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A TRAPPED WAVE MODEL FOR CONFINED VORTEX BREAKDOWN

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ABSTRACT

A trapped wave model for vortex breakdown, which has previously been considered for flow in confined pipes, is applied to breakdown of a torsionally driven cavity flow. A critical-wave model is shown to be applicable in this more physically complex situation.

INTRODUCTION

Vortex breakdown is said to occur when a concentrated vortex core changes suddenly in structure to produce a stagnant region of flow, often called a breakdown bubble. It has often been associated with a change from laminar to turbulent flow.

Since first being observed by Peckham (1957), many mechanisms have been put forward to explain vortex breakdown. Notable explanations include

1. Analogy with boundary layer separation e.g. Hall (1967)

2. A feedback mechanism generating vorticity e.g. Brown & Lopez (1990).

3. A transition from super-critical to sub-critical flow state e.g. Squire (1960), Benjamin (1962). This has been extended to include the idea of wave trapping e.g. Leibovich (1970), Darmofal (1994).

Darmofal used numerical results for swirling pipe flow to demonstrate that vortex breakdown can be seen as a change in state from a super-critical to sub-critical flow, and that trapping of small perturbations is responsible for the growth of vortex breakdown bubbles. With the exception of Brown and Lopez (1990), the previously mentioned papers have concentrated on flow through pipes. The purpose of this paper is to test the applicability of the ideas of the third mechanism to a confined torsionally driven cylinder flow.

The torsionally driven cavity apparatus, studied in detail by Escudier (1984), consists of a cylinder with fixed walls and roof, and a spinning base. The coordinate system used is shown in Figure 1. This apparatus has the advantage that there are only two free parameters, the Reynolds number (based on the velocity of the spinning base) and the aspect ratio of the cylinder. Compare this with the pipe apparatus, where inflow profiles and pipe profiles can vary greatly. The swirling cavity is also an appropriate experiment to study mixing problems which are an important application of vortex breakdown.

NUMERICAL TECHNIQUE

The Navier-Stokes equations are solved in cylindrical-polar coordinates for non-dimensionalised velocity components u_r, u_θ and u_z for $(0 \le r \le 1)$ and $(0 \le z \le H/R)$. The Reynolds number is defined such that the axis velocity $u_\theta = 1$ at $r = 1$, $z = 0$. The flow is assumed to be axisymmetric, which is is consistent with experimental evidence for the aspect ratios and Reynolds numbers considered in this work. A finite element code is employed, using quadrilateral velocity elements. A penalty formulation is used used to eliminate the pressure p, with the penalty parameter set to 10^7. Newton-Raphson iteration is performed on the resulting non-linear equations; the solution is considered converged when the maximum change in velocity is smaller than 10^{-7}. The problem is initially solved at low Reynolds numbers, with these solutions used as initial guesses for higher Reynolds number problems. The Reynolds number is stepped up in intervals of 100.

VORTEX BREAKDOWN

After solving the steady problem for a number of H/R and Re values, Figure 2 shows the bubble existence domain, which corresponds well with both the experimental results of Escudier (1984) and the previous numerical results of Lopez (1990). Note that unlike these studies, two merged bubbles is indicated as a single bubble in the existence graph.

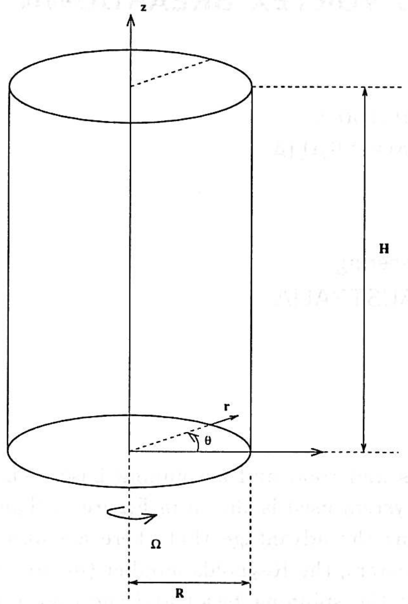


Figure 1:
Schematic diagram of experimental set-up.

