impacting grains. We can criticize this argument on the grounds that it constitutes a hypothesis that cannot be proved false. From our point of view there is the further difficulty that it provides no reason for expecting the actual spin velocities to be so close to the instability values. It is difficult to see why the orbital eccentricities, and hence the encounter velocities, should be so adjusted that for the primordial Earth, for Jupiter, Saturn, Uranus and Neptune, the ratios of actual spin rate to the limiting rate for rotational stability all lie between about 1/3 and 1/6.

The problems are even greater in the case of the asteroids, for 10 of which the spin sense has been determined. Two techniques have been used for the determination of spin, namely visual photometry and infrared radiometry. Both techniques have been used in the case of 2 Pallas, 4 Vesta and 433 Eros, and each technique indicates prograde spin for all three asteroids (Schroll *et al.* 1976; Taylor 1973; Dunlap 1976; Hansen 1977; Morrison 1976). Prograde spins have also been measured for 1 Ceres and 19 Fortuna by infrared radiometry (Hansen 1977) and for 6 Hebe, 624 Hektor and 1685 Toro by visual photometry (Gehrels and Taylor 1977; Dunlap and Gehrels 1969; Dunlap *et al.* 1973). Retrograde spins have been determined for 5 Astraea and 1620 Geographos by visual photometry (Taylor 1978; Dunlap 1974). Thus 8 of these 10 asteroids are believed to have prograde spin. Furthermore, all those asteroids of diameter greater than about 200 km for which the spin sense has been determined—namely 1 Ceres, 2 Pallas, 4 Vesta, 6 Hebe, 19 Fortuna and 624 Hektor—spin in the prograde sense. Tedesco and Zappala (1980) suggest that asteroids of diameter greater than about 200 km may be primordial '. . . possibly even in the sense of showing primordial spin rates'. In order to account for prograde spin the theory of Giuli (1968) and Harris (1977) requires the embryo to be in an almost circular orbit. In fact, of course, asteroid orbits have substantial eccentricities, though it is possible that they may have originally been circular. This aside, the fact that almost all asteroids spin close to the instability limit must itself pose a difficulty for accretion theory.

A recent version of the accretion theory which goes some way towards meeting these objections is due to Prentice (1980). Prentice's theory takes into account the effect on the impacting particle of drag due to gas and predicts prograde spin. It is clear from the author's comments however that more work needs to be done on this theory before it can be regarded as definitive.

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Titan and the Dispersal of the Proto-Saturnian Nebula

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Introduction

The NASA Voyager spacecraft missions to the outer planets have provided a wealth of observational data with regard to the physical and chemical properties of the attendant satellite systems. Although the gaseous nebula discs from which these satellites are suspected to have formed are no longer present, these data provide at least some basis from which the characteristics of individual nebula can be deduced.

Coupled with the observations, two fairly recent theoretical developments have resulted in a far better understanding of nebula evolution and disc/satellite interactions. These are the application of viscous accretion disc theory to protoplanetary and protosolar nebula (Lin and Papaloizou 1980) and the application of density wave theory to gravitational interactions between a gaseous disc and a satellite (Goldreich and Tremaine 1980). The latter theory has been successfully applied to the formation of divisions in Saturn's rings (Goldreich and Tremaine 1979). Here, we examine some of the ramifications that arise when these theories are applied to the proto-Saturnian nebula. In particular, we are interested in exploring whether the presence of Titan in a viscously evolving nebula leads to any restrictions being placed on the timescales of nebula dispersal and satellite accretion.

The Proto-Saturnian Nebula

The structure of the proto-Saturnian nebula may be estimated using the chemical and physical details of the presently observed satellite system. The total mass of the Saturnian satellites is approximately 1.5×10^{26} g, consisting in general of an ice/rock

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mixture. The 'minimum mass nebula' of cosmic composition required to provide the observed satellite mass is then $\simeq 0(10^{28}$ g) or $\simeq 2\times10^{-2}$ M_s, where M_s is the mass of Saturn. If the 'minimum mass nebula' were uniformly spread over a circular area of 30 R_s (where R_s is Saturn's radius) which represents the region occupied by the regular satellites, then the mean surface density would be $\bar{\sigma}_g \simeq 10^5$ gcm^{-2}. The mass of the largest satellite Titan normalized to that of Saturn is $\mu_T \sim 2\times10^{-4}$, it has orbital angular velocity $\Omega \sim 4\times10^{-6}$s^{-1}, orbital radius $R_T \sim 20$ R_s and mean density $\varrho \sim 2$ g cm^{-3}.

