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THREE-DIMENSIONAL NUMERICAL INVESTIGATION OF FLOW PAST A ROTATING SPHERE

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ABSTRACT

The flow past a rotating sphere is investigated numerically using a spectral element/spectral direct numerical simulation. The effect of sphere rotation on transition regimes is analysed for Reynolds numbers of $100 < Re < 300$, where Re is the Reynolds number based on freestream velocity U, sphere diameter d and kinematic viscosity ν. The results show that the Reynolds numbers for the first transition to three-dimensionality, Re_1, and the second transition to time-dependence, Re_2, are functions of the angular velocity of the sphere Ω (normalised by the sphere radius and freestream velocity) and correspondingly occur at different Re than is observed for a fixed (non-rotating) sphere.

1. INTRODUCTION

Previous investigations of the motion of a spinning sphere have mainly dealt with small particle Reynolds numbers, namely Re_p much less than unity. However, in recent years experiments have been performed on the lift of spinning spheres at intermediate Re. Oesterle and Bui Dinh (1998) looked at Reynolds numbers less than 140, and proposed an empirical expression to estimate the lift coefficient within this range and for dimensionless angular velocities varying from 1 to 6. In spite of this, they could not explain the behaviour of the lift coefficient in terms of Re and Ω and acknowledge that further information is required concerning the flow structure around the sphere.

Ece (1992) considered the unsteady boundary-layer flow past an impulsively started translating and spinning axisymmetric body of general shape, in which the stream function and swirl velocity were expanded in series in powers of time. They found that the rotation reduces the friction drag and increasing the spin rate causes a sooner onset of flow separation. Also, the point of separation advances upstream initially very fast and then asymptotes toward its steady-state value slowly. For any given time, the angle of separation is larger for a higher spin rate. However, these results are limited to the early stages of the boundary layer flow, and numerical integration of the boundary layer equations is necessary to extend the solutions to larger values of time.

A finite volume formulation was used by Salem and Oesterle (1998) to investigate a shear flow around a spinning sphere for $Re < 40$. Compared with uniform flow past a sphere, their results indicate that the drag is not altered by the rotation of the sphere, provided that the Reynolds number is low enough. Moreover, the lift and drag coefficients were found to be significantly sensitive to the grid parameters, so that the reported results were restricted to small Re.

Kurose and Komori (1999) performed three-dimensional computations of the flow around a rotating sphere in a linear shear flow. The rotation rates investigated were in the range $0 < \Omega < 0.25$, whereas the Reynolds number ranged from 1 to 500. In this study, it was found that the drag increases with increasing rotation. Also, the sign of the lift coefficient remain unchanged with increasing Re in contrast to a fixed sphere in a linear shear flow, and approached a constant value for $Re > 200$ for a given rotational speed. This asymptotic value of the lift coefficient increased with increasing rotation rate, as did the Strouhal number St. Expressions for lift and drag were proposed for the parameter range investigated.

It is the purpose of the present paper to analyse the effect of sphere rotation on the primary transition regimes of the flow around a sphere. In particular, computations were carried out for $100 < Re < 300$ and $0 < \Omega < 0.25$. The effects of varying spinning orientations were clarified by rotating the sphere about all three primary axes. Finally, wake vortex structures are presented in the form of isosurfaces of the second largest eigenvalue of $\boldsymbol{S}^2 + \boldsymbol{R}^2$, where $\boldsymbol{S}$ and $\boldsymbol{R}$ are the symmetric and antisymmetric components of the velocity gradient tensor $\nabla \boldsymbol{u}$, as suggested by Jeong and Hussain (1995).

2. NUMERICAL METHOD

The velocity-pressure field was solved in primitive variable form using a hybrid spectral element/spectral method incorporated for axisymmetric problems. The procedure involves solving the time-dependent incompressible Navier-Stokes and continuity equations:

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u} \cdot \nabla \boldsymbol{u}) - \nabla p + \frac{1}{Re} \nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0 .$$

In all the simulations presented, eighth order tensor product Lagrange polynomial expansions were used for all elements in the *s-r* plane, and a conventional Fourier expansion is utilised to extrapolate the two-dimensional mesh into three dimensions. A three-step time-splitting technique was used for unsteady simulations. Further details of the method may be found in Thompson *et al.* (1996).

