Fluid Dynamics Research 3 (1988) 349–352
North-Holland

Prediction of vortex shedding from bluff bodies in the presence of a sound field

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Abstract. The separated flow around a rectangular cylinder, in the presence of a transverse duct resonant acoustic mode, is modelled using a vortex method. The instantaneous transfer of power between the mean flow and the acoustic field is predicted using Howe's theory of aerodynamic sound. Whether the net acoustic energy per cycle generated is positive or negative depends on the phase of the acoustic cycle at which vortex clouds arrive at the trailing edge of the cylinder.

1. Introduction

Acoustic resonances are found to occur in heat exchangers and turbomachines and can cause substantial damage to tubes and blades. The determination of the sources of the sound even in an experimental rig can be difficult. The simplest representation of such resonances, a single cylinder in a duct, has been investigated by Welsh et al. (1984), Welsh and Gibson (1979), Stokes and Welsh (1986) and Parker (1966). In the case of a rectangular cylinder with chord-to-thickness ratio 5, acoustic resonance in the duct was found to be excited over discrete velocity ranges by Welsh and Gibson (1979). This flow was further investigated by Stokes and Welsh (1986) with cylinders of different chord-to-thickness ratios and a mathematical model constructed using Howe's (1975) theory of aerodynamic sound. It was found by Stokes and Welsh (1986) that the velocity ranges over which the predicted flow of energy to the sound field was positive corresponded to the ranges of observed resonant sound. The phase of the sound cycle at which vortex clouds shed from the leading edge corners of the cylinder arrive at the trailing edge was found to be an important factor in determining whether there were net sources in the flow.

The model employed in Stokes and Welsh (1986) represented each vortex cloud by a single point vortex, with the time of formation of vortices a free parameter. In the present model, for a rectangular cylinder having chord-to-thickness ratio 5, vortices are released over the entire surface in the presence of an transverse oscillating field; no ad hoc assumptions about the time of formation of vortex clouds are required. The sources of acoustic power are determined through the application of the theory of Howe (1975).

2. Description of model

In the present vortex model, the flow is assumed to be two-dimensional, incompressible, inviscid, isentropic and irrotational except at points where elemental line vortices are embedded in the flow. The boundary layer around the bluff body is replaced by discrete surface vortices which provide a zero velocity at the surface. At each time step, the newly created surface vortices are free to be convected into the flow and new surface vortices are created. The elemental vortices are potential vortices with smoothed cores.

The velocity at any point is the combined value of the irrotational velocity, the velocity induced by the vortices and an acoustic particle velocity. The acoustic field was obtained as a finite element solution of the Helmholtz equation for the Parker (1966) β-mode for duct and cylinder dimensions used in the experiments of Stokes and Welsh (1986). Here, the acoustic particle velocity amplitude at the leading and trailing edges is set at 40% of the freestream velocity, representing loud resonant sound. The wavelength of the sound is far greater than the dimension of the flow structures, allowing the assumption of incompressibility in the vortex model.

The instantaneous acoustic power generated by a vortex having net circulation Γ, velocity v in a sound field with local acoustic particle velocity $u \sin(f\tau)$ is given by (Howe 1975) $P = Q(\tau) \sin(2\pi f\tau)$, where $Q(\tau) = \rho \Gamma v u \cos(\alpha)$. Here, ρ is the fluid density, τ is time, α is the angle between the vortex velocity and local acoustic particle velocity, and f is the acoustic frequency. For a vortex starting with zero circulation and moving in an acoustic field which decreases exponentially along the duct, the integrated value of the power P over all time for a vortex is identical to the integrated value for the function W given by $W = (1/(2\pi f))(\mathrm{d}/\mathrm{d}\tau)(Q(\tau)) \cos(2\pi f\tau)$. The advantage of considering the function W, instead of integrating P, is that it localizes regions of the flow where net energy is transferred over a sound cycle, allowing acoustic source regions to be determined. The acoustic power generated by a vortex cloud is taken as the sum of the powers generated by the individual elemental vortices that constitute the cloud.

