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The Unsteady Wake of a Circular Cylinder near a Free Surface

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Abstract. The behaviour of the wake Strouhal number for flow past a cylinder close to a free surface at both low and moderate Froude numbers is investigated numerically. For the low Froude number case (i.e., gravity-dominated), the results obtained are similar to those for flow past a cylinder close to an adjacent no-slip boundary. As the distance between the wall and the cylinder is reduced, the Strouhal number, as measured from the time varying lift, increases to a maximum at a gap ratio of 0.70. Further gap reduction leads to a rapid decrease in the Strouhal number, with shedding finally ceasing altogether at gap ratios below 0.16. The agreement between the results for a free surface and a no-slip boundary suggests that the mechanism behind the suppression of vortex shedding is common. For flow at a fixed gap ratio and a moderate Froude number, two distinctly different wake states are observed with the flow passing over the cylinder tending to switch from a state of attachment to the free surface, to one of separation from it, and then back again in a pseudo-periodic fashion. Even though there is a significant difference in Reynolds number, the predicted numerical two-dimensional behaviour is found to compare favourably with the experimental observations at higher Reynolds number.

Key words: bluff-body flows, free surface, metastability, unsteady wake.

1. Introduction

Flow past a cylindrical body close to a free surface is of immense importance in the design of offshore structures, marine vessels, pipelines and water-based power generation systems. Despite these economically important applications, to the authors' knowledge little work has been done in this area to date (see [7, 13, 15, 17]), hence a further comparison will be made with the more widely investigated but closely related case of flow past a cylinder close to a plane no-slip boundary. This comparison may highlight some of the distinct similarities and differences induced by replacing a solid surface with a free surface at low Froude numbers. For the interaction of vortices with a free surface, it has been shown by Ohring and Lugt [14] that at low Froude numbers, the wake behaviour is similar whether the surface is free or fixed. In addition, because of the low Froude number, there is little or no surface deformation observed. This suggests that a comparison between the two cases is warranted.

Miyata et al. [13] have examined flow past a cylinder close to a free surface both experimentally and numerically. Their investigation was conducted at a Reynolds number of 4.96×10^4 and at a Froude number (based on cylinder diameter) of approximately 0.24. They note a step-like change in the flow behaviour as the submergence depth (gap ratio) of the cylinder is altered. In particular, sharp changes in the Strouhal number are noted. The large jump occurs in conjunction with a notable weakening in the intensity of the spectra, along with the occurrence of a broader range of frequencies; they suggest that the shedding at the smaller gap ratio is less remarkable, which is assumed to mean weaker. While their visualization suggests that the flow changes with time, there is no explicit evidence that shedding is observed, although no mention of any suppression of shedding is made either.

Sheridan et al. [17, 18] and Hoyt and Sellin [7] examined flow past a cylinder close to a free surface using particle image velocimetry (PIV) and a dye tracer technique, respectively. They illustrate the flow behaviour at larger Froude numbers between 0.47 and 0.72. As the surface is capable of supporting much larger surface curvature at these Froude numbers, their results provide limited insight into the low Froude number cases being examined here, although they do indicate the impact that the Froude number has on the results.

Taneda [19] examined the problem of flow past a cylinder close to a solid surface at a Reynolds number of 170. A towing tank was used to eliminate the influence of the wall boundary layer that develops when a similar problem is considered in a wind or water tunnel. In particular, Taneda [19] noted that at certain submergence depths (e.g., a gap ratio of 0.60), that regular vortex shedding was observed, but at smaller gap ratios (0.10) only a single layer of vortices was seen to be shed. In the latter case, the wavelength of the shed vortices tended to increase with downstream distance, with the wake becoming unstable and breaking down after only a few wavelengths.

Others such as Angrilli et al. [1], Bearman and Zdravkovich [4], Grass et al. [5], and Lei et al. [11] have all examined flow past a cylinder close to a no-slip wall, with all except [4] observing substantial changes in the Strouhal number with gap ratio.