The pressure scale height of the nebula, neglecting the self-gravity of the disc, is given by

$$h = \sqrt{(2kT/\mu\Omega^2)},$$

where k is Boltzmann's constant, T is the gas temperature, μ is the molecular weight and Ω is the local gas angular velocity. For temperatures appropriate to the chemical composition of the Saturnian satellites, $h/r \sim 0(10^{-1})$, where r is the orbital radius.

Although these estimates of the nebula properties are necessarily crude, they nevertheless may be used to provide order-of-magnitude estimates of some important timescales associated with the Saturnian satellite system.

Nebula Dispersal Timescale

The presence of viscous shear stresses in a nebula disc leads to a couple via which angular momentum is transported radially outwards. The dissipation as heat of energy stored in the shear leads to an ever-increasing amount of nebula material falling into the primary. At the same time, a decreasing proportion of matter moves away from the primary, conserving the system's angular momentum (Shakura and Sunyaev 1973; Lynden-Bell and Pringle 1974).

In a turbulent Keplerian disc, the viscous couple assumes the form

$$g = 3\pi\, \nu\sigma_g r^2 \Omega,$$

where ν is the effective turbulent viscosity, σ_g is the local surface density of the disc, and Ω is the local orbital angular velocity. According to the alpha-model of Shakura and Sunyaev (1973), the turbulent viscosity may be represented by

$$\nu = \alpha\, h^2\, \Omega,$$

where α is a scaling constant of order less than unity.

A number of mechanisms for generating turbulence in a gaseous disc have been suggested (Cameron 1978; Lin and Bodenheimer 1982). The latter authors argue that convective-driven turbulence due to grain opacity results in gas eddies that couple radial motions over distances of the scale height h. Additional opacity may be provided by the Rosseland mean gaseous opacity, which is insignificant in the solar nebula context but more relevant for the far higher surface densities expected in the nebula discs of the giant planets (Lunine and Stevenson 1982).

The characteristic timescale of viscous dissipation of the nebula is then

$$\tau_N \sim R_N^{\,2}/\nu,$$

where R_N is a representative radial dimension of the disc.

Orbital Drift of an Embedded Satellite

The orbital velocity of a satellite does not precisely match that locally of the gas as a result of radial pressure gradients in the nebula. Aerodynamic drag then leads to a steady decay of the satellite's orbit. The drag force exerted on a large satellite of radius R by gas streaming past at a relative velocity v_g is given by

$$F = C_D \frac{1}{2} \rho_g v_g^{\,2} \pi R^2,$$

where $C_D \simeq 0.44$ is the drag coefficient and ϱ_g is the gas density. The time scale of orbital decay due to gas drag is then

$$\tau_{drag} \sim \frac{32}{3} (C_D\, \Omega)^{-1} (r/h)^3 (\rho_p R/\sigma_g),$$

where ϱ_p is the satellite density. For the nebula parameter values given in section one, the gas drag induced orbital decay time for Titan is found to be of the order 10^5 years.

A mechanism that can provide far greater rapidity of orbital drift in the large satellite range is the tidal torque that arises from satellite-disc interactions. The Keplerian flow of the gaseous disc is perturbed by the gravity field of a satellite. Density waves are launched at the Lindblad resonances in the disc where the perturbations are most pronounced (Goldreich and Tremaine 1980). A mutual torque is established that transfers angular momentum between the satellite and the disc. Close to the satellite, the resonances are densely packed. A reasonable approximation to their net effect is provided by a continuous expression for the torque density (Lin and Papaloizou 1979).

$$\frac{dT}{dr} \sim 9/4\, f \frac{G^2 M_s^{\,2}\, r\, \sigma_g}{(\Omega_p - \Omega_g)^2 x^2}\, sgn\, (\Omega_p - \Omega_g),$$

where M_s is the satellite mass, x is the distance from the satellite, Ω_p and Ω_g are the orbital angular velocities of the satellite and the gas, respectively, and f is a constant of order unity. This form breaks down at distances x less than about a scale height h. Inside this distance, the torque density is approximately constant.