3. Results and discussion

The predicted acoustic particle velocity amplitudes for the β-mode around the rectangular cylinder are shown in fig. 1. Also shown are the pressure isobars. Of note is the decrease in acoustic particle velocity amplitudes near the mid-chord position of the cylinder; according to the Howe formula, a vortex cloud is less able to generate acoustic power in this region.

An instantaneous 'snapshot' of the velocities of the elemental vortices is shown in fig. 2. The vortex clouds that are shed from the leading edge of the cylinder form the dominant structures in the wake. For the large acoustic velocity perturbation to the flow, the shedding frequency of these vortex clouds is locked to the acoustic frequency, with the clouds being shed alternately from the two leading edge corners.

The acoustic power generation history of an individual vortex cloud formed at the leading edge of the cylinder is shown in fig. 3 for two different values of the acoustic Strouhal number $\mathrm{St}_a = ft/v_\infty$, where t is the cylinder thickness and v_∞ is the flow velocity in the absence of the

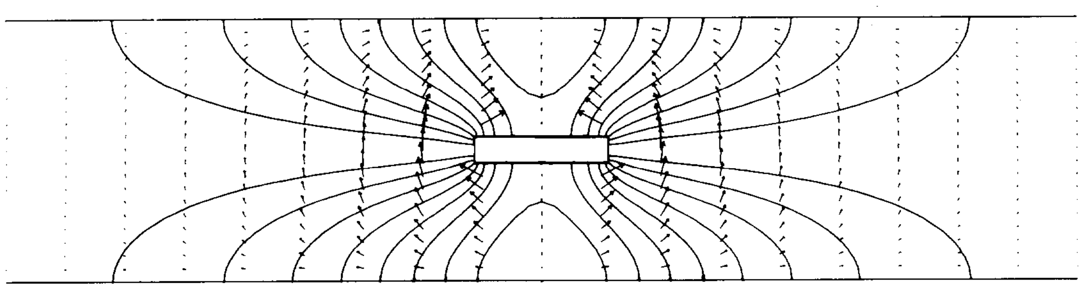

Fig. 1. Predicted acoustic particle velocity magnitudes and directions for a Parker β-mode. Relative magnitudes are indicated by length of arrows. Predicted isobars are shown by the solid lines.

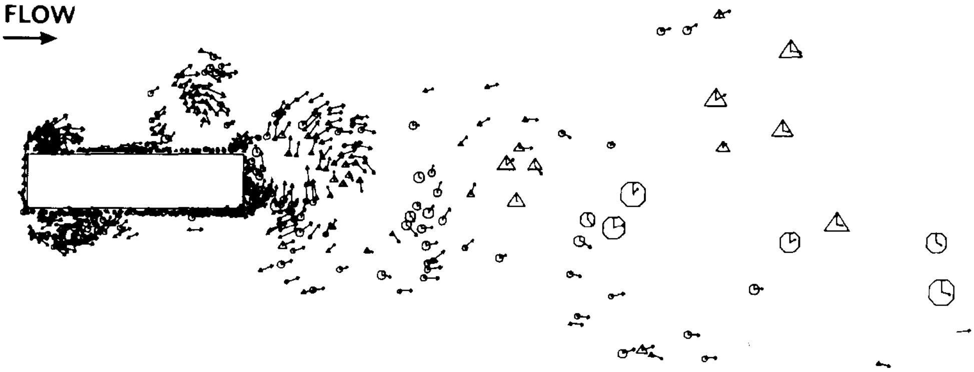

Fig. 2. Snapshot of the positions and velocity vectors of the elemental vortices for $St_a = 0.15$. The relative magnitudes of the velocities are indicated by the arrow lengths. △, clockwise rotating vortices; ○, anticlockwise rotating vortices.