For moderate Froude numbers, it has been shown experimentally by Sheridan et al. [17, 18], and more recently numerically by Reichl [15], that the wake of a cylinder close to a free surface may exhibit more than one state at a fixed point in parameter space. The behaviour of the wake under these circumstances was deemed by Sheridan et al. [17] to be metastable, as each wake state displayed only a limited degree of stability.

Prior to discussing the metastable wake states observed by Sheridan et al. [17, 18] and Reichl [15], attention is focused on the types of flow behaviour that have been observed in the region of parameter space considered in this paper. For the moderate Froude number case, the study will restrict itself to flow at a Froude number of 0.55 and a gap ratio of 0.40.

Sheridan et al. [17] note that at a Froude number of 0.60 and for gap ratios between 0.75 and 0.24, the flow passing over the cylinder, which in the following will be referred to as a ‘jet’, tended to progressively move from being attached to the free surface through to being almost attached to the rear of the cylinder.

Although the particle image velocimetry (PIV) technique adopted by Sheridan et al. [17, 18] yields detailed instantaneous flow fields, limited detail was given of the transient behaviour of the flow states. Hoyt and Sellin [7] also examined this flow at a Froude number of 0.53, with their dye tracer technique indicating that the flow field was indeed time-dependent, with von Kármán vortex shedding clearly noted at a gap ratio of 0.75.

For a couple of key cases, Sheridan et al. [17, 18] found that two distinct wake states could be observed at a fixed gap ratio and Froude number. For these cases the wake spontaneously underwent transformations between the states in a pseudo-periodic manner. They indicated that the frequency was roughly two orders of magnitude lower than that of von Kármán vortex shedding for a fully submerged cylinder.

The metastable behaviour observed by Sheridan et al. [17] at a gap ratio of 0.45 and a Froude number of 0.60 involved the ‘jet’ transiently switching between a state of attachment to the free surface and a state of separation from it, such that it was directed into the region of space between the free surface and the cylinder. For this case they note that the transition between the two states could be artificially induced by transiently piercing the free surface at a position downstream to a depth of approximately 0.4 of a cylinder diameter. They also mention that it was possible to induce hysteretic effects by altering the flow velocity, with this variation typically resulting in a change of wake state.

Sheridan et al. [17] also noted that meta-stable behaviour occurred at a Froude number of 0.60, for gap ratios of both 0.31 and 0.59. Their observations at the smaller gap ratio indicated that the ‘jet’ switched between a state of attachment to the rear of the cylinder, and detachment from it such that it occupied a region of space in between the free surface and the cylinder. At the larger gap ratio, they note that the ’jet’ flips between the latter state observed above (i.e., occupying a region in between the free surface and the cylinder) and attachment to the free surface.

Of relevance to the current study are investigations of vortex interactions with free surfaces. Yu and Tryggvason [22] investigated the surface signature of two-dimensional vortex flows through numerical modelling. They found that the classification is a strong function of Froude number, with the vortex interaction with the free surface leading to minimal surface deformation in low Froude number cases; in effect the free surface acts somewhat like a fixed surface (as previously described). Conversely, for higher Froude numbers, substantial deformation can occur. Sarpkaya [16] summarises many of these studies and describes the effect of surfactants which can alter surface boundary conditions to such an extent that the surface effectively appears to lie between no slip and free slip.

Ohring and Lugt [14] have shown for the interaction of a vortex pair with a free surface that increasing the Froude number leads to a greater level of surface deformation. For the present case, the increased curvature generated at a moderate Froude number, combined with the inherent time-dependent shedding of vortices from the cylinder, may affect in a time-dependent manner the ability of the flow wake to shed. That is, this coupling may alter the nature of the absolute instability in the wake. This is perhaps not surprising as it is well known that the wake of a fully submerged cylinder is absolutely unstable [9], and it has been shown by Triantafyllou and Dimas [20] that the wake of a floating cylinder is convectively unstable.

