

unionized gas,  $m_u$  is the mass of one molecule of the gas,  $m_g$  is the mass of a grain,  $r_g$  is the radius of a grain, and  $R$  is the radius of the cloud.

The larger  $B$ , the slower the gas responds to the grain motion;  $A$  affects only the density profile.

The solution of the equation of motion yield a finite collapse time

$$t_c = \left( \frac{r_g^2}{Gm_g} \right) \left( \frac{R}{2\pi\mu} \right)^{1/2} T^{1/2} \ln \left[ \frac{1 + \rho/\rho_g}{\times - 4\pi/(T\rho C)} \right]$$

where  $C = 128\pi GR r_g^4/(9\mu m_g^2)$

by which time all the grains have migrated to the centre of the cloud. (Here  $\rho$  is the initial gas density and  $\rho_g$  the initial space averaged grain density.)

The dynamical consequence of the existence of grains in the cool globules of the interstellar gas can be outlined as follows: The grains are rapidly accelerated to their terminal velocity and then for a time, migrate steadily toward their common centre of mass. As the grains collapse the gas cloud is "squeezed", but remains in approximate hydrostatic equilibrium at each stage. This quasi-static stage, however, does not persist indefinitely.

As the contracting grain cloud grows smaller in size, the increase in its mean density and velocity leads to a rapid increase in the drag force experienced by the gas. It was found that when the grain density reaches the critical value of  $B\rho_i/T^{1/2}$  (where  $\rho_i$  is the density of ionised gas), the gas interior to the grain cloud is "swept up", and begins to co-collapse with the grains. Outside the grain cloud the gas is left much compressed compared to its initial density.

The physical implications of this for stellar evolution may be briefly outlined as follows: It is apparent that the purely dynamic effect of the existence of grains in interstellar gas clouds will lead to the collapse of globules in a time much shorter than is generally appreciated. The evolution from globule to protostar may take as little as  $5 \times 10^7$  years in fairly ordinary cases ( $m_g/r_g^2 = 4 \times 10^{-4} \text{ gm cm}^{-2}$ ,  $T = 5\text{K}$ ,  $\rho/\rho_g = 100$ ) and less than  $10^7$  years if temperatures become low enough for  $\text{H}_2$  ice mantles to form on the grains ( $T \leq 3\text{K}$ ). The size of such protostellar objects is almost unlimited since the collapse time is independent of radius. However, it was noted that an upper limit of about  $10^6 M_\odot$  is implied by the possibility of evaporation of grains by friction.

Now, it was also found that the gas remains approximately stationary until the central grain density reaches about  $B\rho_i/T^{1/2}$ , which is usually much larger than the initial gas density, and then the gas interior to the grain distribution, previously resisted by the ionized component which was assumed frozen to the cosmic magnetic field, collapses extremely rapidly due to direct grain pressure, as well as the increased gravitational attraction of interior matter. Consequently the grain material will form a large proportion of the protostellar core.

Although the major part of the initial globule may eventually form the star, the retardation of the gas referred to above ensures that such a core forms first at the moment when the gas is swept up by the grains, and the gas and grains forming the core will thereafter evolve together.

Thus, any model based on grain/gas cloud collapse will yield

a protostellar core with an enhanced heavy element abundance, and a depleted helium abundance, since the grains contain much of the heavier elements and practically no helium.

When the rotation of the cloud is taken into account there arises the problem of having to find a method of shedding angular momentum. This was solved by Prentice and ter Haar (1971) who realized the grains would lose angular momentum as they collapse. It was shown that a fraction  $1 - S_1$  of the grain clouds angular momentum is lost, where  $S_1$  is the grain cloud radius at which the gas begins to collapse. My work leads to an estimate of  $S_1$  in exact agreement with Prentice and ter Haar; namely

$$\left( \frac{T^{1/2} \rho_g}{B \rho_i} \right)^{1/2}$$

Because the early phase of grain collapse is so much longer than the final collapse of the gas, grains of a given radius need only be very slightly more dense to arrive at the centre before most of the other grains and gas enter their final stage of collapse. Thus, within the protostellar core a chemical composition gradient will be formed with the denser elements (metals) extremely overabundant at the centre. This composition gradient, if it survives later evolution, as it well may, will require a change in stellar models. It has already been pointed out by Prentice (1973) that this may explain the low solar neutrino flux.