cylinder. In fig. 4, the time history of the function W, which designates regions of acoustic power generation, is shown. The two different acoustic Strouhal numbers result in vortex clouds, shed from the leading edge, arriving at the trailing edge at different phases of the acoustic cycle. Consequently, the trailing edge region is a net source for $St_a = 0.2$ and a net sink for $St_a = 0.175$. For $St_a = 0.2$, the vortex cloud shed from the top leading edge corner arrives at the trailing edge when the acoustic particle velocity is directed in the upward sense, leading to a rapid increase in Q and a net positive contribution to W. For $St_a = 0.175$, the vortex cloud arrives at the trailing edge about half a sound cycle earlier than when $St_a = 0.2$, and a negative contribution to W results. In each case, the contribution to the net power from the region near the leading edge of the cylinder is small.

The net acoustic energy transfer per acoustic cycle for different acoustic Strouhal numbers is shown in fig. 5. For $St_a = 0.2$, net positive acoustic energy is generated by the vortex clouds. For $St_a = 0.225$, the vortex clouds contribute little net energy. For acoustic Strouhal numbers less than 0.2, net energy is removed from the acoustic field and it unlikely that a resonance could be sustained. The region where the energy transfer is highest is consistent with the

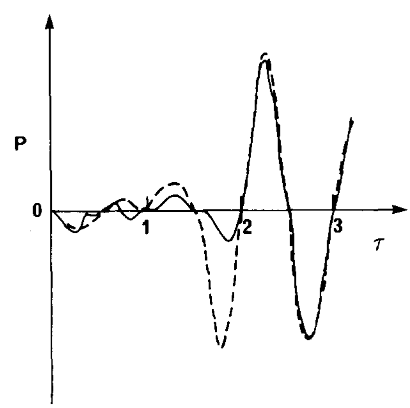

Fig. 3. Time histories of the instantaneous acoustic power P (arbitrary units) for a leading edge vortex cloud. — — —, $St_a = 0.175$; ———, $St_a = 0.2$.

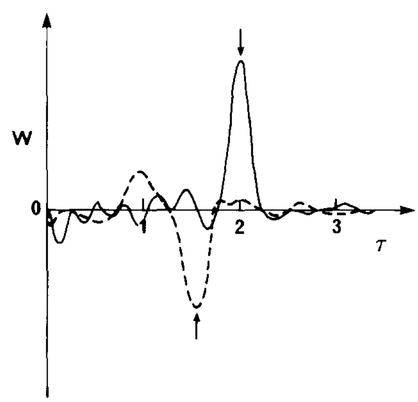

Fig. 4. Time history of W (arbitrary units) for a leading edge vortex cloud. — — —, $St_a = 0.175$; ———, $St_a = 0.2$; ↑, time at which vortex cloud arrives at trailing edge.

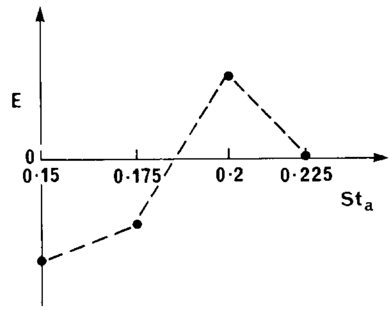

Fig. 5. Net energy E (arbitrary units) per cycle generated for different acoustic Strouhal numbers.

experimentally range of resonance observed experimentally by Stokes and Welsh (1986), $St_a = 0.2$ to 0.24.

4. Conclusions

A discrete vortex model combined with the finite element solution of the acoustic field and Howe's theory of aerodynamic sound is found to predict the sources of sound in the flow over a rectangular cylinder in a duct. The dominant acoustic source region is near the trailing edge of the cylinder. The vortex model shows that resonance depends on the phase of the acoustic cycle that a vortex shed from the leading edge of the cylinder arrives at the trailing edge. When arrival of the vortex coincides with opposition of the acoustic particle velocity to the mean flow at the trailing edge corner, acoustic resonance occurs; this coincidence is dependent on the acoustic Strouhal number.

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