It has been suggested by Reichl [15] that it is the time-dependent skewing of the wake, brought about by the deforming free surface, that causes the wake to switch between the two types of instability. Of relevance are the findings of Koch [10], and the speculation by Huerre and Monkewitz [8], that only a limited degree of asymmetry is required before no time-harmonic resonance (or absolute instability) is possible. To clarify, it is hypothesized that the changes in the wake behaviour as the cylinder is brought closer to the free surface are linked to the associated change in the nature of the wake stability. For a fully submerged cylinder the short formation length and coupled shedding from both sides of the wake are associated with an absolute instability. In contrast as the cylinder is brought closer to the surface, the wake flow becomes convectively unstable with one consequence being a much longer wake formation length.

The primary aims of the current investigation are: (a) to examine numerically the changes in vortex shedding characteristics and the Strouhal number for varying gap ratios between a cylinder and a free surface for low Froude numbers (0.0, 0.2); and (b) to determine the physical mechanism leading to metastability in the wake at a moderate Froude number (0.55).

2. Numerical Method and Setup

With reference to Figure 1, the governing parameters are defined as follows: Reynolds number $\mathrm{Re} = ud/\nu$, where u is the upstream flow velocity, ν is the kinematic viscosity and d is the cylinder diameter; Froude number $\mathrm{Fr} = u/\sqrt{(dg)}$, where g is the gravitational acceleration; gap ratio $h^* = h/d$ (submergence depth); and Strouhal number $\mathrm{St} = fd/u$, where f is the shedding frequency.

The current two-dimensional study was performed with the commercially available computational fluid dynamics code, *Fluent*. The version used was 5.1.18. In order to model the free surface, which in practice is often an interface between water and air, a variant of the *volume-of-fluid* (VOF) method as incorporated within Fluent was employed. The reader is referred to Hirt and Nichols [6] for more details on the VOF method, but essentially it calculates the flow field for a single fluid (with variable density and diffusivity) and determines the degree of mixing between the phases via the void or volume fraction. Using two phases has the

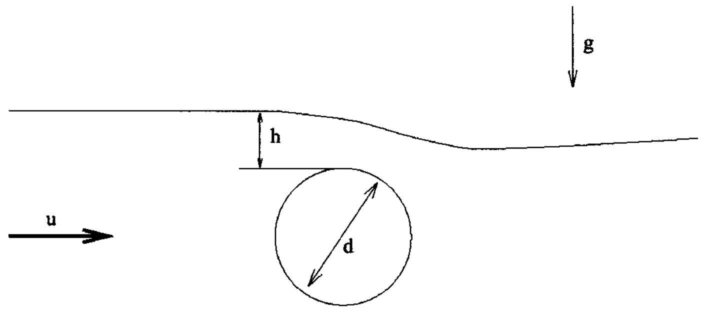

Figure 1. Schematic of the flow arrangement for a submerged cylinder.

advantage that it allows cases to be modeled for which air entrapment occurs. (In line with experimental observations, for numerical simulations over restricted parameter ranges this is found to occur. However, it is not the focus of this paper). The scheme used produces results which are second-order accurate in space for velocity and pressure and first-order accurate for the volume fraction. Although the temporal scheme used is only first-order accurate, there are at least 200 timesteps per period, so good temporal resolution is achieved. The system setup simply involved the two phases entering into the domain with a uniform velocity at inlet and leaving though an outlet boundary where a pressure boundary condition is prescribed. The pressure boundary condition is necessary to maintain the height of the liquid phase.

Initially, the actual densities and viscosities of water and air were used for two free-surface validation studies; flow in a spinning bowl and flow generated by a breaking dam. For the first case an analytical solution exists when the upper phase is a vacuum, whilst for the second case experimental results are available [12]. It was found that as the viscosity and density ratios were varied away from unity, the computation time increased markedly. Both ratios affect the *stiffness* of the problem considerably. Tests using a wide range of different density and viscosity ratio pairs indicated that the accurate results could be obtained with a density ratio of $\rho_1/\rho_2 = 100$ and a viscosity ratio of $\nu_1/\nu_2 = 1$. (In comparison, for an water/air system at standard conditions, $\rho_1/\rho_2 = 811$ and $\nu_1/\nu_2 = 0.074$). For the spinning bowl the predicted free surface height was within 0.05% of the analytical height (in terms of the L_2 norm). Even with a density ratio of 10, the prediction was accurate to within about 0.2%. For the breaking dam the predictions of the surge front location and time-dependent maximum height were both within experimental error. Thus, a density ratio of $\rho_1/\rho_2 = 100$ and a viscosity ratio of $\nu_1/\nu_2 = 1$ were considered adequate to accurately model the target problem of a cylinder near a free surface. In addition, there was little gain, and a large computational cost, in using the real values corresponding to a water/air interface.

