orbit, the drift time for Titan is of the order 10^2 years. Comparing this value with that obtained above in the drag case, it can be seen that the effect of tidal torques dominates that of aerodynamic drag in the large satellite range.

The above result was obtained under the assumption that the nebula disc remained unmodified by the tidal torques. However, if local damping of density waves takes place (see Hourigan and Schwarz 1984), gap clearance may occur. In this event, the orbital angular momentum of the satellite is locked into the transport process of the disc, resulting in serious orbital modification of the satellite on the same timescale as disc dispersal (Ward 1982).

In order then that the orbit of Titan is not severely disrupted, it is necessary that the disc dispersal time is less than the orbital drift time i.e. $\tau_N < \tau_D$. Using the above results, this is only possible in the case where strong turbulence is present in the disc and $\alpha > 0(10^{-2})$.

Discussion

An effective method of nebula disc dispersal may be that due to internal viscous shear stresses. The resulting viscous couple leads to an outward flux of angular momentum and an increasing inflow of matter, which may be accreted on to the primary. Instead of needing to invoke a final 'blow-off' phase, this accretion disc model assumes the contemporaneous removal of the disc with the accretion of the satellites. However, when this viscous couple operates in conjunction with the generation of density waves by a large satellite, certain timescale restrictions are found to emerge.

In the preceding sections, it was established that fairly rapid removal of the proto-Saturnian nebula on the timescale of order 10^2 years is required in order that severe disruption of Titan's orbit is avoided. This also represents an upper limit to the timescale of satellite accretion, which may be difficult to satisfy by previously proposed accretion mechanisms (e.g. Wetherill's (1980) mechanism of collisional accretion resulting from 'pumped-up' orbital eccentricities, which is even less efficient in the presence of drag-inducing gas). The question must also be raised as to whether efficient satellitesimal aggregation is viable in the presence of such strong turbulent overturning of the gas. It should be noted, finally, that the above strict timescales for nebula dispersal and satellite accretion may be relaxed somewhat if Titan is considered to be a captured body, as proposed by Prentice (1983). This is due to the fact that the corresponding value of the mass of the reconstructed nebula is reduced by an order or two of magnitude if Titan's contribution is ignored, leading to an increased timescale of tidal drift.

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Nebula Tides and Gap Formation

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Introduction

An intriguing problem in cosmogony concerns the ability of a planetoid embedded in a nebula disc to clear a gap around its orbit. The application of density wave theory to this problem has demonstrated that a significant exchange of angular momentum can take place between a planetoid and a disc (Goldreich and Tremaine 1980). The torque exerted by the disc on the planetoid can result in orbital drifting of the latter, which may play an important role in the aggregation process (Hourigan and Ward 1983). In fact, in the absence of significant deformation of the nebula, the radial orbital drift rate of a planetoid increases with planetoid mass. In this case, it would be expected that only one or two planetoids would sweep out the nebula, a situation not compatible with present observations. The orbital drift resulting from the generation of density waves therefore requires a limiting mechanism.

One possible resolution of this difficulty may in fact be supplied by the density waves themselves. In the absence of a damping mechanism, these waves transport angular momentum to regions of the disc far from the planetoid. The nebula material close to the planetoid is left relatively undisturbed. On the other hand, if strong local damping is present, angular momentum can be transferred from the density waves to the disc matter, leading to the clearance of a gap which stabilizes the planetoid's orbit (Hourigan and Ward 1984). However, a further dilemma is encountered if the gap is maintained during the process of viscous nebula dispersal. In this case, the orbital angular momentum of the planet is locked into the angular momentum transfer process of the disc and can result in destabilization of its orbit (Ward 1982).

The process of nebula truncation, or gap clearance, is therefore highly relevant to the problems of planetary aggregation and disc dispersal. In the present paper, mechanisms leading to an opposing gap formation are discussed.