The satellite experiences a resultant radial drift if the inner and outer torques do not precisely match as a result of gradients of the disc properties. The drift velocity is of the order

$$\dot{r} \sim f'\, r\, \Omega\, (r/h)^2\, (\sigma_g r^2/M_o)\, (M_s/M_o),$$

where M_o is the mass of the primary and f' is a constant of order unity. This expression holds provided the local disc structure is not significantly modified by the tidal torques.

The characteristic drift time of a satellite is then given by $\tau_D \sim r/\dot{r}$. Using nebula parameter values appropriate to Titan's orbit, the drift time for Titan is of the order 10^2 years. Comparing this value with that obtained above in the drag case, it can be seen that the effect of tidal torques dominates that of aerodynamic drag in the large satellite range.

The above result was obtained under the assumption that the nebula disc remained unmodified by the tidal torques. However, if local damping of density waves takes place (see Hourigan and Schwarz 1984), gap clearance may occur. In this event, the orbital angular momentum of the satellite is locked into the transport process of the disc, resulting in serious orbital modification of the satellite on the same timescale as disc dispersal (Ward 1982).

In order then that the orbit of Titan is not severely disrupted, it is necessary that the disc dispersal time is less than the orbital drift time i.e. $\tau_N < \tau_D$. Using the above results, this is only possible in the case where strong turbulence is present in the disc and $\alpha > 0(10^{-2})$.

Discussion

An effective method of nebula disc dispersal may be that due to internal viscous shear stresses. The resulting viscous couple leads to an outward flux of angular momentum and an increasing inflow of matter, which may be accreted on to the primary. Instead of needing to invoke a final 'blow-off' phase, this accretion disc model assumes the contemporaneous removal of the disc with the accretion of the satellites. However, when this viscous couple operates in conjunction with the generation of density waves by a large satellite, certain timescale restrictions are found to emerge.

In the preceding sections, it was established that fairly rapid removal of the proto-Saturnian nebula on the timescale of order 10^2 years is required in order that severe disruption of Titan's orbit is avoided. This also represents an upper limit to the timescale of satellite accretion, which may be difficult to satisfy by previously proposed accretion mechanisms (e.g. Wetherill's (1980) mechanism of collisional accretion resulting from 'pumped-up' orbital eccentricities, which is even less efficient in the presence of drag-inducing gas). The question must also be raised as to whether efficient satellitesimal aggregation is viable in the presence of such strong turbulent overturning of the gas. It should be noted, finally, that the above strict timescales for nebula dispersal and satellite accretion may be relaxed somewhat if Titan is considered to be a captured body, as proposed by Prentice (1983). This is due to the fact that the corresponding value of the mass of the reconstructed nebula is reduced by an order or two of magnitude if Titan's contribution is ignored, leading to an increased timescale of tidal drift.

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Nebula Tides and Gap Formation

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Introduction

An intriguing problem in cosmogony concerns the ability of a planetoid embedded in a nebula disc to clear a gap around its orbit. The application of density wave theory to this problem has demonstrated that a significant exchange of angular momentum can take place between a planetoid and a disc (Goldreich and Tremaine 1980). The torque exerted by the disc on the planetoid can result in orbital drifting of the latter, which may play an important role in the aggregation process (Hourigan and Ward 1983). In fact, in the absence of significant deformation of the nebula, the radial orbital drift rate of a planetoid increases with planetoid mass. In this case, it would be expected that only one or two planetoids would sweep out the nebula, a situation not compatible with present observations. The orbital drift resulting from the generation of density waves therefore requires a limiting mechanism.

One possible resolution of this difficulty may in fact be supplied by the density waves themselves. In the absence of a damping mechanism, these waves transport angular momentum to regions of the disc far from the planetoid. The nebula material close to the planetoid is left relatively undisturbed. On the other hand, if strong local damping is present, angular momentum can be transferred from the density waves to the disc matter, leading to the clearance of a gap which stabilizes the planetoid's orbit (Hourigan and Ward 1984). However, a further dilemma is encountered if the gap is maintained during the process of viscous nebula dispersal. In this case, the orbital angular momentum of the planet is locked into the angular momentum transfer process of the disc and can result in destabilization of its orbit (Ward 1982).

The process of nebula truncation, or gap clearance, is therefore highly relevant to the problems of planetary aggregation and disc dispersal. In the present paper, mechanisms leading to an opposing gap formation are discussed.

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