An investigation of the effect of computational domain size was undertaken. It was found that using an inflow length of $10d$, an outflow length of $30d$ and a cross-flow length of 30d was sufficient to allow prediction of Strouhal numbers and drag coefficients to within 3 and 5%, respectively, for the fully submerged case. A resolution study was also performed. The Strouhal number and force coefficients were predicted for compressed meshes containing approximately 50000 and 90000 node points. Using Richardson extrapolation it was possible to estimate the accuracy of these parameters. For the 50000-node mesh, the Strouhal number and drag were estimated to be within 3%, and the fluctuating lift within 6% of the resolution independent values.

Another important choice is the selection of the Reynolds number for the computations. This required careful consideration. Typically, the previous experiments (described above) were carried out at Reynolds numbers of order 10^3–10^4. At these Reynolds numbers, while the flow is predominantly two-dimensional, there is some three-dimensionality, and the wake state will be turbulent. From the computational viewpoint, it is extremely expensive to perform three-dimensional simulations with a free-surface. Instead, it was decided to perform two-dimensional simulations which are still likely to capture the main physics, since the experiments show the wake is predominantly two-dimensional. The Reynolds number chosen was 180; this is just below the accepted value, $\mathrm{Re}_{\mathrm{crit}} = 189$, at which the flow becomes three-dimensional [2, 21]. It makes sense not to use a higher value because the wake formation length for two-dimensional simulations reduces rapidly (and unphysically) with Reynolds number without the possibility of three-dimensional flow. In addition, lower Reynolds numbers lead to excessive diffusion. Finally, it is found experimentally that the Strouhal number remains approximately constant at 0.2 for Reynolds number in the range $200 < \mathrm{Re} < 10^5$.

3. Results and Discussion

3.1. LOW FROUDE NUMBER CASE (FR = 0.00, 0.20)

Two low Froude numbers were considered, $\mathrm{Fr} = 0.00$ and 0.20. The zero Froude number (or free-slip) case was considered as it represents the limiting situation in which the free surface is unable to deform. This is achieved numerically by adjusting the gravitational force so that it dominates the inertial force completely (i.e. $g \to \infty$), and when this is the case, the surface cannot deform and the zero tangential stress condition at the surface requires that it should act like a free-slip wall.

Simulations were performed for both Froude numbers at the following gap ratios: 0.10, 0.13, 0.16, 0.19, 0.22, 0.25, 0.40, 0.55, 0.70, 0.85, 1.00, 1.50, 2.50, and 5.00. The time variation of the lift force acting upon the cylinder was then analyzed and notable changes in Strouhal number were observed as the gap ratio was altered. The first point to note is that as the cylinder is moved closer to the surface, the Strouhal number increases to a maximum at a gap ratio of 0.70 before decreasing rapidly as the gap is reduced further. The general trend observed compares well with the results of Angrilli et al. [1], with the normalized Strouhal