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Torque Density

Radial mobility of planetoids embedded in a nebula disc appears to be possible through the action of tidal torques (Hourigan and Ward 1984). The nebula's response to the satellite's gravitational perturbations involves complex wave phenomena capable of redistributing angular momentum. Spiral waves are launched from the Lindblad resonances and couple to the non-spiral potential of the planetoid to produce a mutual torque. Waves generated at inner Lindblad resonances carry negative angular momentum whilst the outer waves carry positive angular momentum. If the damping length of the waves is long compared to the resonance spacing, the torque density close to the planetoid, where the resonances are densely packed, assumes the continuous form (Lin and Papaloizou 1979, Goldreich and Tremaine 1980).

$$\frac{dT}{dr} \sim sgn(x)\ f\ \frac{G^2\ M^2\ \sigma_g r}{\Omega^2\ x^4},$$

where x is the radial distance from the satellite of mass M, σ_g is the local disc surface density, Ω is the orbital angular velocity at radius r, and f is a constant of order unity. This expression for the torque density is only valid for $x \gtrsim h$, where $h \sim c/\Omega$ is the pressure scale height of the disc and c is the gas sound speed. Inside this distance, the x^{-4} dependence is not maintained and the torque density profile flattens out.

Nebula Truncation

Imbalance of the inner and outer torques resulting from gradients in the nebula disc can lead to radial drifting of the planetoid's orbit. A crude estimate of the time for a planetoid to drift a distance x is given by (Hourigan and Ward 1984).

$$\tau_D \sim [f'\ r\ \Omega\ (M/M_\odot)\ (r/h)^2\ (\sigma_g r^2/M_\odot)\ r/x]^{-1}$$

where $M_\odot$ is the solar mass and f' is the constant of order unity.

Radial drift may be terminated if sufficiently strong local damping of the density waves occurs and a gap is cleared in the nebula. Ignoring diffusion for the moment, smooth damping can clear a gap of width $x > h$ (the minimum gap width supportable by the nebula) in a time of order (Goldreich and Tremaine 1980).

$$\tau_{gap} \sim (\mu^2\ \Omega)^{-1}\ (x/r)^5, \qquad (1)$$

where μ is the mass of the planetoid normalized to that of the Sun.

We have constructed simple particle models of disc/planet interactions. The nebula is represented by a two-dimensional disc of particles, initially moving along circular orbits about a central mass. A point mass representing the planet is embedded in the disc and gravitationally disturbs the orbits of the disc particles. Wave-damping is modelled by imposing a certain degree of inelasticity on the collisions between disc particles. It should be noted that the torque density is fairly insensitive to the degree of damping (Greenberg 1983). Figure 1 shows a snap-shot of the positions of the disc particles after 28 orbital revolutions of the planet of normalized mass 5×10^{-4}, around the central body. Initially, the particle surface density of the disc was uniform. The effect of the gravitational encounters between the disc and the planet is that of orbital excitation of the disc particles. Upon damping of these excitations through inelastic collisions, angular momentum is tranferred to particle orbits, resulting in the clearing of a gap. The azimuthally averaged radial density profile is shown in Figure 2 for the same instant of time as Figure 1. The timescale of gap clearance in this case is consistent with that obtained from equation (1), using the present parameter values of normalized mass and gap width. More realistic numerical models incorporating pressure gradients

