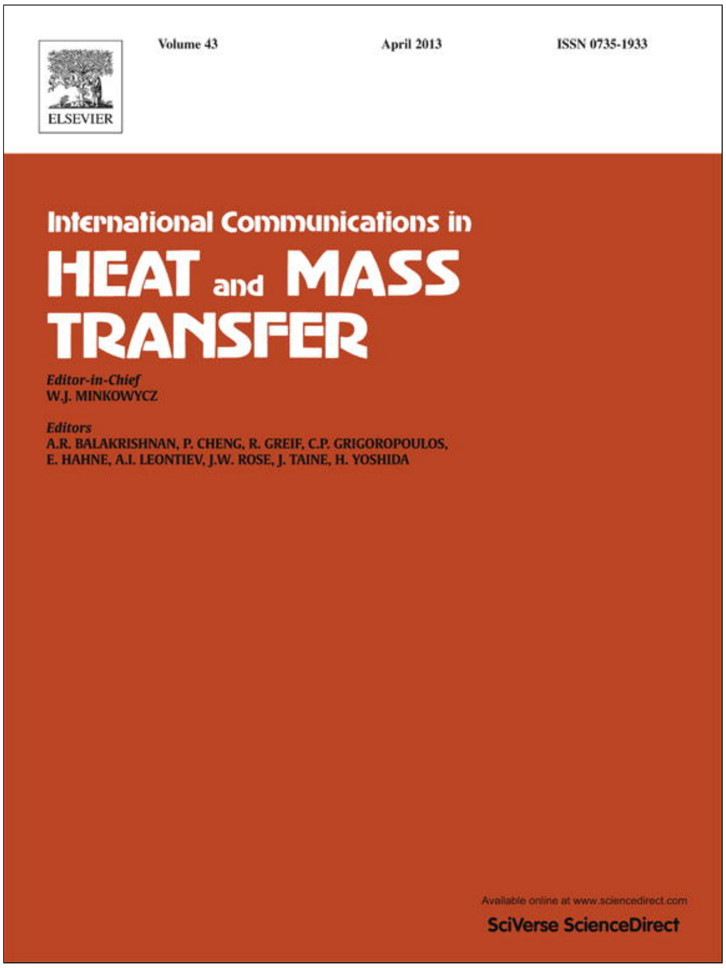
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Validation of thermal equilibrium assumption in forced convection steady and pulsatile flows over a cylinder embedded in a porous channel☆

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ABSTRACT

The validity of the local thermal equilibrium assumption in the forced convection steady and pulsatile flows over a circular cylinder heated at constant temperature and embedded in a horizontal porous channel is investigated. For this purpose, the Darcy–Brinkman–Forchheimer momentum and the local thermal non-equilibrium energy models are solved numerically using a spectral element method. Numerical solutions obtained over broad ranges of representative dimensionless parameters are utilized to present conditions at which the local thermal equilibrium assumption can or cannot be employed. For steady flow, the circumstances of a higher Reynolds number, a higher Prandtl number, a lower Darcy number, a lower microscopic and macroscopic frictional flow resistance coefficient, a lower Biot number, a lower solid-to-fluid thermal conductivity ratio, a lower cylinder-to-particle diameter ratio and a lower porosity, are identified as unfavorable circumstances for the local thermal equilibrium *LTE* condition to hold. For oscillatory flow, the degree of non-equilibrium can be decreased as pulsating amplitude increases or pulsating frequency decreases; however, such flows do not have a significant influence on satisfying the *LTE*.

1. Introduction

Forced convection heat transfer in porous media has been extensively investigated. This interest stemmed from the variety of engineering applications associated with the presence of a porous medium in the path of the working fluid. Excellent review articles and monographs have been provided by, for example, Vafai [22], Pop and Ingham [17] and Nield and Bejan [14]. In the literature, two models are adopted to describe the thermal behavior of porous systems. These are the so-called the single-phase and the two-phase Schumann models. The main distinction between these models is that local thermal equilibrium (*LTE*) is assumed in the former model while no such assumption is made in the later model. Therefore, the single-phase model reduces to one governing energy equation, whereas in the two-phase model which is often called the (*LTNE*) model, there are two governing energy equations with each of them possessing an inter-phase heat transfer term.