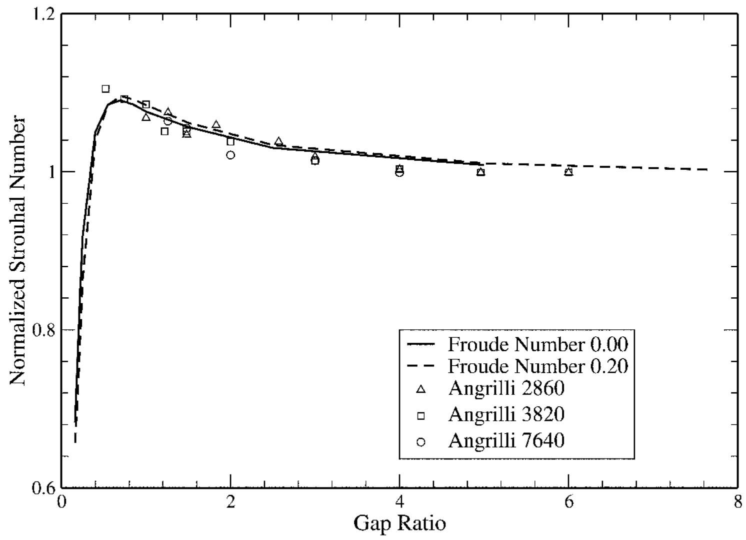

Figure 2. Variation of Strouhal number (normalized with respect to Strouhal number of the reference cylinder, i.e. St/St_0) with gap depth h^*, for both the Fr = 0.00 and Fr = 0.20 cases. Also shown are the results of Angilli et al. [1].

number and asymptotic distance (distance over which normalized Strouhal number approaches unity) in good agreement, as shown in Figure 2. The labels Angrilli-2860, Angrilli-3820 and Angrilli-7640 in Figure 2, refer to the results given in [1] for three different Reynolds numbers, namely Re = 2860, 3820 and 7640. At smaller gap ratios (which were not considered in [1]), there is a marked drop in Strouhal number and the strength of the time varying lift acting upon the cylinder becomes considerably weaker. An investigation of the flow field and the spectra suggests that a weak form of shedding is still observed down to a gap ratio of 0.16, with no shedding observed at smaller gaps. The change in the wake behaviour with gap ratio is shown in Figure 3. The significant increase in the wake formation length as the gap ratio is reduced is consistent with experiments (e.g., [17]).

Lei et al. [11] suggest that the behaviour of the root mean squared (RMS) lift coefficient is a better measure for determining when shedding ceases. The trends observed here correspond well with those observed by Lei et al. [11] for a cylinder close to a no-slip wall, with both showing similar behaviour over a range of gap ratios. It is only at very small gap ratios that the behaviour differs, with [11] observing a slight upswing in the RMS lift, which is not seen here at small gap ratios. In addition the gap ratio at which shedding ceases tends to be smaller for the free-slip/free-surface case. The behaviour of the normalized RMS lift is shown in Figure 4. It is suggested by Miyata et al. [13] that the difference in the gap ratio at which shedding ceases is due to the ease with which fluid may be entrained upstream from previous shed vortices. This process is made significantly easier when the adjacent boundary is a free-slip/free-surface one, although the basic

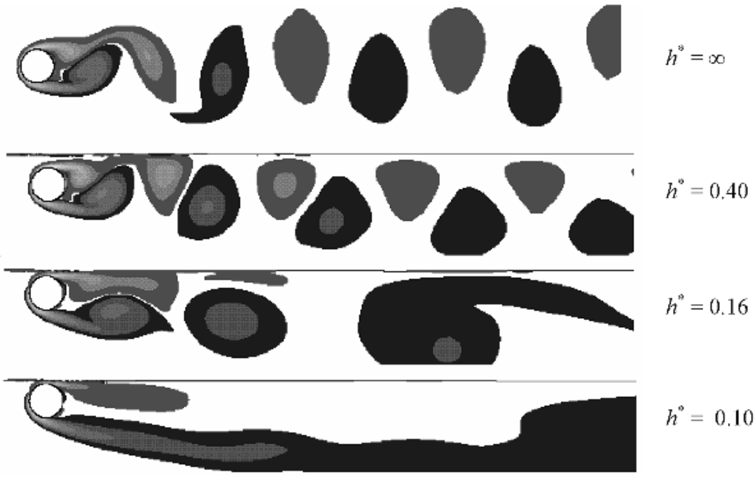

Figure 3. Vorticity plots for the reference cylinder (top) and for decreasing gap ratios, h^*, for Fr = 0.20. (Original in colour)

