Scaling of streamwise vortices in wakes

C. H. K. Williamson
Mechanical and Aerospace Engineering, Upson Hall, Cornell University, Ithaca, New York 14853
J. Wu
CSIRO Division of Building, Construction and Engineering, Highett, Victoria 3190, Australia
J. Sheridan
Mechanical Engineering, Monash University, Clayton, Victoria 3168, Australia

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In this Letter, we demonstrate the coexistence of two distinct systems of streamwise vortices in a bluff body wake. It appears that there exist conditions to amplify streamwise vorticity in bluff body wakes, by vortex stretching, in both the separating shear layers from the sides of the body and also in the vortex street wake. The length scale governing the streamwise vortices in the shear layer has a $1/\sqrt{\mathrm{Re}}$ dependence, whereas the scale of such structures in the wake is independent of Reynolds number, Re (over a large range of Re). The proposition that there should exist two distinct, and possibly disparate, spanwise length scales in the cylinder wake is well supported by compiled measurements, particularly those of Williams and co-workers (Mansy *et al.* [J. Fluid Mech. **270**, 277 (1994)]), as well as those from Chyu and Rockwell (submitted to J. Fluid Mech.). © *1995 American Institute of Physics.*

There has recently been a surge of activity regarding bluff body wakes, which has been fueled by the present capacity to compute by direct numerical simulation the wake of a body, as well as to determine experimentally the structure of wakes using cutting-edge techniques such as Particle-Image-Velocimetry (PIV) and the scanning laser anemometry technique of Williams and co-workers.[1,2] It is the emergence of these computational and experimental techniques that has partly triggered the present paper.

For Reynolds numbers above the "laminar regime" (Re >180), it was shown by Hama[3] that the shedding primary "Kármán" vortices behind a cylinder exhibited a small-scale spanwise waviness, which Gerrard[4] later described as "fingers of dye." These structures were shown by Williamson[5–7] to be manifestations of streamwise vortices, amplified by vortex stretching in the "braid" regions between primary Kármán vortices, closely similar to those streamwise vortices studied in mixing layers (Bernal and Roshko;[8] Corcos and Lin[9]), and in unseparated wakes (Meiburg and Lasheras[10]). In the transition regime of vortex shedding, extending over a range Re=180–260, it was shown (Williamson[5–7]) that there exist two discontinuous changes in the wake formation as one increases Re through this range, each stage being associated with a distinct spanwise length scale of the streamwise vortex structure. In the range Re=180–245, one observes predominantly the formation of vortex loops during wake formation, with a spanwise wavelength of around three diameters, in what has become known as "mode A" shedding. For Re>245, the predominant structures are finer-scale streamwise vortex pairs with a spanwise length scale of closer to one diameter, in what is known as "mode B" shedding. These vortex pairs are clearly seen in the visualization of Fig. 1, taken using laser-induced fluorescence in our XY Towing Tank. Both of the modes A and B, with their distinct spanwise wavelengths, have recently been computed clearly in direct numerical simulation by Thompson *et al.*[11]

Although the spanwise wavelengths associated with the above shedding modes were measured in Williamson,[5] and included later in Fig. 2, it can be seen that such measurements were coarse. The recent use of new techniques of PIV (Wu *et al.*;[12] Lin and Rockwell;[13] Lin *et al.*[14]) and scanning-laser anemometry (introduced to this flow by Williams and Economou[2]) now allow us to measure much more accurately the spanwise length scales. In fact, it is the extension of such measurements up into the Reynolds number range of order 10^4, that has triggered the suggestions in Williamson[15] and Wu *et al.*[16] that one should find more than one distinct spanwise length scale for the streamwise vortices, depending on the downstream location where a measurement is made. The suggestions and evidence for the existence of these two spanwise length scales at moderate Re forms the basis of the present paper.

In this Letter, we should briefly outline a central mechanism to generate and amplify such vortices. Wei and Smith[17] hypothesized that the streamwise vortices are produced by distortion of shear layer vortices, which form out of an instability of the separating shear layers from the sides of the body. It was alternatively shown in Williamson[5,6] that the streamwise vortices in a cylinder wake originate from vorticity stretching in the "braid" region lying between primary Kármán vortices. The principal point that we make in this paper is that the conditions for such streamwise vortex generation can exist both in the vortex street wake and also in the separating and unstable shear layers coming from the sides of the body.