Depending on the nature of the transient process and the thermo-physical properties of the individual phases, the phase temperatures can be different. In several industrial applications, see Nield and Kuznetsov [15], such as nuclear rods placed in a coolant fluid bath and thermal energy storages in underground reservoirs, the temperature difference between the local fluid and solid phases is essential and the transport process is inherently unsteady, hence, the performance of the device depends on the degree of non-equilibrium between the two phases. More and more studies, for example by Vafai and Sözen [23] and Kaviany [8], have found the *LTE* assumption invalid for a number of applications involving convection in porous media. They mentioned that when there is a significant difference between advection and conduction mechanisms in transferring heat, the deviation between the solid and fluid phase temperatures increases, and therefore the *LTE* model becomes progressively less valid. It was also noted by Pop and Cheng [16] that this approach may be questionable when the particle size in the solid porous matrix is comparable to or exceeds the thermal boundary layer thickness. In addition, Quintard [18] argued that assessing the validity of the assumption of *LTE* is not a simple task since the temperature difference between the two phases cannot be easily estimated. He suggested that the use of the *LTNE* model is the possible approach to solving the problem.

By using the *LTNE* model, much research has been conducted to determine the applicable region of the *LTE* model for forced convection through porous media. Whitaker and his co-workers, Carbonell and Whitaker [4], Whitaker [25], Quintard and Whitaker [19] and Quintard and Whitaker [20], have performed a pioneering work in this regard. Based on the order of magnitude analysis, they proposed a criterion in the case where the effect of conduction is dominant in porous channel. Lee and Vafai [12] presented a practical criterion for the case of Darcian flow in channel subjected to a constant heat flux. Later, Kim and Jang [10] presented a general criterion for the *LTE* expressed in terms of

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Nomenclature

A	dimensionless oscillating amplitude of axial inlet velocity.
a_{sf}	specific interfacial area (m^{-1}).
Bi	Biot number, $Bi = h_{sf} a_{sf} D_{cy}^2 / k_s$.
d_p	particle diameter (m).
D_{cy}	cylinder diameter (m).
Da	Darcy number, $Da = K/D_{cy}^2$.
$\acute{f}$	pulsating frequency (Hz).
C_F	inertial coefficient, $C_F = 1.75/\sqrt{150\varepsilon^3}$.
h_{sf}	interfacial heat transfer coefficient ($W/m^2 \cdot K$).
H	channel height (m).
k	thermal conductivity, ($W/m \cdot K$).
k_r	solid/fluid thermal conductivity ratio, $k_r = k_s/k_f$.
K	permeability of the porous medium, $K = \varepsilon^3 d_p^2 / 150(1-\varepsilon)^2$ (m^2).
L	channel length (m).
P_f	dimensionless fluid pressure, $P_f = \acute{P}_f / \rho_f \acute{u}_o^2$.
Pr	Prandtl number, $Pr = \nu_f / \alpha_f$.
Re_D	Reynolds number, $Re_D = \acute{u}_o \rho_f D_{cy} / \mu_f$.
St	dimensionless oscillating frequency parameter, Strouhal number, $\acute{f} D_{cy} / \acute{u}_o$.
$\acute{t}$	time (s).
t	dimensionless time, $t = \acute{t} \acute{u}_o / D_{cy}$.
$\acute{T}$	temperature (K).
$\mathbf{u}$	dimensionless vectorial fluid velocity, $\mathbf{u} = \acute{\mathbf{u}} / \acute{u}_o$.
u	dimensionless horizontal velocity component.
$\acute{u}_o$	inlet horizontal fluid velocity (m/s).
v	dimensionless vertical velocity component.
$\acute{x}, \acute{y}$	horizontal and vertical coordinates (m).
x, y	dimensionless horizontal and vertical coordinates, $x = \acute{x}/D_{cy}$, $y = \acute{y}/D_{cy}$.