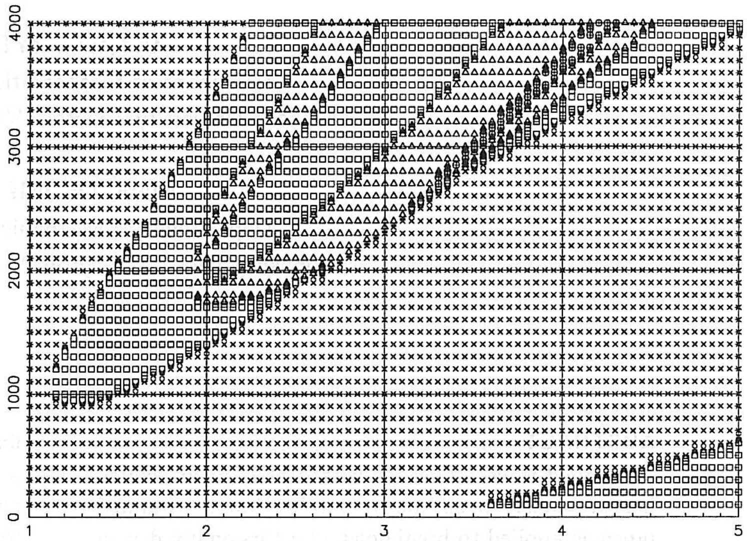


Figure 2:
Existence of breakdown bubbles
$\times$ **0 bubbles,** $\square$ **1 bubble,** $\triangle$ **2 bubbles,** $\oplus$ **3 bubbles**

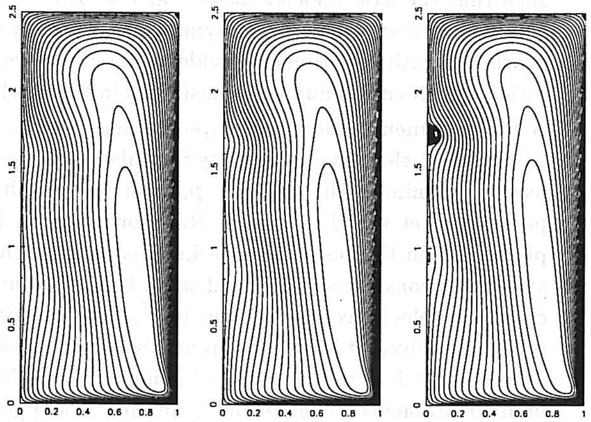


Figure 3
Stream function for breakdown $H/R = 2.5$
(a) Re = 1740, (b) Re = 1840, (c) Re = 1900

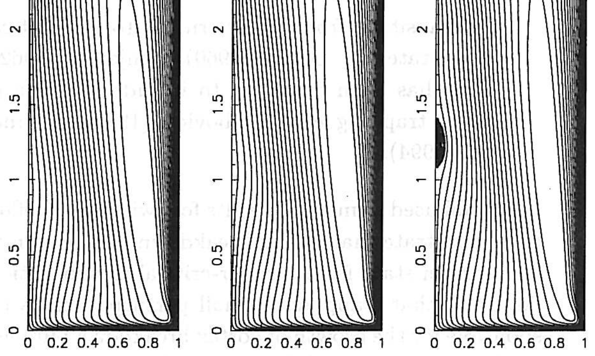


Figure 4
Stream function for breakdown $H/R = 4.0$
(a) Re = 2900, (b) Re = 3100, (c) Re = 3300

Two typical examples of the onset of breakdown in the confined flow as the Reynolds number is increased are shown in figures 3 and 4. These figures demonstrate the onset of vortex breakdown for two different aspect ratios. Contour levels of the stream-function ψ are displayed with 20 positive and 20 negative values, with intervals cubicly stretched towards zero.

WAVE MODEL

Considering a steady mean flow without breakdown with a velocity field $u = (U(r,z), V(r,z), W(r,z))$. The flow is assumed to be quasi-cylindrical i.e. $\frac{\partial U}{\partial r} \ll \frac{\partial U}{\partial z}$. If we consider an axial length scale L and a length scale for radial perturbations of λ, such that $\lambda/L \ll 1$, we can use the assumption of Hall that perturbations to the stream-function be defined as

$$\psi = f(r)\exp(\gamma z) \tag{1}$$

where γ is an axial wavenumber. The governing equations for the perturbation field (Hall, 1967) is

$$r\frac{d}{dr}\left(\frac{1}{r}\frac{\partial f}{\partial r}\right) + \left[\gamma^2 - \frac{r}{W}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial W}{\partial r}\right) + \frac{1}{r^3W^2}\frac{\partial \Gamma^2}{\partial r}\right] f = 0 \tag{2}$$

where $\Gamma = rV(r,z)$ is the mean flow circulation. The minimum local eigenvalue γ_0 at each axial node is calculated.

Darmofal used this model for the the pipe problem, and showed that the eigenvalues γ_0^2 of the system become negative just before breakdown occurs. It is argued that this represented a change in nature of the flow, from a super-critical flow with information traveling downstream only, to a sub-critical flow in which information (and in particular small perturbations) could move back up stream and be trapped. The point of this trapping is said to coincide with the breakdown point.