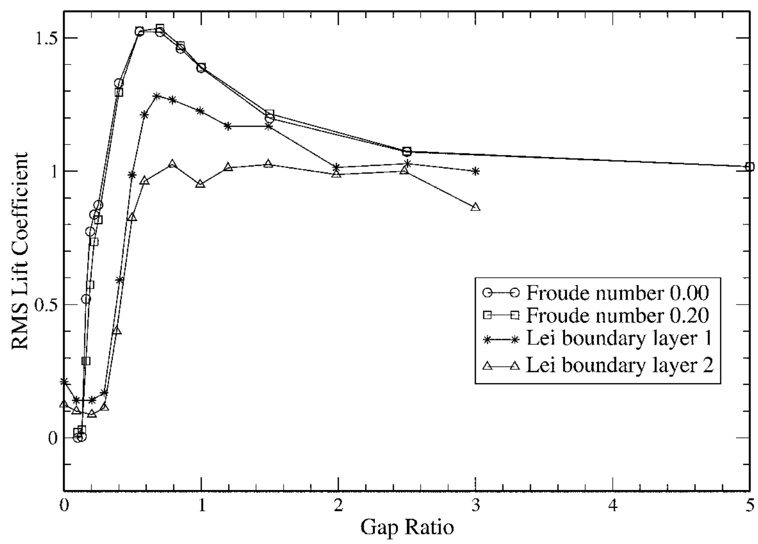

Figure 4. Plot showing the variation of RMS lift coefficient with gap ratio h^* for the free slip (Fr = 0.00) and Fr = 0.20 cases. Also plotted are the results of Lei et al. [11] (for their boundary layers 1 and 2). (Original in colour)

mechanism is believed to be the same for the free surface and the no-slip surface cases. A discussion of the proposed mechanism is beyond the scope of this paper.

The variation of the RMS lift with gap ratio observed in Figure 4 is believed to be due to the movement of the formation position (i.e., the position at which discrete vortices roll-up). This is an extension of the concept of the formation length used for unbounded flow past a cylinder (e.g., [3]). Due to the inherent asymmetry

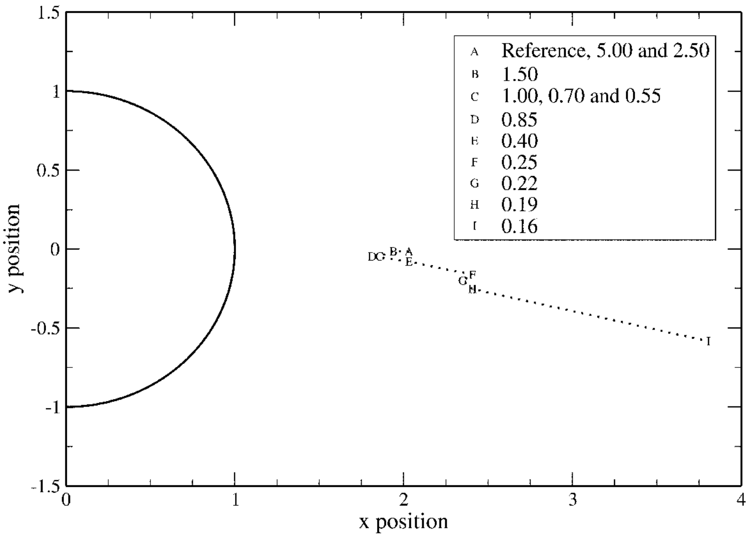

Figure 5. Position of the vortex formation position calculated using the standard deviation in the vertical velocity for a Froude number of 0.20 and different gap ratios $h^* = h/d$.