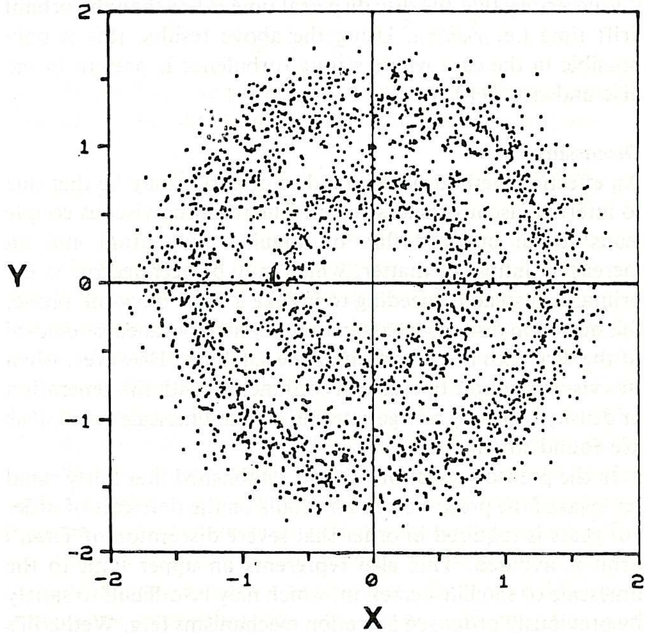

Figure 1 A snap-shot of the disc particle positions after a period of 26 orbits for $\mu = 5 \times 10^{-4}$. The co-ordinates are normalized to the fixed radius of the planetoid.

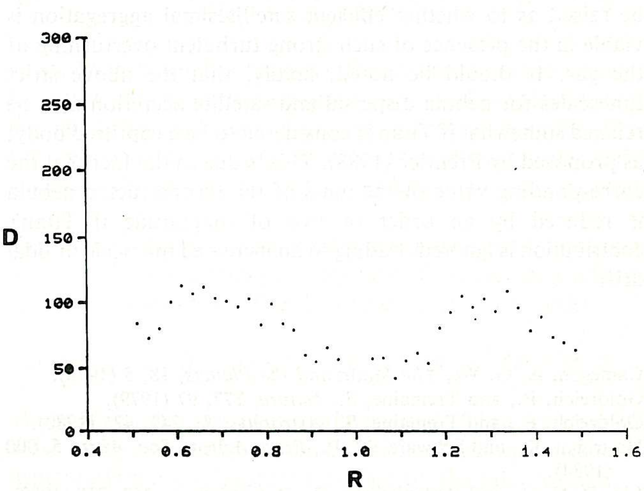

Figure 2 The azimuthally averaged disc particle density versus the normalized radius corresponding to Figure 2.

and shock wave development are presently being constructed.

If the nebula does possess a non-zero kinematic viscosity ν, the time scale of closure of a gap of width x by diffusion is

$$\tau_{dif} \sim x^2/\nu.$$

In order that a gap can be opened, it is then required that

$$\tau_{gap} < min\ (\tau_D,\ \tau_{dif}).$$

This expression may be rewritten in terms of the normalized mass μ.

The condition to be satisfied for gap formation is then

$$\mu > max\ (\mu_I,\ \mu_\nu),$$

Here the inertial mass limit is given by

$$\mu_I \sim f_2\ \sigma_g h^2,$$

where f_2 is a constant of order unity.

The viscous mass limit is

$$\mu_\nu \sim (\nu/r^2\Omega)^{\frac{1}{2}}\ (h/r)^{3/2}. \quad (2)$$

It is interesting to note that for reasonable values of the nebula parameters, the inertial mass limit μ_I is of the same order as those observed throughout the planetary system. That is, one may speculate that present planetary core dimensions may be attained through sweeping up of planetesimals by a drifting embryo, this process terminating as a result of gap formation when the inertial mass limit is reached (Hourigan and Ward 1983). Furthermore, in the case of strong local damping, quite vigorous turbulence is required to close a gap in the neighbourhood of a body the size of Jupiter, according to the viscous mass limit. Failure of the nebula material to be able to flow past such a body during the dispersal stage would lead to severe destabilization of the Jovian orbit (Ward 1982). Similar arguments apply to Titan and the proto-Saturnian nebula (Hourigan 1984). Therefore, the question as to whether planetoids can open gaps in nebula discs is extremely relevant to problems of planetary formation and orbital stability.

Wave Damping

The results of the previous section are dependent in part on whether local wave damping does in fact take place. In its absence, angular momentum is carried far away from the satellite by density waves, leaving the nearby parts of the nebula relatively undisturbed. Two possible damping agents that have been proposed are those involving turbulent viscous action and non-linear wave development (Goldreich and Tremaine 1980).