Greek symbols

α_r	thermal diffusivity ratio, $\alpha_r = \alpha_s / \alpha_f$.
ε	porosity.
θ	dimensionless temperature, $\theta = \left(\acute{T} - \acute{T}_o\right) / \left(\acute{T}_h - \acute{T}_o\right)$.
μ_f	fluid dynamic viscosity ($kg/m \cdot s$).
ρ_f	fluid density (kg/m^3).
ν_f	fluid kinematic viscosity (m^2/s).
φ	angular coordinate (°).

Subscripts

eff	effective.
f	fluid.
o	inlet of the channel.
s	solid.

important engineering parameters such as Darcy, Prandtl and Reynolds numbers. Their criterion which was checked to be applied for conduction and/or convection heat transfer in porous media, is more general than the previous one suggested by Whitaker and his co-workers above. Nield [13] clarified the circumstances under which the *LTNE* may be important in a saturated porous channel using an analytical approach for phase temperature fields and Nusselt number. The applicability of the *LTE* model for the micro-channel sink, which was modeled as a porous medium, was also analytically assessed by Kim et al. [11] for the case where the bottom surface is uniformly heated by constant heat flux and the top surface is insulated. Numerically, Vafai and Sözen [23] assessed the validity of the *LTE* assumption for a forced convective gas flow through a packed bed of uniform spherical particles by presenting an error contour maps based on qualitative ratings for three types of materials; lithium–nitrate–trihydrate, sandstone and steel. Following, Amiri and Vafai [3] examined this validity in a packed bed confined by walls at constant temperature, using the same qualitative error maps used by Vafai and Sözen [23]. Khashan and Al-Nimr [9] presented quantitative *LTE–LTNE* region maps for the problem of non-Newtonian forced convective flow through channel filled with porous media for a broad range of representative dimensionless parameters such as Péclet number, Biot number, power-law index, fluid/solid thermal conductivity ratio and microscopic and macroscopic frictional flow resistance coefficients, to examine whether the *LTE* assumption can or cannot be employed. Celli and Rees [5] analyzed analytically and numerically the effect of the *LTNE* on forced convective thermal boundary layer flow in a saturated porous medium over a horizontal flat plate. This effect was found to be at its strongest close the leading edge with $O(x^{-1})$ in magnitude, but the maximum discrepancy between the temperature fields decreases with the distance from the leading edge, and the *LTE* condition is attained at large distance.

The current literature reveals that the assessment of the validity of the *LTE* assumption and the corresponding one-phase energy model in forced convection regime has been surprisingly restricted to only the application of porous channel or flat plate, with far fewer numerical than analytical work. Therefore, the present study aims to examine this validity for forced convective steady and pulsatile flows over heated bodies of high complexity, such as a circular cylinder, embedded in a porous medium. The departure from local thermal equilibrium is to be captured by solving the problem numerically using the *LTNE* model that incorporates the effect of thermal dispersion. A Darcy–Brinkmann–Forchheimer (*DBF*) model that incorporates non-Darcian effects, i.e., including the solid boundaries and inertia effects, is adopted to accurately predict the hydrodynamic behavior.

2. Mathematical formulation

The analysis is carried out for steady and oscillatory two-dimensional forced convective laminar flows over a circular cylinder embedded in a homogenous and isotropic porous medium confined by two impermeable horizontal walls as shown in Fig. 1. The cylinder is assumed to be heated at constant temperature $\acute{T}_h$ and cooled by the incoming external flow at $\acute{T}_o$. The channel walls are considered to have the same

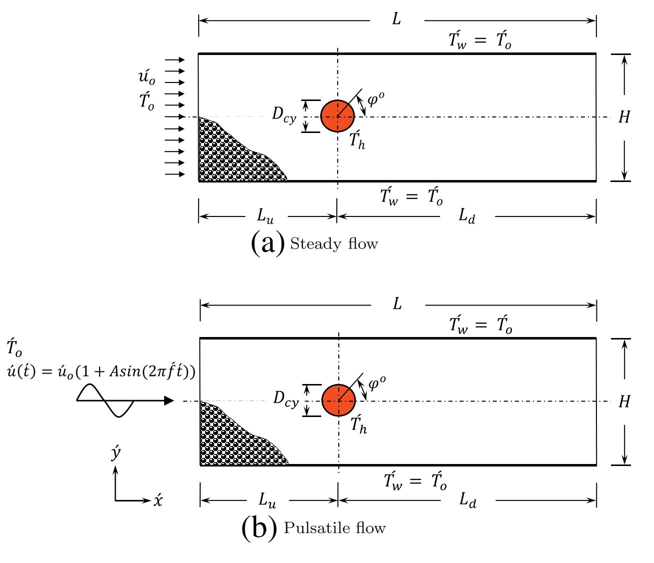

Fig. 1. Flow configurations.