introduced by the adjacent free surface, the formation length could not be measured in the standard way (i.e., by calculating the point along a line of symmetry at which the standard deviation in the vertical velocity was a maximum). In the current investigation three different approaches were adopted, and while each yielded a slightly different result, all produced the same trends. The first approach involved calculating the standard deviation of the vertical velocity component at every point in the flow field and then locating the point at which this was a maximum. The second approach was similar except that it involved calculating the point of the maximum standard deviation in the velocity azimuthal to the cylinder. The third method was slightly different as it involved calculating the point (away from the body) at which the standard deviation in the vorticity was a maximum. The first two methods yielded similar positions, while the third produced points closer to the cylinder. Figure 5 shows the variation of the formation length as a function of gap ratio using the first approach. Comparing with Figure 4, it is clear that the variation of the RMS lift and the formation location correlate well. The movement of the formation length also explains the observed change in the Strouhal number, with [5] indicating that the period for a fully submerged cylinder at lower Reynolds numbers depends upon the time taken for sufficient vorticity to accumulate outside the region of high shear stress (i.e., in a region from which it can be shed). However, the lack of change in the Strouhal number with gap ratio observed for a cylinder close to a no-slip wall at much higher Reynolds numbers [4] suggests that the movement of the formation length with gap ratio may also be Reynolds number dependent.

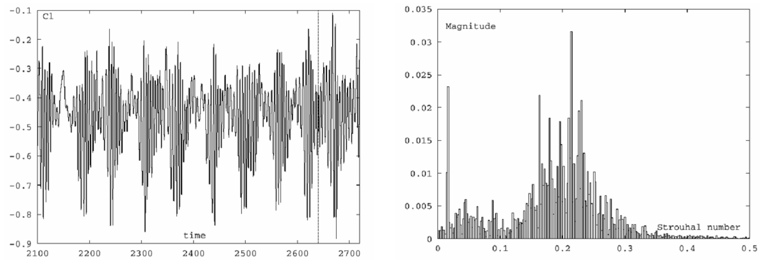

Figure 6. Lift history (left) and power spectrum (right) for a gap ratio of 0.40 and Froude number 0.55.

3.2. MODERATE FROUDE NUMBER ($\mathrm{fr} = 0.55$): METASTABILITY

The metastable behaviour, being intrinsically time-dependent, was most clearly illustrated via animations that show the time-dependent wake development. For the case considered here (i.e., gap ratio 0.40 and Froude number 0.55) the flow from above the cylinder (i.e. the ‘jet’) switched between a state of attachment to the free surface and oblique separation from it (see Figures 6–8).

It is proposed that this metastable behaviour represents a form of feedback loop, in which the shedding of discrete vortices and their interaction with the free surface induce significant surface curvature, with this curvature then skewing the wake. These changes subsequently alter the conditions which give rise to shedding in the first place, and the absolute instability is weakened or perhaps in some cases extinguished. The weakening of the absolute instability (via the skewing of the wake) hinders the formation of discrete vortices and in doing so it removes the source of the surface distortion, hence allowing the absolute instability to reassert itself. This behaviour is pseudo-cyclical and it is dependent upon the response of the free surface to the underlying vorticity distribution, which is highly time dependent. It appears that it is the presence of the vortical structures that form downstream of the cylinder that help govern the degree of skew in the wake. For this reason, it is not surprising Sheridan et al. [17] found that external disturbances in the region downstream caused a switching between states.

The changes in the behaviour of the wake influence the lift force acting upon the cylinder. Such time-dependent changes can then be measured in terms of a non-dimensionalized shedding frequency or Strouhal number. Figure 6 shows the behaviour of the lift trace and its Fourier transform, while Figure 7 shows a close up view of the lift signal with the labels denoting the frames (illustrating characteristic flow states during the pseudo-cycle), a sample of which are shown in Figure 8.

The metastable behaviour has a clear impact upon the lift with the separation of the ’jet’ from the free surface resulting in a decrease in the lift force acting upon the cylinder. This behaviour is shown in both Figures 7 and 8. Figure 8 shows both the particle transport and the vorticity fields at key instants in the metastable cycle.