In the case of gaseous discs, the damping length due to turbulent viscosity of density waves created at the most important resonances (i.e. near $x \sim h$) can be written as (Ward 1984).

$$x_\nu \sim \alpha^{-2/3}\ h, \quad (3)$$

where α is the constant in the alpha-model of turbulent kinematic viscosity $\nu = \alpha h^2\Omega$, $\alpha < 1$. From the expressions for the viscous mass limit (equation (2)) and the viscous damping length (equation (3)), it is found that a consistent solution for gap clearing and wave damping is marginally possible in the case of a Jovian-size body. However, such damping is unlikely in the case of the terrestial planets.

A second mechanism that may lead to local wave damping is that resulting from non-linear wave development. Using the Lin-Shu dispersion relation for density waves, the non-linear development length of density waves generated at the most important resonances ($x \sim h$) in a pressure-supported disc is given by (ignoring and relatively small distance separating the Lindblad resonance and the reflecting edge of the forbidden zone)

$$x_{NL} \sim 3r\ (h/r)\ (\sigma_g\ h^2/M_\odot)\ \mu^{-2}. \quad (4)$$

For Jupiter and the protosolar nebula, this implies a length (Ward 1984).

$$x_{NL}/r \sim 0.2\ (\sigma_g/100\ g\ cm^{-2})\ (T/100K),$$

where T is the local gas temperature.

It is expected that the steepening gradients accompanying non-linear wave development lead to much higher rates of damping. Thus, it would appear that local wave damping and gap clearing are plausible for a Jovian-size body. However, equation (4), suggests that this is not the case for the smaller plantets.

Discussion

The role of density waves in transferring angular momentum from the orbit of a planetoid to a gaseous nebula, and vice versa, has been outlined above. Two mechanisms proposed for wave damping, viz. turbulent viscosity and non-linear wave development, may possibly be effective for density waves generated in the proto-solar nebula by bodies of Jovian-size. Their effectiveness for smaller bodies still needs to be established. It should be noted, however, that these mechanisms have been investigated only in the context of waves confined to the ecliptic plane. That is, the vertical dimension of the disc is essentially ignored in consideration of density wave development and propagation. In the case of a gaseous nebula, further attention needs to be paid to vertical resonances, recalling that the most important horizontal or Lindblad resonances occur at a distance of only a pressure scale height from the planetoid. At this position, the disc no longer appears two-dimensional to the planetoid. Furthermore, the exponential profile of gas density with disc height may provide shock conditions for waves possessing a non-zero vertical wave number. Research into these considerations is progressing.

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The Origin of Inner Rings in Barred Spiral Galaxies

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A large number of barred spiral galaxies contain a ring-like structure surrounding the bar. This is known as an inner ring if the bar terminates at the ring. Structures which are not closed but which appear to be related phenomena are given the name pseudo-inner rings by de Vaucouleurs (1959). Inner rings can take a variety of shapes ranging from the near circular to ones with quite sharp corners on the bar major axis (almost diamond shaped ◇). The average intrinsic axial ratio is found from statistics of apparent axial ratio to be 0.8 (Schwarz 1984a, Buta 1984). Several observations suggest that inner rings are formed by gas dynamic processes: they are almost always to some extent spiral (i.e. not perfectly closed); HII regions are associated with the rings; and they are blue, although in some cases there might also be a broader redder ring at the same position (Buta 1984). [In early gas-poor galaxies, the inner ring is sometimes replaced by the edge of a lens or possibly a broad stellar ring.]