We may expect one length scale based on the three-dimensional (3-D) instability of the Kármán vortices, which, judging by existing measurements (see Fig. 2, in particular, the measurements of Williams and co-workers[1] for $x/D=10$) and the relatively small changes in the character of vortex shedding over a surprisingly large range of Re, would have a spanwise wavelength λ_{ZK} of around $1D$ or $\frac{1}{5}\lambda_K$, where

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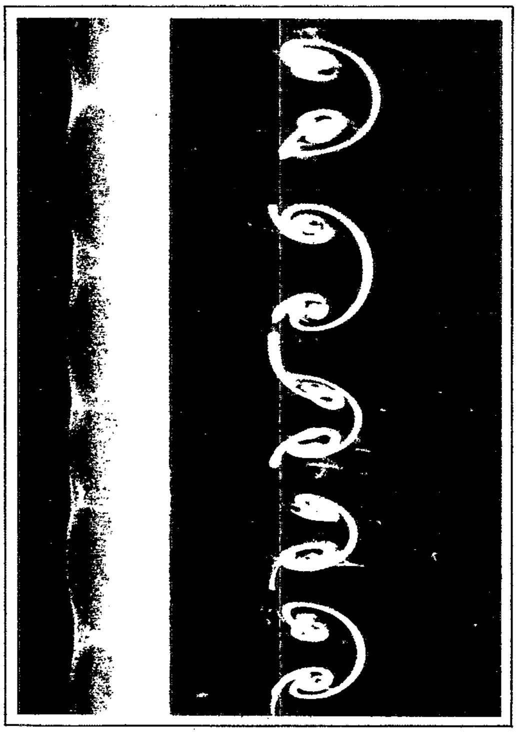

FIG. 1. Streamwise vortex pairs in the cylinder wake, Re=300. The vortex pair structure is clearly shown from this laser-induced fluorescence visualization in a plane close to the wake centerplane, taken from our XY Towing Tank facility. The streamwise vortex pairs, which actually reside (and are stretched) in the "braid" region between primary Karman vortices (vertically oriented but not seen in this image), pass through the light sheet plane at an angle of around 45°. The flow is from left to right. The vertical body surface is seen to the left.

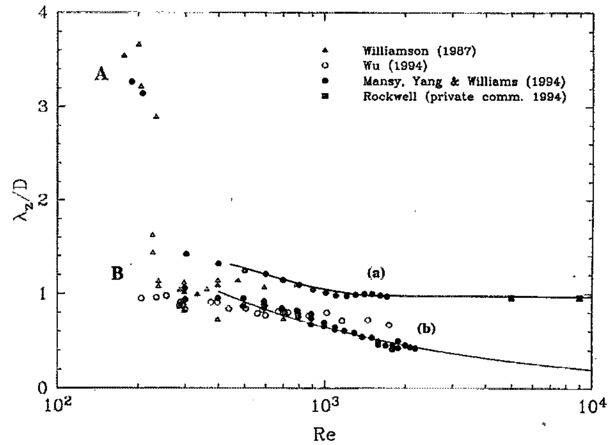

FIG. 2. Two spanwise length scales shown from a plot of normalized spanwise wavelength (λ_Z/D) versus Reynolds number (Re). Curve (a) represents the streamwise vortex scale in the wake, which is reasonably independent of Re. Curve (b) represents the streamwise vortex scale generated in the separating shear layers from the sides of the body. Their curve represents the relation: $\lambda_Z/D \sim 22/\sqrt{\text{Re}}$, which is close to the predicted spanwise scale based on the separating shear layer. The data near A and B represent those shedding modes found from Williamson.[5–7]

λ_K=streamwise wavelength of the Kármán vortices. (We may note that we are not discussing the mode A three-dimensional instability of Williamson,[6] since the latter mode is only observed at Re close to incipient wake transition, in the range Re=190–245.) One may surmise that such a wavelength scales on the dimensions of the primary (turbulent) vortex street, yet is independent of Reynolds numbers, based on a similar independence of streamwise vortex scale discovered from turbulent mixing layer studies by Bernal and Roshko.[8] It would appear that in both of these flows, the streamwise vortex spacing would scale on the thickness of the "braid" vorticity, which, it is suggested, varies only little with Reynolds number.

We may expect a second length scale from the 3-D instability of the shear layer vortices. It has long been known, since the work of Schiller and Linke in 1933,[18] that the shear layers separating from the sides of a cylinder become turbulent at sufficiently high Reynolds numbers. However, it was not until Bloor[19] that the frequency of the shear layer instability waves were detected. She demonstrated that the shear layer instability frequency scaled approximately with $\sqrt{\text{Re}}$, by considering the thickness and velocity of the separating laminar boundary layer. One would expect the shear layer frequency f_{SL} to scale on a velocity near separation U_{SEP}, and on a dimension that we take to be the shear layer momentum thickness θ_{SL}. This is consistent with the work of Michalke,[20] who showed that

$$f_{SL}=0.017\,U_{SEP}/\theta_{SL}. \tag{1}$$

The momentum thickness is expected to scale on the thickness of the laminar boundary layer at separation, which Bloor suggested would vary as