It should be emphasized that the ‘jet’ tends to spend most of its time separated from the surface, with the animation for this case showing only limited periods

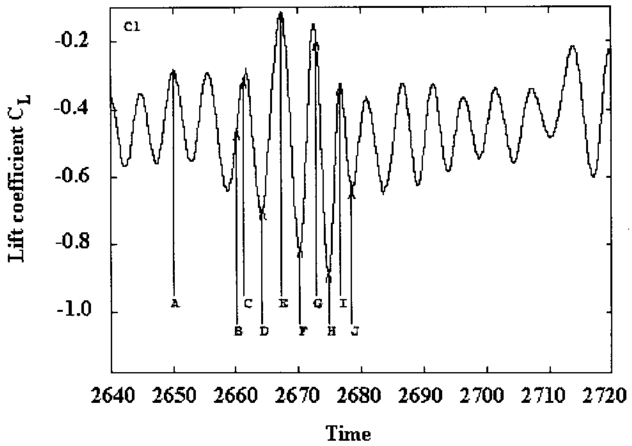

Figure 7. Close up view of the lift trace for a gap ratio of 0.40 and Froude number 0.55. (Original in colour)

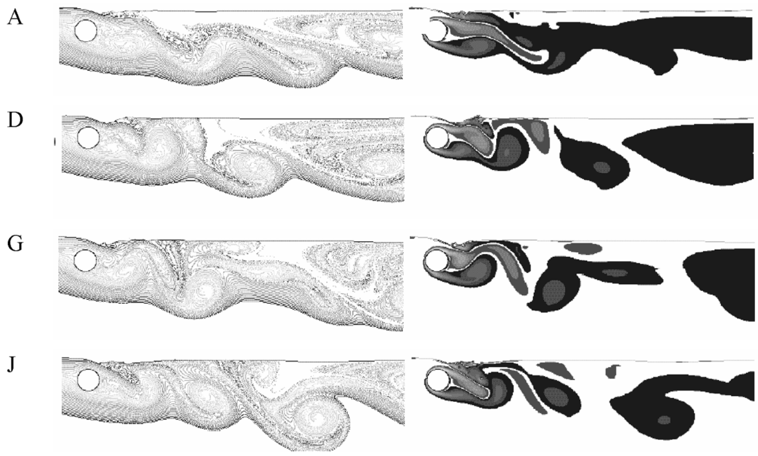

Figure 8. Passive tracer particle transport (left) and vorticity plots (right) showing flow state at some of the instants of time A-J denoted in Figure 7.

of attachment. The other interesting point is that even when the 'jet' is detached, a limited form of shedding still persists. However, the negative vortices decay very rapidly with downstream distance and are no longer observed after only 5 diameters.

When the 'jet' separates from the free surface, the particle transport animations highlight the more recirculatory nature of the wake, with much of the fluid passing over the cylinder being transported downstream via entrainment into larger scale positive vortical structures. The recirculatory behaviour results in a significant slowing and, at some points, in a reversal of the flow close to the free surface at positions downstream of the cylinder. The fluid within this zone has a particularly long residence time, with its dominant mode of removal being via entrainment into the larger scale vortical structures, which tend to form at locations further downstream. These larger-scale structures typically form from the coalescence of two or more positive vortices from beneath the cylinder.

The formation of a large recirculation bubble is largely consistent with the notion that the wake is becoming convectively unstable. It appears as if the metastable wake states represent a loose border in parameter space between an absolute and a convective instability; the level of surface deformation and hence skew at any particular instant determines the wake behaviour.

4. Conclusions

Flow past a cylinder close to a free surface at low Froude numbers is found to display behaviour common with flow past a cylinder close to a no-slip wall, at least in terms of behaviour of the Strouhal number. This suggests that the observed changes (which have been noted by others to occur over a wide range of Reynolds numbers) are primarily governed by the basic geometry, and not by the peculiar characteristics of the adjacent boundary condition.

It is suggested that the metastable wake states represent a loose form of boundary between an absolute and a convective instability. This assertion is supported by the findings of Koch [10] and Huerre and Monkewitz [8] that suggest only a limited degree of asymmetry is required before an absolute instability is lost. For this case, the switching between the two types of instability occurs as a result of the altered surface curvature, which introduces sufficient skewing of the near wake to alter transiently the nature of the instability.

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