Transition in Bluff Body Wakes: Is the Initial Transition Elliptic?

MARK C THOMPSON[a], THOMAS LEWEKE[b] AND CHARLES H K WILLIAMSON[c]

[a]Department of Mechanical Engineering,
Monash University, Australia

[b]Institut de Recherche sur les Phénomènes Hors Equilibre
12, ave Général Leclerc, 13003 Marseille, France

[c]Department of Aerospace and Mechanical Engineering
Cornell University, Ithaca, NY, USA.

The three-dimensional transition of the wake of a circular cylinder involves two distinct instability modes, known as mode A and B (Williamson, 1988). Mode A occurs first at a Reynolds number of about 190 and has a wavelength of about 4 cylinder diameters (Henderson and Barkley, 1996, Williamson, 1988). Mode B occurs later and dominates the wake at the higher Reynolds numbers. At least for two-dimensional bodies, these instability modes are probably somewhat generic in that equivalent topological modes occur for other body shapes. For example, for the square cylinder the transition occurs through a mode A instability at about the same Reynolds number (Robichaux et al, 1999), and for long elliptical leading-edge plates this mode also occurs in the wake transition process (Tan et al, 1998).

It is tempting (and perhaps useful) to be able to relate this instability to known physical instability mechanisms. Leweke and Provansal (1995) attempted to relate it to a Benjamin-Feir instability of a set of oscillators, Brede et al (1996) to a centrifugal instability and Williamson (1996), and Leweke and Williamson (1998) to an elliptic instability of the vortex cores. The evidence presented in the former two studies is either insufficient or disagrees with experiment. The latter mechanism has generated some lively debate amongst researchers in the area. Henderson (1997) suggests that it does not make sense to classify the instability as an elliptic instability, especially since Floquet analysis seems to indicate that the maximum amplification occurs between the wake vortices rather than in their cores. Nevertheless, as pointed out by Leweke and Williamson (1998), elliptic instability theory is surprising good at predicting the wavelength of the most unstable instability mode. Further supporting evidence is advanced in Leweke and Williamson (1998).

In this paper, we examine some more evidence that suggests although the instability may not be purely elliptic, it does have some elliptic character. Of course, this is not surprising given the wake is clearly much more complicated than (a set of) isolated vortices for which elliptic instability theory was developed.

Figure 1 shows a color contour plot of the perturbation vorticity at a two stages in the shedding cycle calculated using Floquet instability analysis. The lefthand plot shows that the instability appears to develop within the cores of the wake vortices while they are forming. The righthand plot is a short time later. It shows that the nascent instabilities in each core have migrated towards the region in between the vortex cores, consistent with the findings of Barkley and Henderson (1996). The instability is then strongly amplified. This suggests that elliptic instability may at least play some role in the initial triggering of the instability.

To investigate further, we split the floquet mode at into two mutually exclusive parts. For the first (component A), the velocity field was zeroed in regions where elliptic instability indicated that the flow was *not* elliptic. For the second (component B), the reverse was done. Hence, we effectively split the velocity field into an elliptic component (A) and a hyperbolic component (B). These two fields were then evolved using direct simulation. After many shedding cycles both components A and B will evolve back to the Floquet mode. The simulations revealed that component A did this more quickly. In fact, this can be quantified. A comparison of simulations indicate that the mode A instability is 2/3 elliptic: 1/3 hyperbolic!

Further investigations will be presented at the conference.

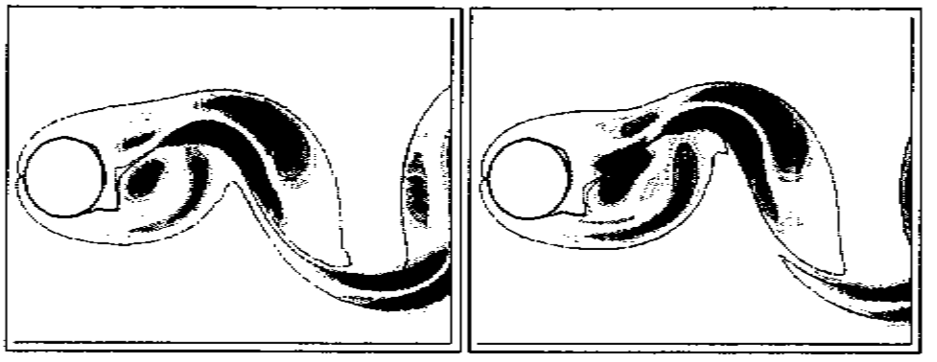

Figure 1: Left: Color Contour plot of the (perturbation) vorticity in the mode A instability showing the development of the instability in the vortex cores just after their formation. Right: Slightly later the core instabilities have migrated and amalgamated in the position between the cores.

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