PLANETARY AGGREGATION AND NEBULA TIDES.

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According to density wave theory, the disturbing gravitational potential of a planet embedded in a fluid disc leads to a mutual torque that transfers angular momentum between the planet and disc (1-2). The continuous functional form for the torque density can be written $dT/dr \sim \pm f\, G^2 M_p^2\, r\, \sigma/\Omega^2 x^4$ (2-3), where x is the distance from the planet of mass M_p, Ω is the orbital angular velocity of the planet, f is a constant of order unity, and the - and + signs refer to the regions of the disc interior and exterior, respectively, to the planet's orbit. The torque density cuts off for $x < h$, where h is the scale height of the disc. Via this torque density, gas on either side of the planet's orbit tends to withdraw, clearing a zone. This formation of a gap in the nebula is opposed by the action of viscous shear stresses in the disc. Concomitantly, the semi-major axis of the planet undergoes alteration due to torque imbalances resulting from asymmetries in the disc properties.

It turns out, the density wave mechanism implies a severe restriction on the timescale of nebular dispersal. This is true whether or not a planet clears a zone. In the absence of a gap, Jupiter's semi-major axis varies on a timescale of less than $0(10^4)$ years as a result of probable disc asymmetries (1). On the other hand, the existence of a gap, which furnishes a barrier to the flow of nebular materials, provides no permanent stabilizing effect either, because the orbital angular momentum of the planet becomes locked into the angular momentum redistribution process of the disc as a whole and substantial radial drift of the Jovian orbit ensues (4). Only rapid nebula dispersal [i.e., $0(10^{4-5}$ yrs)] seem capable of preventing this. Such inconsistency in timescales presents a principal obstacle to cosmogonic models. The above implied nebula life time is much shorter than typical estimates of accretion times, i.e., $\sim 0(10^7$ yr) for the terrestrial planets and $\gtrsim 0(10^9$ yrs) for the outer planetary cores (5,6). Especially noteworthy is that fact that the latter estimate is dangerously close to the total age of the solar system.

A partial resolution to this problem could be furnished by density waves themselves. It is generally acknowledged that objects of order 10^{25} gms can accumulate quickly [i.e., $\lesssim 0(10^4$ yr)] via strong local gravitational encounters (7). It is the terminal stages of planetary growth to $\gtrsim 0(10^{27}$ gms) that requires a comparatively protracted interval. This is a direct consequence of the low collision frequency as fewer and fewer objects are assumed to be distributed more or less uniformly throughout a region. Shorter accretion times would, of course, be possible if planetesimals were brought into greater proximity through changes in their differential semi-major axes rather than by increased eccentricities. Although gas drag has long been recognized as a potential agent of radial migration, timescales associated with $\gtrsim 10^{25}$ gm objects become prohibitively long (8). Hence a significant portion of this material must avoid participating in the rapid local growth phase long enough to migrate great distances--a situation which seems less than certain. Here we call attention to the fact that orbital drift generated by disc-planet tides,

$$\dot{r} \sim f'\, r\Omega (r/h)^2 (\sigma r^2/M_\odot)(m/M_\odot) \qquad (1)$$

dominates aerodynamic drag in the large size range and for reasonable parameters implies a characteristic growth time, $\tau_m \approx m/\dot{m} \approx m/2\pi \sigma_p r \dot{r}$, of order 10^5–10^6 yrs everywhere in the disc. In the above, σ and σ_p are surface densities for gas and accretable material, respectively; r is the heliocentric distance, $M_\odot$ is the solar mass, and f' is a constant of order unity. Of course, eqn. (1) is not valid if a tidal zone has opened. Viscous diffusion closes a zone if the characteristic evolution timescale for the nebula $\tau_N = r^2/\nu \lesssim \Omega^{-1} \mu^{-2} (h/r)^3$. Were it not for Jupiter (and Saturn) it would be possible to choose a ν that would yield a self-consistent model allowing for both accretion and nebula dispersal. [In particular, accretion in the extreme outer portion of the system would not require intervals comparable to the age of the solar system.] However, τ_m is still considerably longer than the critical dispersal time of the nebula needed to ensure stability of Jupiter's orbit. Hence, the motivation for considering a two-stage nebula history persists (4). Although a two-stage history seems regrettably cumbersome, it would clearly be contrived to assume a changing viscosity of just the required magnitude to close tidal zones throughout the accretion process but not so large as to disperse the nebula too quickly. In this context we also point out that diffusion effects may not be the only mechanism to suppress zone formation. If the timescale for an object to drift a distance comparable to the scale height h via eqn. (1) is less than the time necessary for a zone of such width to form, i.e., $\tau_z \sim \Omega^{-1} \mu^{-2} (h/r)^5$, the zone may indeed fail to open. This is equivalent to the condition $m < O(\sigma h^2)$, which in the terrestrial zone is $\sim O(10^{27}$ gm), i.e., suspiciously close to planetary size. Thus, in a very quiescent nebula, objects would tend to grow to a critical size before gap formation inhibits further migration. In the outer solar system the limiting size is about an order of magnitude larger, again comparable to planetary (or giant planet core) size in that region. In this scheme then, a final vigorous stage of nebula evolution is still required to initiate gas accretion by the giant planets and disperse the remaining disc without destabilizing the system.

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