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## Numerical Experiments on Planetesimal Aggregation during the Formation of the Solar System

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### Introduction

A major area of difficulty in the cosmogony of the solar system is understanding how a large number of small planetesimals, which have condensed from the primordial gas, can aggregate into the ordered planetary system present today. Theories involving aggregation within a gaseous disc [e.g. Cameron (1973)] suffer the common difficulty that the particles, once condensed, are no longer supported by the



radial gas pressure gradient and spiral rapidly in towards the Sun. Most of the planetesimals are dragged in to the central body in times several orders of magnitude less than would be required for larger bodies to accrete (Goldreich & Ward 1973).

Alfven (1970) neglects gas drag effects but postulates that inelastic collisions will tend to focus the planetesimals into a concentrated toroid-shaped 'jetstream'. This would seem to be a necessary intermediate configuration of the bodies for subsequent planetary formation. Otherwise, the accretion time would be excessively long and a large fraction of small bodies would never have aggregated. A number of attempts to show a focusing effect due to inelastic collisions have been performed by Trulsen (1972) and Brahic (1975). However, no such effect has been obtained and, indeed, the opposite result has emerged.

Prentice (1974, 1978) has shown how a discrete system of gaseous rings may be detached from the collapsing solar nebular at the present orbital radii of the planets. In the following, the early stages of planetesimal aggregation within such a gaseous ring are investigated.

**Numerical Model**

The Earth's mass is divided up into spherical planetesimals, each of radius  $r_p = 4$  cm. and having mean Earth density, and spread around the Earth's orbit. To perform a numerical simulation of this system, only a limited number of bodies can be considered — in this case 100. To faithfully represent the actual planetesimal system by these few simulation bodies, the rate of collision per body must be retained. This is achieved by appropriate scaling of the radii of the simulation bodies in order to keep the product of particle number by geometrical cross section approximately constant.

The initial distribution of bodies has the form (cf. Trulsen (1972))

$$F(a, e, i) = (1.05 - a)(a - 0.95)(0.01 - e^2) \left(1 - \frac{\sin^2 i}{\sin^2 9^\circ}\right)$$

where  $a$  is semi-major axis in A.U.,  $i$  is inclination and  $e$  is eccentricity. This is shown in Fig. 1(a). The distribution in the azimuthal direction is taken to be random.

After each collision, the kinetic energy in the centre-of-mass frame of the colliding bodies is reduced by a factor  $\eta$  — the coefficient of inelasticity.

The structure of the gaseous ring detached at the Earth's orbit is given by Prentice (1974, 1978)

$$\rho_g(s, z) = \rho_0 \exp\left[\alpha\left(\frac{1}{r} - \frac{1}{2s^2} - \frac{1}{2}\right)\right]$$

where

$\rho_0$  is density on central circular axis

$s, z$  are radial and lateral cylindrical co-ordinates in A.U.'s

$$r = (s^2 + z^2)^{1/2}$$

$$\alpha \approx 4$$

The equation of motion for a body in between collisions is

$$\ddot{\mathbf{r}} = -\left(\frac{GM}{r^2}\right)\hat{\mathbf{r}} + \frac{\gamma}{m_p} |\mathbf{v}_g - \dot{\mathbf{r}}| (\mathbf{v}_g - \dot{\mathbf{r}})$$

where

$\gamma$  is drag coefficient for a sphere,  $\gamma = \pi \rho_g r_p^2$

$\mathbf{v}_g$  is gas velocity,  $\mathbf{v}_g = \left(\frac{H_0}{S}\right)\hat{\phi}$  ( $H_0 = \text{constant}$ )

$G$  is gravitational constant

$M$  is mass of Sun,  $m_p$  is mass of planetesimal.