$$\theta_{SL}\propto\sqrt{\nu D/U_\infty} \tag{2}$$

(where D=cylinder diameter, U_∞=free-stream velocity; ν=kinematic viscosity). Over a large range of Reynolds number, a rough estimate for the separation velocity may be given by $U_{SEP}/U_\infty=\sqrt{1-\text{Cpb}}$=constant (roughly 1.4), where Cpb is the base pressure coefficient. If we estimate the Strouhal number to be roughly constant over this range of Re, such that the Kármán vortex shedding frequency $f_K\propto(U_\infty/D)$, then the above equations give

$$f_{SL}/f_K\propto\sqrt{\text{Re}}, \tag{3}$$

as first considered by Bloor, and confirmed by her measurements, for Re>1300. A typical relationship from Kourta *et al.*[21] and Braza *et al.*[22] gives

$$f_{SL}/f_K=0.095\sqrt{\text{Re}}. \tag{4}$$

The streamwise wavelength of the shear layer vortices is given by

$$\lambda_{SL}=1/2\,U_{SEP}/f_{SL}, \tag{5}$$

which can be combined with (4), and an estimate for the Strouhal number $S=f_K\,D/U_\infty$=0.2 over a large range of Reynolds number, to give

$$\lambda_{SL}/D\sim 37/\sqrt{\text{Re}}. \tag{6}$$

Based now on the results of Bernal and Roshko,[8] one might expect a spanwise wavelength λ_{ZSL} of around $\frac{2}{3}\lambda_{SL}$. In summary, one might expect the following distinct spanwise scales (given that an estimate of primary vortex street wavelength, $\lambda_K \sim 5D$):

(a) Streamwise vortices in separating shear layer:

$$\lambda_{ZSL}/D \sim 25/\sqrt{\text{Re}} \quad \text{or} \quad \lambda_{ZSL}/\lambda_K \sim 5/\sqrt{\text{Re}}; \tag{7}$$

(b) streamwise vortices in wake:

$$\lambda_{ZK}/D \sim 1 \quad \text{or} \quad \lambda_{ZK}/\lambda_K \sim \tfrac{1}{5}. \tag{8}$$

Some spanwise length scale measurements are compiled in Fig. 2. It can be seen clearly that the data support the proposition of two distinct scales. This is particularly evident from the data made using the laser scanning anemometer technique developed by Williams and co-workers (data from Mansy *et al.*[1]), which admitted some spatial averaging. It appears more precise than the data of Williamson[5] or Wu *et al.*,[16,23] who used simple counting of streamwise structures. The measurements of Mansy *et al.* show a spanwise length scale of around $1-1.3D$ if a measurement is made beyond $x/D=10$, which is the scale based on the vortex street wake. Interestingly, a smaller length scale is measured upstream at $x/D=3$, which can be shown to have the approximate relation (using a least-squares fit to their data points)

$$\lambda_Z/D \sim 22/\sqrt{\text{Re}}. \tag{9}$$

(In their paper, Mansy *et al.*[1] had already proposed from their own curve fit, the relation $\lambda_Z/D \sim 20/\sqrt{\text{Re}}$.) Their upstream measurement, represented here by Eq. (9), is close to the estimate made in Eq. (7) above, suggesting that this streamwise structure scales on the separating shear layer. The above results, plotted in Fig. 2, appear to support well the concept that two distinct spanwise length scales exist in the cylinder wake.

Mention should be made of the ongoing work of Rockwell and co-workers at Lehigh University, who have developed a digital PIV technique, and are carefully studying wake vorticity. Chyu and Rockwell[24] have found that spanwise scales of between $0.85-1.07D$ are measured at Re $=5000$ and 10 000, and the average of these values have been plotted in Fig. 2, lending strong support to the existence of a Reynolds-number-independent streamwise vortex scale for the vortices generated in the wake. Their study also finds the existence of smaller scales right near the body, and suggests, in good accordance with the present conclusions, that different length scales could be associated with vortex stretching in the shear layers and in the wake.

In conclusion, it appears that there exist conditions to amplify streamwise vorticity in bluff body wakes by vortex stretching in both the separating shear layers from the sides of the body and also in the vortex street wake. The length scale governing the streamwise vortices in the shear layer has a $1/\sqrt{\text{Re}}$ dependence, whereas the scale of such structures in the wake is independent of Re, both of these conclusions being based on previous work in mixing layers. The proposition that there should exist two distinct, and possibly disparate, spanwise length scales in the cylinder wake is well supported by measurements, particularly those of David Williams and co-workers,[1] as well as those from Donald Rockwell's research group (Chyu and Rockwell[24]).

ACKNOWLEDGMENTS

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