In numerical simulations of gas flow in a bar field we have shown that a ring of the above type can form just inside corotation (CR) if the pattern speed is low enough (Schwarz 1979, 1984b). The ring will appear to be at the ends of the bar if, as is generally believed (see e.g. Sellwood 1981), the bar terminates near CR. The rings thus formed have sharp corners on the bar major axis as does that in NGC 1433. Figure 1 shows a typical example. The small ring near the inner Lindblad resonance (I.L.R.) has been discussed previously (Schwarz 1984b) as has the formation near the outer Lindblad resonance (O.L.R.) of a structure which we identify with outer rings (R). Since no other convincing mechanism for the formation of inner rings has been proposed, we attempt here to understand what causes the rings in the gas simulations, and what affects their shape.

The Gas Flow

The models in Figure 1 were computed exactly as described in Schwarz (1984b) except that the bar used is a 1:4 axis ratio homogeneous prolate spheroid ending at CR. Only the 2θ components of the field are retained for this calculation – the effect of restoring the higher harmonics is described later. The pattern speed is $\Omega_p = 0.06$ and the bar force is about 10% of the axisymmetric force at CR.

Before we study the gas flow in this model in detail, we can get an idea of what causes the ring in the simulations by varying some of the bar parameters. For example if we move the ends of the bar away from CR (either inward or outward) the ring remains near CR. It is thus a resonance phenomenon and not directly related to the bar cutoff. Of course in reality the extent

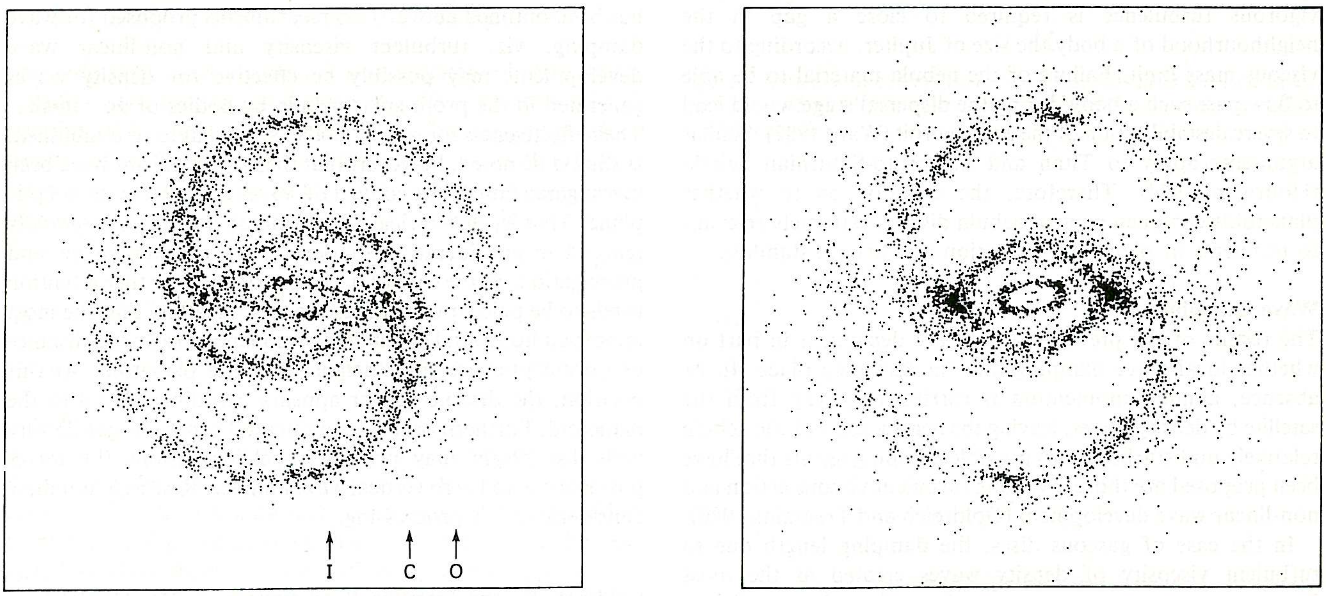

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Figure 1 Particle distributions at times two and four bar rotations. As in all subsequent figures, the bar lies along the x-axis, and the resonant radii are indicated appropriately by I, C, O.