temperature $\acute{T}_w$ as the flow at the inlet. The blockage ratio of the channel is $D_{cy}/H = 0.25$, where D_{cy} and H are the cylinder diameter and the channel height, respectively.

The transient governing equations of the average-volume mass, *DBF* momentum, and *LTNE* energy which are presented in Nield and Bejan [14] and Kaviany [8], are transformed into the following vectorial non-dimensional form employing the dimensionless variables given in the nomenclature:

$$\nabla\cdot\langle\mathbf{u}\rangle = 0, \tag{1}$$

$$\frac{\partial\langle\mathbf{u}\rangle}{\partial t} + \frac{1}{\varepsilon}\langle(\mathbf{u}\cdot\nabla)\mathbf{u}\rangle = -\frac{\varepsilon}{Re_D\cdot Da}\langle\mathbf{u}\rangle - \frac{C_F\varepsilon}{\sqrt{Da}}|\langle\mathbf{u}\rangle|\langle\mathbf{u}\rangle + \frac{1}{Re_D}\nabla^2\langle\mathbf{u}\rangle - \varepsilon\nabla\left\langle P_f\right\rangle, \tag{2}$$

$$\frac{\partial\left\langle\theta_f\right\rangle}{\partial t} + \langle\mathbf{u}\rangle\cdot\nabla\left\langle\theta_f\right\rangle = \frac{1}{\varepsilon Re_D Pr}\nabla\cdot\left[\frac{k_{f\cdot eff,(x,y)}}{k_f}\nabla\left\langle\theta_f\right\rangle\right] + \frac{Bi\cdot k_r}{\varepsilon Re_D Pr}\left(\langle\theta_s\rangle - \left\langle\theta_f\right\rangle\right), \tag{3}$$

$$\frac{\partial\langle\theta_s\rangle}{\partial t} = \frac{\alpha_r}{(1-\varepsilon)Re_D Pr}\nabla\cdot\left[\frac{k_{s\cdot eff}}{k_s}\nabla\langle\theta_s\rangle\right] - \frac{Bi\cdot\alpha_r}{(1-\varepsilon)Re_D Pr}\left(\langle\theta_s\rangle - \left\langle\theta_f\right\rangle\right), \tag{4}$$

where, $\langle\mathbf{u}\rangle$ is the velocity vector field, t is the time variable, $\langle P_f\rangle$ is the pressure field and $\langle\theta\rangle$ is the temperature field, C_F is the inertial coefficient, and the operator $\langle\ldots\rangle$ denotes local volume average of a quantity. Also, k_r and α_r are the solid/fluid thermal conductivity and diffusivity ratios, k_s/k_f and α_s/α_f, respectively. Subscripts f and s denote the fluid and solid phases, respectively. In addition, the key governing non-dimensional groups: the Reynolds, Darcy, Prandtl, and Biot numbers are defined respectively, as:

$$Re_D = \frac{\acute{u}_o D_{cy}\rho_f}{\mu_f}, \quad Da = \frac{K}{D_{cy}^2}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad \text{and } Bi = \frac{D_{cy}^2 h_{sf} a_{sf}}{k_s}, \tag{5}$$

here, the primes refer to dimensional quantities. The following are other fluid and medium properties: μ_f and ν_f are the fluid dynamic and kinematic viscosities, respectively, ρ_f is the fluid density, K is the permeability, a_{sf} is the specific surface area of the packed bed and h_{sf} is the interfacial heat transfer coefficient. The permeability of the porous medium K and the inertial coefficient C_F in the momentum equation are based on the Ergun's experimental investigation [6] who expressed them in terms of porosity ε and particle diameter d_p as follows:

$$K = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2}, \quad C_F = \frac{1.75}{\sqrt{150\varepsilon^3}}. \tag{6}$$

