Forced Response of the Subcritical Landau Equation: Application to Vortex Shedding from a Cylinder

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The complex Landau equation models a prototypical Hopf bifurcation in which, when the control parameter exceeds a critical value, the null solution bifurcates into a finite amplitude time-periodic solution. This equation has been widely used in the past to model the shedding of vortices in the two-dimensional wake of a cylinder at low Reynolds numbers. In particular, the different coefficients of the model can be directly measured from experiments [1] and from numerical simulations [2]. In addition, 2D-flows past oscillating cylinder have also been the subject of many experiments [3]. The present study is thus devoted to the calculation of the response of the forced Landau equation and to the comparison between its analytical solutions with experiments and numerical simulations. We will focus our attention on the subcritical regime, where the periodic solution is damped. To our knowledge, the only attempt in modeling the periodically forced wake by a forced Landau equation under threshold has been by Provansal et al. [1]. In this case, the forcing term which is added to the model is a simple harmonic term, with a given amplitude and frequency. It should be noted that above threshold, additional third-order terms are involved in the amplitude equation associated with the forced Hopf bifurcation. The solution is much more intricate in that case [4]. Searching for locked solutions, we restrict our analysis to a frequency range around the natural frequency where locking is expected. Due to the cubic nonlinearity of the Landau equation, we demonstrate that the resonance curve can, in a certain range of parameters, exhibit hysteresis. This behavior is similar to the mechanical response of a periodically-forced pendulum [5]. Figure 1(a) presents a sketch of this resonance curve.

The next part of our study is devoted to checking these predictions against experiments and numerical simulations. In both cases, we study the wake of a circular cylinder subject to cross-flow oscillations. Figure 1(b) presents a visualisation of such a wake in a water tunnel, at a Reynolds number of 40. The cylinder has a diameter of 2 mm and is mounted on a support which can oscillate at a chosen frequency by the use of an electric motor. Visualisations are made by oxidation of a tin wire. A strong resonance can easily be discerned while scanning the frequencies around the natural frequency of the wake.

On the computational side, simulations were performed using the spectral-element method. The critical Reynolds number was first determined as $Re = 46.5$ consistent with previous numerical and experimental studies. The system was then forced by oscillating the plate in the crossflow direction at different frequencies for $Re = 44$. Each simulation was integrated forward in time until the flow reached an asymptotic state. Also, the simulation at the next highest frequency was started from the periodic flow at the previous frequency. The system response was measured by monitoring the cross-stream velocity at a point in the wake. Figure 2(a) shows the vorticity in the wake for a forcing amplitude of 0.7 times the natural frequency. Figure 2(b) presents the resonance curves at the same Reynolds number for forcing amplitudes of 1, 3, 10 and 30%. (Here, the forcing amplitude corresponds to the velocity amplitude of the plate.) Although an asymmetry—characteristic of the presence of odd terms in the dynamical system—is clearly visible, no hysteresis can be observed. Current experimental and numerical works are in progress to understand the reasons for the mismatch between the predictions of the Landau model and the forced wake.

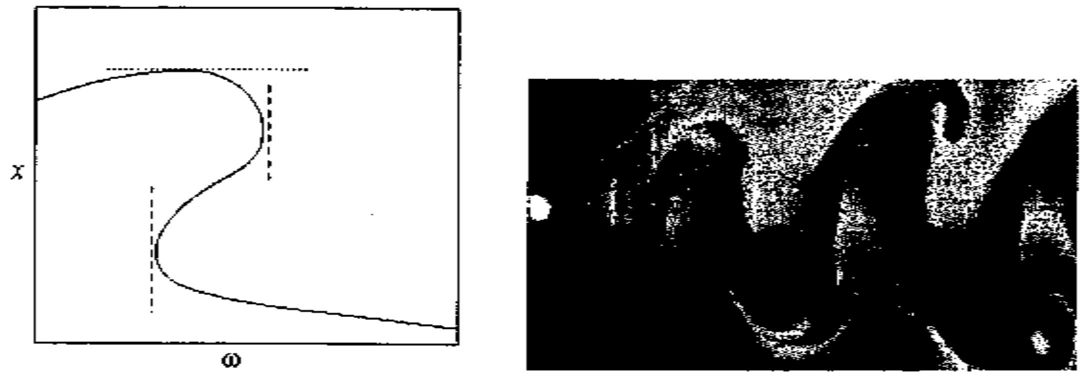

Figure 1: (a) The frequency-response curve, Amplitude x versus forcing frequency ω near the hysteretic loop. (b) Visualisation of an oscillating wake at a Reynolds number of 40 and a frequency of 1.1 times the natural frequency.

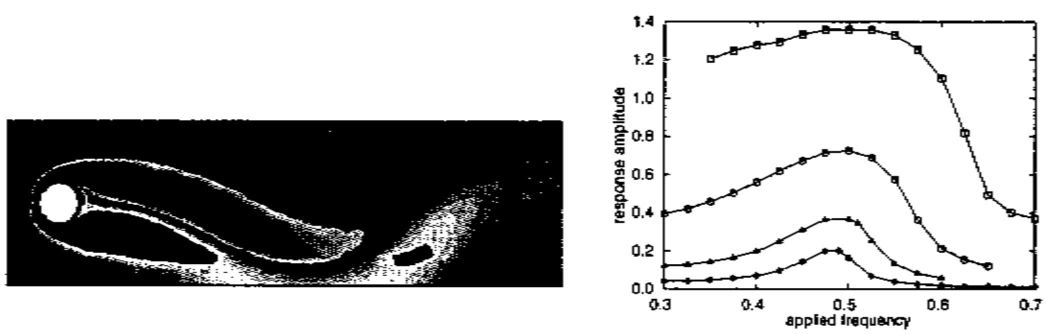

Figure 2: (a) Simulation of an oscillating wake at a Reynolds number of 44 and at a frequency of 0.7 times the natural frequency. (b) Numerical resonance curve for different forcing amplitudes.

References

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