To extend this idea to the confined cylinder it is necessary to choose a region of the flow that is similar to a pipe, and hence meets the necessary requirements for the use of equation (3). The region $0 \leq r \leq \frac{1}{2}$ and $.1 \leq z \leq H - .1$ is used to define the base flow. At any given Reynolds number, a finite element approximation to equation 3 is solved using the LAPACK routine DGEEVX at each axial position z.

RESULTS

Figures 5 and 6 indicate the axial velocities and eigenvalues γ_0^2 of the flow for two different aspect ratios. It can be seen that the flow becomes sub-critical before the flow has broken down, but that the physical location of the breakdown points correspond well with the location of vortex breakdown bubbles, which appear for $-w(0,z) < 0$.

Figure 7 shows the minimum axial velocities and eigenvalues as the Reynolds number is increased. It is apparent that the flow becomes sub-critical at a Reynolds lower than that required for flow reversal to occur. This is consistent with the results of Darmofal in the pipe apparatus. Note that when $\gamma_0^2 = 0$, upstream perturbations of infinite wavelength can be trapped. The length of wave perturbation possible is given by Hall as $L = 2\pi/\sqrt{-\gamma_0^2}$. Work is currently underway to determinate a value for γ_0^2 at breakdown, and hence the shortest wavelength that is predicted to be trapped.

If Figure 7 is compared with the stream-functions in Figures 3 and 4, the three plots correspond to (a) positive eigenvalues without breakdown, (b) negative eigenvalues with breakdown and (c) negative eigenvalues with breakdown. In the second case, a large amount of waviness is evident in the stream-functions, without breakdown having occurred.

CONCLUSION

The wave trapping models originally put suggested by Squire, and further enhanced by Hall, Leibovich and Darmofal have successfully been applied to a torsionally driven cavity. The success of this model in a very different physical domain to that normally considered suggests that the wave trapping model provides a robust explanation for vortex breakdown.

REFERENCES

BENJAMIN, T., "Theory of vortex breakdown phenomenon", *J Fluid Mechanics,* **14,** 593-629, 1962

BROWN, G.L. and LOPEZ, J.M., "Axisymmetric vortex breakdown Part 2. Physical mechanisms", *J. Fluid Mechanics,* **221,** 553-576, 1990

DARMOFAL, D.L. and MURMAN, E.M., "On the Trapped Wave Nature of Axisymmetric Vortex Breakdown", *AIAA Paper,* 94-2318, 1994

ESCUDIER, M.P., "Observations of the flow produced in a cylindrical container by a rotating endwall", *Experiments in Fluids,* **2,** 189-196, 1984

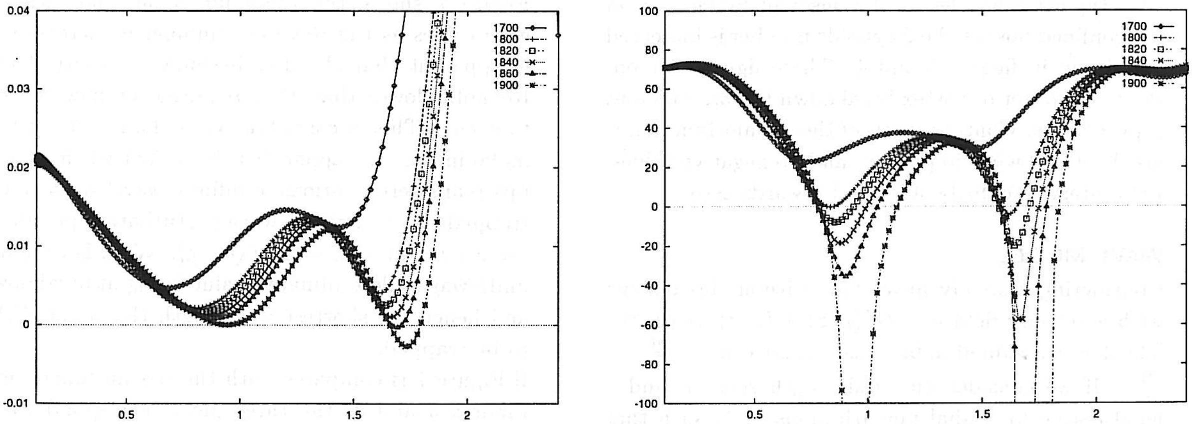
HALL, M., "A new approach to vortex breakdown.", In *Proc. 1967 Heat Transfer Fluid Mech. Inst.,* 319-340, 1967