3. RESULTS AND DISCUSSION

3.1 GRID RESOLUTION STUDY

An extensive study was performed to determine the effects of varying the grid parameters on the numerical solution. Table 1 summarises these results. A point immediately downstream of the sphere and located close to the separating shear layer was chosen, and streamwise velocity fluctuations were measured for Re = 100. This location was chosen due to the high velocity gradients experienced there, and gave a good indication of the adequacy of the resolution there. Initially, a test grid (Test) was used which was then modified by increasing the number of elements in the vicinity of the sphere as well as in the downstream wake (Mesh1). Blockage effects were also examined by using a mesh whose inlet and radial dimensions were half the dimensions of the original mesh (Mesh 2 in Table 1), as well as a mesh whose dimensions were double that of the original mesh. To examine the effect of the outlet position, another mesh was constructed in which the outlet was distanced twice as far as the original mesh, whilst maintaining the same mesh concentration (and hence more elements), and it was observed that very little change in solution occurred. It is evident in Table 1 that for all meshes, increasing the order of the tensor products only increased the accuracy marginally. A difference in solution of approximately 0.3% was found between Mesh1 and Mesh2, relating the fact that blockage did not have a significant effect. However, for the purposes of flow visualisation, Mesh1 was chosen with N = 8 for all simulations (Figure 2).

N	Test	% Difference	Mesh1	% Difference	Mesh2	% Difference
7	1.0030452	8.91664E-05	1.0030653	0.000105878	1.0060042	2.90335E-05
8	1.0029429	1.28061E-05	1.0029445	1.45021E-05	1.0059533	2.15294E-05
9	1.0030042	4.82823E-05	1.0030102	5.09642E-05	1.0059796	4.66612E-06
10	1.0029558	0	1.0029591	0	1.005975	0

Table 1 Grid resolution study for an axisymmetric sphere.

Author	Method	St (Re = 300)
Present (2000)	Spectral-Element	0.134
Johnson & Patel (1999)	Finite-Volume	0.137
Kurose & Komori (1999)	Finite-Difference	0.128
Tomboulides *et al.* (1993)	Spectral-Element	0.136
Tomboulides *et al.* (2000)	Spectral-Element	0.136
Ormieres & Provansal (1999)	Experimental	0.123
Sakamoto & Haniu (1990)	Experimental	0.15 - 0.18

Table 2 Comparisons of Strouhal number St at a Reynolds number of 300.

3.2 FIXED SPHERE COMPUTATIONS

Initially, computations were performed for the case of a fixed sphere in order to validate the numerical procedure. The parameter of choice to monitor was the vortex shedding frequency f at a Reynolds number of 300. This parameter is known to be sensitive to the grid dimensions (and in particular to blockage), and in non-dimensional form is known as the Strouhal number St. Table 2 summarises the Strouhal numbers obtained at $Re = 300$ from the present and previous studies. It also demonstrates that the present numerical code is well suited to the problem of bluff body vortex shedding.

3.3 ROTATING SPHERE COMPUTATIONS

Simulations were performed for the rotating sphere with dimensionless angular velocities varying from $0.05 < \Omega < 0.25$. Initially, the sphere was forced to rotate about the z-axis, and lift, drag and side-force coefficients were computed. Figure 1 presents lift and drag coefficients as a function of angular velocity for a Reynolds number of 200. For comparison, the results of Kurose and Komori (1999) are also depicted. The results show that both the lift and drag increase with an increase in sphere rotation, in accordance with previous findings (see, for example, Oesterle and Bui Dinh (1998)).

Table 3 shows the contributions of lift, drag and side forces for selected rotation rates at a Reynolds number of 200. It is evident that a streamwise rotation of the sphere results in zero lift and side force. It is also interesting to note that the drag of a streamwise-rotating sphere is less than the drag of a sphere which is rotating about the other axes. Furthermore, although not shown here, it was found that the contribution to the drag from viscous forces was of the same order as the contributions from the pressure, for $Re > 100$. In contrast, the viscous force contributions to the lift and side forces were an order of magnitude lower than the pressure contributions.

Figures 2a and 2b illustrate the vortical structure of the wake of a spinning sphere, rotating at $\Omega = 0.05$ about the z-axis and y-axis respectively at a Reynolds number of 200. Clearly visible is the double-threaded wake which is observed for a fixed sphere after $Re \approx 212$. It appears that the non-streamwise rotation of the sphere promotes conversion of azimuthal vorticity into streamwise vorticity, caused by the tilting of fluid rings as they pass close to the surface of the sphere (Thompson *et al.* (2000)). However, a rotation about the streamwise (x) axis forces the fluid immediately adjacent to the sphere to flow downstream parallel to the axis of symmetry. As a result, the fluid rings do not tilt as they pass close to the sphere's surface, and the double-threaded wake is not observed. Indeed, recent computations have observed that for a relatively small streamwise rotation, the wake remains axisymmetric up to a Reynolds number approaching 280 which, for a non-rotating sphere, is characterized by the presence of periodic vortex loops.