Gravitational attraction between bodies was neglected.

The model was run for the cases  $\eta = 1, 0.5$  and  $0.1$  both for the case where  $\rho_0 = 10^{-8} \text{g-cm}^{-3}$  and also for no gas present.

A fourth-order Runge-Kutta integration scheme was used with 1000 steps per Earth year. A small ( $\approx 3\%$ ) correction to body cross section was included to compensate for collisions missed due to the finite step length.

**Results**

In the absence of gas, the results are similar to those obtained by Trulsen (1972) and Brahic (1975). These are shown in Figs. 1(d), 1(e).

For elastic collisions (i.e.  $\eta = 1$ ), there is a transfer of energy from the ordered motion to random motion. The eccentricities of the orbits increase and a large radial spread results, thereby destroying the initial thin stream configuration. As the degree of inelasticity increases, less energy is extracted from the ordered motion. However, since each collision now represents an energy sink, the energy of the system is steadily drained and nearly all of the bodies fall in towards the central body. A few bodies are left beyond the Earth orbit to conserve the system's total angular momentum. This picture is contrary to the observed situation.

Consider now Figs. 1(b), 1(c) where we have taken into account the presence of the gas from which the bodies originally condensed. As can be observed, the motions of the bodies off the central Keplerian axis of the ring are continuously being dissipated. The dispersive effects of collisions are now overwhelmed by the radial and lateral focusing effect of frictional drag due to the gas. The bodies migrate towards the central circular axis of the gaseous ring where the local Keplerian velocity matches that of the gas.

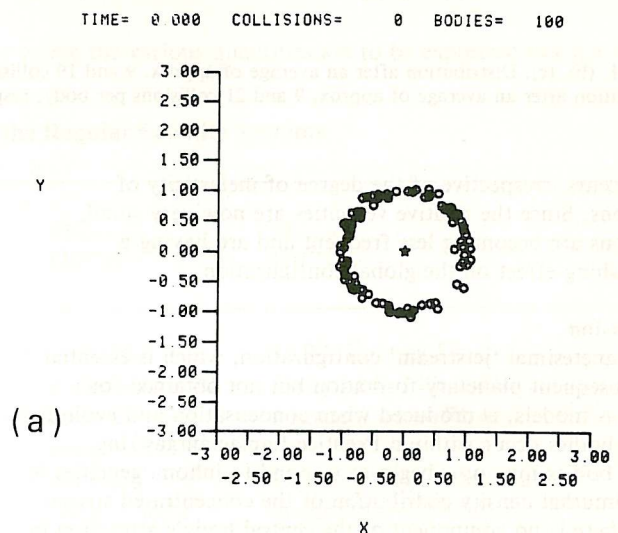


Figure 1. (a). Initial distribution of the bodies in the ecliptic.



