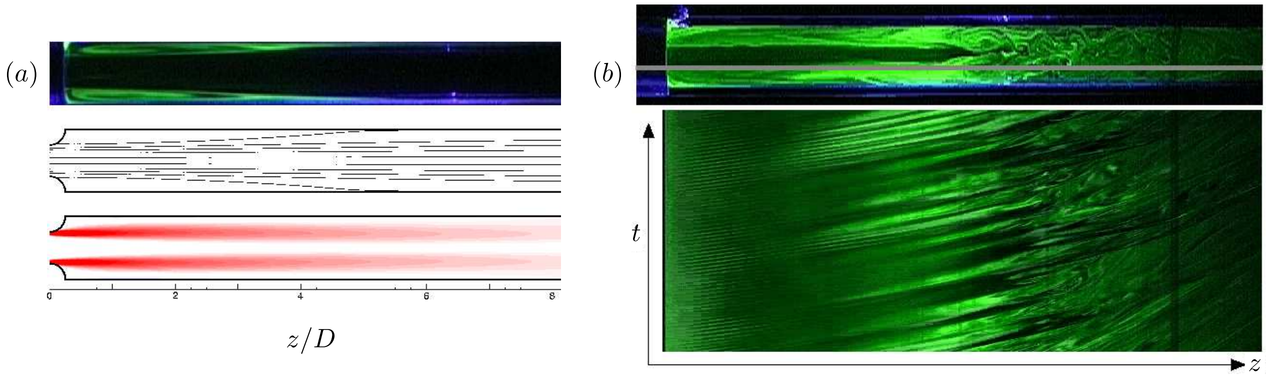


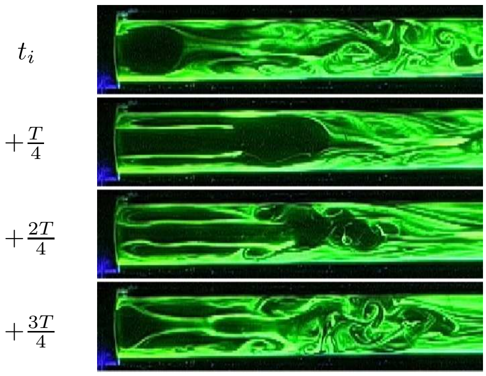
Figure 1. (a) Dye visualisation and numerical streamlines and vorticity for $S = 0.75$ and $Re = 194$. A reflection of the laser in the middle of the image on the bottom portion of the tube is present, evident in the fi gure as a thin, blue shade. Flow is from left to right. (b) At top, the unsteady flow for $S = 0.90$, $Re = 194$. For the spatio-temporal diagram, at bottom, the waves seen in the shear layer are evident as straight lines to the left of the lower diagram.

numbers. Figure $1(b)$ presents a still of a film of a dye-visualisation of the flow for $S = 0.90$ and $Re = 194$. The flow is unstable to convective shear layer instability. Small waves in the shear-layer can be seen immediately downstream of the stenosis, growing as they propagate and meeting an area of unsteadiness at $z \approx 5D$. The second part of figure $1(b)$ shows a spatio-temporal diagram, constructed from the line of pixels along the grey line of the first image. In this diagram, the waves in the separating shear layer are visible as straight lines to the left of the spatio-temporal diagram. From such diagrams we can calculate the period of the waves in the shear layer that the instability creates. The period (non-dimensionalised) of the shear layer waves in these self-sustaining unstable cases is found to vary with stenosis degree, but is roughly constant with Reynolds number.

By using the piston on the experimental rig, flows of Reynolds number less than critical were subjected to a high-frequency, low-amplitude forcing, allowing a peak forcing period to be determined; that is, the forcing period which induced the greatest response in the shear layers downstream of the stenosis. These peak forcing periods were found to be significantly higher than the instability periods of the self-sustaining unstable flows at higher Reynolds number. A possible explanation for this lies in the turbulence and unsteadiness generated downstream in the higher Reynolds number flows, which may alter the noise profile in the rig, amplifying some particular frequencies ahead of others.

Pulsatile Flow

Figure $2(a)$ presents dye visualisations over one pulse period of the experimental flow for stenosis degree $S = 0.75$, Reynolds number $Re = 206$, pulse amplitude $A = 0.75$ and pulse period $T = 2.43$. The flow consists of a vortex ring which forms each pulse period downstream of the stenosis, then detaches and propagates downstream. In the first image, we see the flow at the end of the vortex formation phase, with a large body of clear fluid from upstream rolling up into the main vortex ring, which subsequently travels downstream. Figure $2(b)$ presents critical Reynolds numbers for stability as determined from Floquet stability analysis. A period-doubling instability of wavenumber $m = 1$ dominates for most of the pulse period range, however, higher wavenumber modes dominate for lower pulse periods. The Floquet



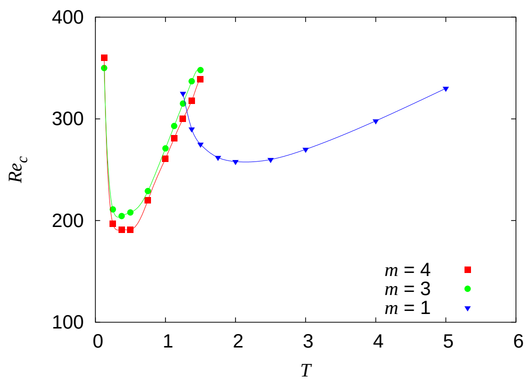


Figure 2. (a) Coloured-dye visualisations of the flow over one pulse period, for $S = 0.75$, $Re = 206$, $A = 0.75$ and $T = 2.43$, (a) critical Reynolds number for linear absolute instability as a function of pulse period, for stenosis degree $S = 0.75$.

stability analysis of figure $2(b)$ predicts a critical Reynolds number $Re_c \approx 260$ for a period-doubling ($m = 1$) linear absolute instability. However, the experimental flow shown in figure $2(a)$ is unstable, breaking down at approximately 4 to 5 diameters downstream of the stenosis. The waves, or roll-ups, in the separating shear layer seem relatively gentle in comparison with the breakdown which occurs further downstream; shear layer roll-up in the wake of the main vortex ring does not appear to be the main flow instability mechanism. It is possible an interaction exists, whereby the noise dependent convective instability induces the absolute linear instability at a lower Reynolds number, as suggested in [2].

CONCLUSIONS

Steady flow in the stenotic geometry consists of an axisymmetric jet and recirculation zone downstream of the stenosis. Experimentally, the flow becomes unstable to a Kelvin-Helmholtz shear-layer convective instability, the period of which is roughly constant with Reynolds number. For pulsatile flow, convective instability appears to play a role, although there is some indication of linear absolute instability at play. The results show that even under carefully-controlled experiments of flows in stenotic geometries, convective instability plays an important role, appearing at Reynolds numbers much lower than predicted by linear absolute stability analysis.

References

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