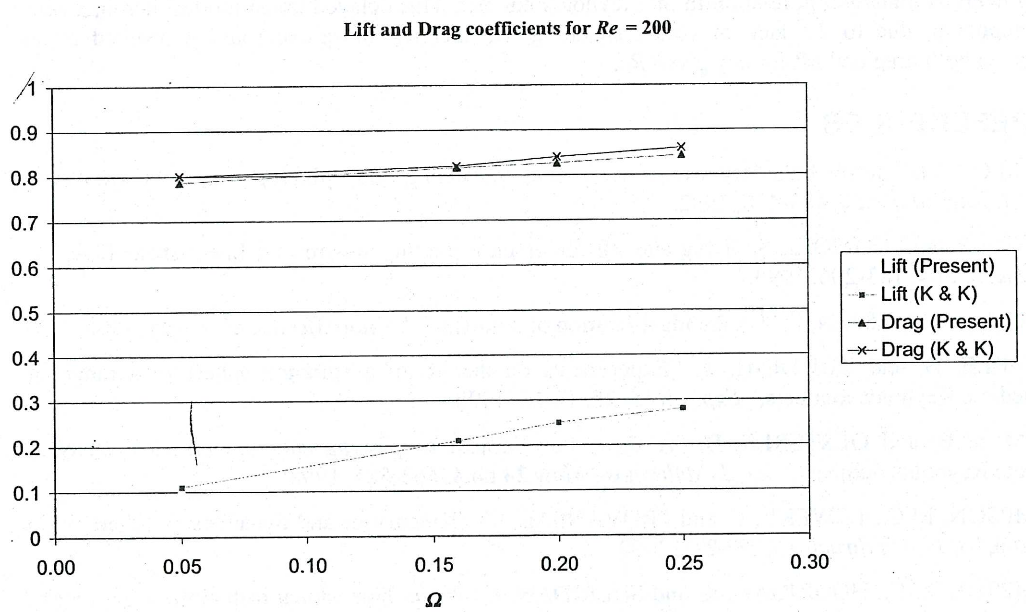
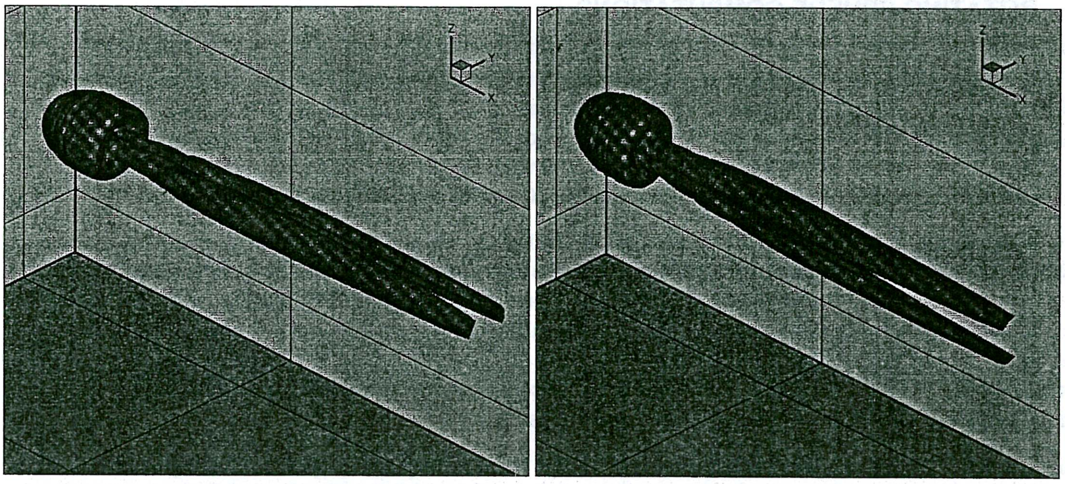


Figure 1 Lift and drag coefficients as a function of sphere rotation.

	Ωx			Ωy			Ωz		
Ω	Cd	Cl	Cs	Cd	Cl	Cs	Cd	Cl	Cs
0.05	0.7753	0	0	0.7862	0	0.1013	0.7862	0.1013	0
0.15	0.7762	0	0	0.8102	0	0.2147	0.8102	0.2147	0
0.16	0.7767	0	0	0.8164	0	0.2264	0.8133	0.2250	0

Table 3 Effect of sphere rotation on forces for selected angular velocities.



(a) **(b)**

Figure 2 "Double-thread" vortex structures in the wake of a rotating sphere, with (a) $\Omega_y = 0.05$, and (b) $\Omega_z = 0.05$. Note that the surface of the sphere is obstructed from view by the $-\lambda_2$ isosurface in accordance with Jeong and Hussain (1995).

4. CONCLUSION

The flow about a rotating sphere was analysed numerically using a three-dimensional spectral element/spectral method for axisymmetric geometries. It was found that a non-streamwise sphere rotation caused an earlier transition to non-axisymmetry, whereas a streamwise rotation resulted in a greatly delayed transition in relation to the Reynolds number. This delayed transition resulted in a zero lift component, due to the lack of vortex shedding. An increase in sphere rotation resulted in an increase in both drag and lift for any given *Re*.

5. REFERENCES

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