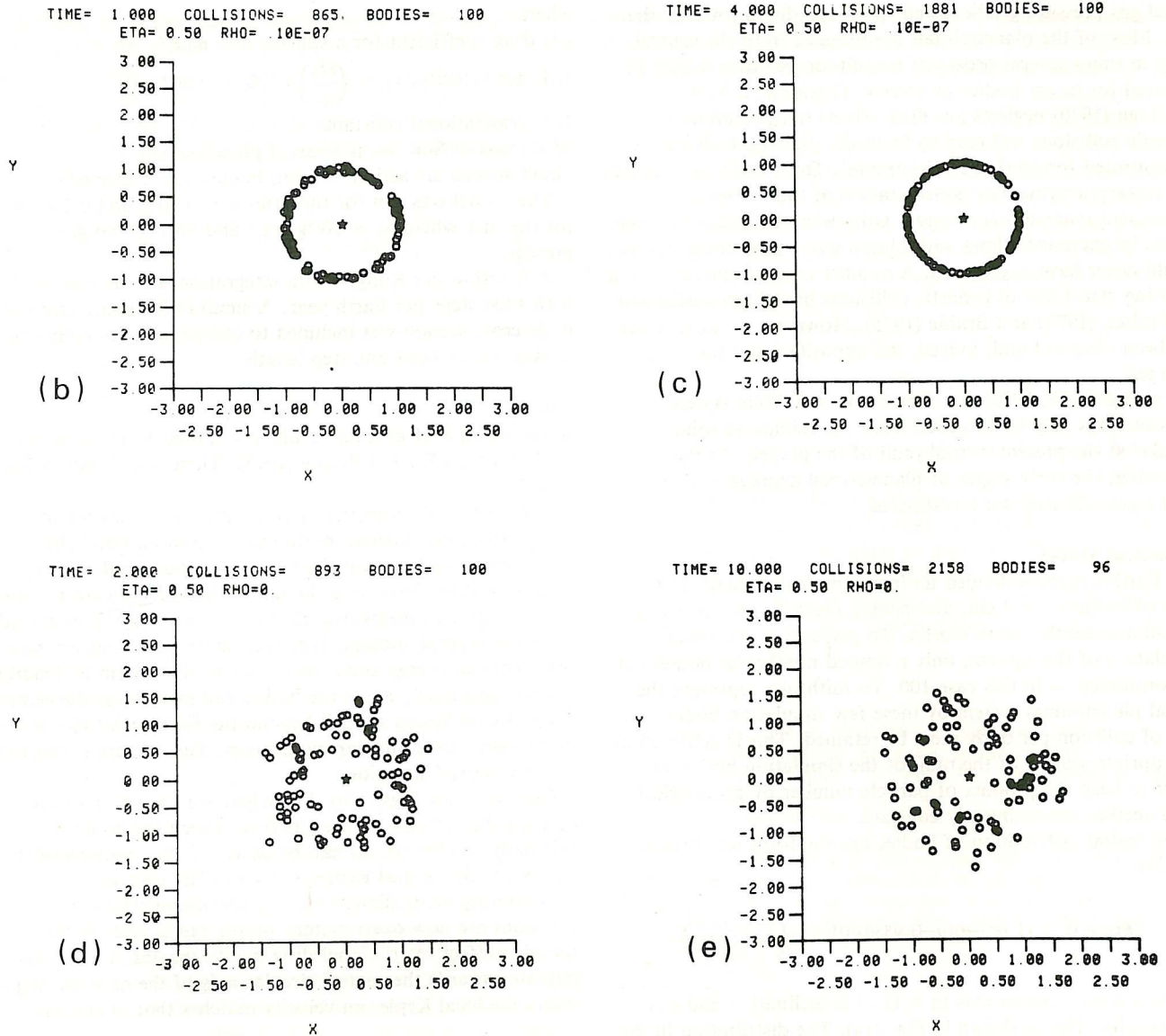


Figure 1. (b), (c). Distribution after an average of approx. 9 and 19 collisions per body, respectively, with gas present ( $\eta = 0.5$ ). (d), (e). Distribution after an average of approx. 9 and 21 collisions per body, respectively with no gas present ( $\eta = 0.5$ ).

This occurs irrespective of the degree of inelasticity of collisions. Since the relative velocities are now very small, collisions are becoming less frequent and are having a diminishing effect on the global configuration.

### Conclusion

The planetesimal 'jetstream' configuration, which is essential for subsequent planetary formation but not obtained for previous models, is produced when condensation and evolution of the bodies occur within a Prentice-Laplacian gas ring.

The bodies may now begin to respond to inhomogeneities in the azimuthal density distribution of the concentrated stream since there is no component of the central body's attraction in this direction. With the damping out of excess kinetic energy,

there may be a gradual migration of planetesimals towards these regions of higher density.

This subsequent azimuthal aggregation phase is currently being investigated numerically, taking into account the relatively small but important self-gravitational properties of the planetesimal stream.

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