LEIBOVICH, S., "Weakly non-linear waves in rotating fluids", *J. Fluid Mechanics,* **42(4),** 803–822, 1970

LOPEZ, J., "Axisymmetric vortex breakdown part 1. confined swirling flow.", *J. Fluid Mechanics,* **221,** 533-552, 1990

PECKHAM, D.H. and Atkinson, S.A., "Preliminary results of low speed wind tunnel tests on a Gothic wing of aspect ratio 1.0", *Aeronaut. Res Counc.,* CP 508, 1957

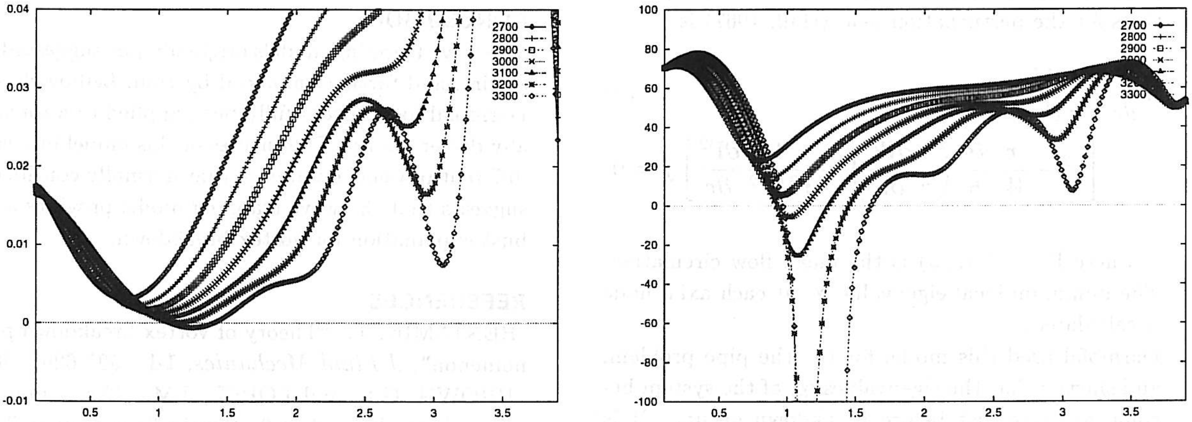
SQUIRE, H.B., "Analysis of the 'vortex breakdown' phenomenon. Part 1.", *Aero. Dept., Imperial Coll., London* Tech. Report 102, 1962



(a) (b)

Figure 5

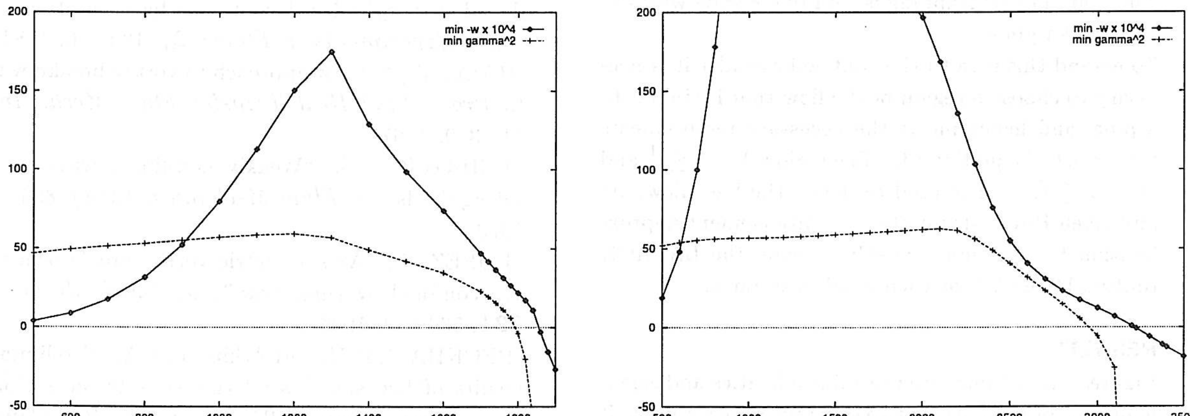
(a) Axial velocities $-w(0,z)$ and (b) Minimum eigenvalues $\gamma_0^2(z)$ for $H/R = 2.5$



(a) (b)

Figure 6

(a) Axial velocities $-w(0,z)$ and (b) Minimum eigenvalues $\gamma_0^2(z)$ for $H/R = 4.0$



(a) (b)

Figure 7

Minimum $-w(0,z)$ and eigenvalues γ_0^2 for (a) $H/R = 2.5$ (b) $H/R = 4.0$