The effective thermal conductivity of the fluid phase emerges as a combination of the conductivities of two constituents; a stagnant component k_{st} and a dispersion component k_d,

$$k_{f\cdot \mathrm{eff},(x,y)} = k_{st} + k_{d(x,y)}. \tag{7}$$

In the ongoing investigation, the stagnant component is based on the experimental findings of [27] as follows:

$$\frac{k_{st}}{k_f} = \left(1-\sqrt{1-\varepsilon}\right) + \frac{2\sqrt{1-\varepsilon}}{1-\lambda B}\times\left[\frac{(1-\lambda)B}{(1-\lambda B)^2}\ln(\lambda B) - \frac{B+1}{2} - \frac{B-1}{1-\lambda B}\right], \tag{8}$$

where $\lambda = 1/k_r$ and $B = 1.25[(1-\varepsilon)/\varepsilon]^{\frac{10}{9}}$. Whereas, the dispersion conductivity is determined in both longitudinal and transverse directions based on the experimental correlation reported by [24] given by:

$$\frac{k_{dx}}{k_f} = 0.5Pr\left(\frac{\rho_f|\langle\mathbf{u}\rangle|d_p}{\mu_f}\right), \quad \frac{k_{dy}}{k_f} = 0.1Pr\left(\frac{\rho_f|\langle\mathbf{u}\rangle|d_p}{\mu_f}\right). \tag{9}$$

The solid effective thermal conductivity has a stagnant component only since it is stationary:

$$k_{s\cdot \mathrm{eff}} = (1-\varepsilon)k_s. \tag{10}$$

In accordance with the problem description, the Neumann boundary conditions are imposed on the inlet and solid boundaries, while the Dirichlet boundary conditions are imposed on the outlet. The dimensionless initial and boundary conditions are presented as:

$$\begin{aligned}
&\text{at } t = 0: u = v = \theta_f = \theta_s = 0 \\
&\text{at } t > 0: u = 1 \quad (\text{for steady flow}), \\
&u(t) = 1 + A\sin(2\pi Stt) \quad (\text{for pulsatile flow}), \\
&v = \theta_f = \theta_s = 0 && \text{at } (x = 0, 0<y<H) \\
&\frac{\partial u}{\partial x} = \frac{\partial\theta_f}{\partial x} = \frac{\partial\theta_s}{\partial x} = v = 0 && \text{at } (x = L, 0<y<H) \\
&u = v = 0 = \theta_f = \theta_s = 0 && \text{at } (0<x<L, y = 0 \text{ and } H) \\
&u = v = 0, \theta_f = \theta_s && \text{at cylinder boundary,}
\end{aligned} \tag{11}$$

where A is the pulsating flow amplitude of the axial inlet velocity, and $St = \acute{f}D_{cy}/\acute{u}_o$ is the dimensionless pulsating frequency parameter (Strouhal number). It is assumed that the fluid and solid phases share the same temperature as that of the isothermal cylinders surface, i.e. thermal equilibrium assumption is imposed on the heated boundary conditions, Alazmi and Vafai [2] and Wong and Saeid [26].

3. Numerical method of solution

A spectral-element method, which is essentially a high-order finite-element method, was used to numerically solve the non-dimensional system of governing Eqs. (1)–(4). For brevity, this method is described in detail in Karniadakis et al. [7] and Thompson et al. [21], and was used in our work Al-Sumaily et al. [1]. In order to validate the implementation, results from our code were compared to previously published numerical results for the problem of a cooling air jet impinging on an isothermal heated surface immersed in a confined porous channel under the *LTNE* condition performed by Wong and Saeid [26]. Fig. 2 shows this comparison for the *LTNE* parameter, defined as:

$$LTNE = \frac{\Sigma_N\left|\theta_s - \theta_f\right|}{N}, \tag{12}$$

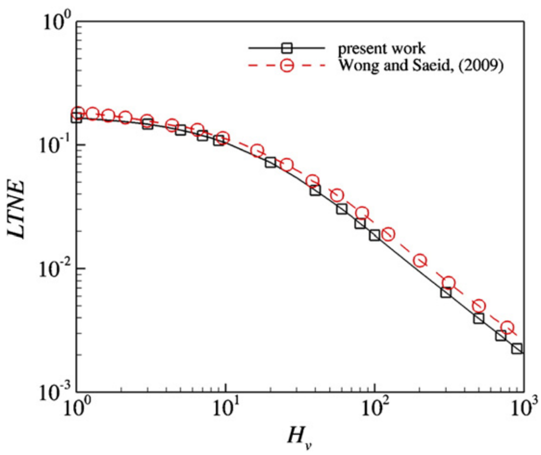

Fig. 2. Comparison between two numerical models: the present model and the model used by Wong and Saeid [26], for the variation of the *LTNE* parameter against the dimensionless interfacial convective heat transfer coefficient H_v, at Péclet number $Pe = 100$ and porosity-scaled fluid-to-solid thermal conductivity ratio $\gamma = 1.0$.

where N is the total number of nodes in the domain. This is a simple measure of the mean difference between the fluid and solid over the domain. Excellent agreement between the two numerical results can be seen in this figure.

4. Results and discussion

The validity of the *LTE* assumption and the corresponding one-equation energy model was tested in this parametric study under various flow conditions and porous medium thermal and structural properties. This was achieved by introducing a number of dimensionless groups such as Reynolds number $1.0 \leq Re_D \leq 250$, Prandtl number $Pr = 0.7$, 7.0 and 100, solid-to-fluid thermal conductivity ratio $0.01 \leq k_r \leq 1000$, Biot number $0.01 \leq Bi \leq 100$, cylinder-to-particle diameter ratio $100 \leq D_{cy}/d_p \leq 10$ and porosity $0.5 \leq \varepsilon \leq 0.95$ for steady non-pulsating flow, and dimensionless oscillating flow amplitude $0.2 \leq A \leq 3.0$ and Strouhal number $0.1 \leq St \leq 2.0$ for pulsatile flow. The value of the solid-to-fluid thermal diffusivity ratio α_r is fixed on unity throughout the entire study. The degree of non-equilibrium is calculated by using the *LTNE* parameter defined in Eq. (12) and used by Wong and Saeid [26]. In essence, to validate the *LTE* assumption, the local temperature difference between the fluid and solid phases should be negligibly small. Thus, the *LTE* condition is declared to be satisfied when the *LTNE* parameter becomes less than 0.05.

4.1. Steady flow

Fig. 3 shows the results of the effect of Reynolds number Re_D on the *LTNE* parameter at different values of thermal conductivity ratio k_r, and on isotherms of the local temperature difference between the fluid and solid phases inside the porous channel and around the heated cylinder. These results are obtained at $Pr = 7.0$ for water as a working fluid, $Bi = 1.0$, $D_{cy}/d_p = 20$ and $\varepsilon = 0.5$. It is clear from this figure that the decrease in Re_D causes the *LTNE* to decrease, and approaching towards thermal equilibrium between the two phases. The elevation of Re_D values corresponds to high speed flows and subsequently higher thermal convection heat transfer rates. Also, the effect of increasing Re_D value seems insignificant at the low k_r value of less than unity. The energy transport by the solid phase is quite high at this stage which suppresses the sensitivity to Re_D variation. As the magnitude of k_r value is increased; however, the significance of Re_D variation becomes noticeable. This is likely attributed to the increase in the solid effective thermal conductivity over its fluid counterpart for $k_r > 1$. The isotherms which are obtained at $k_r = 100$ show that the domain of the local *LTNE* condition appears to extend over the cylinder from the upstream towards the downstream direction with the increase in Re_D value. Apparently, higher speed flows do not allow sufficient energy communication between the phases to retain *LTE* status in the vicinity of the cylinder and in the downstream region of the porous channel.

Next, the implications of varying the Re_D on the *LTNE* condition for $k_r = 1.0$ and $Bi = 1.0$ using different fluids, i.e. air with Prandtl number $Pr = 0.7$, water with $Pr = 7.0$ and engines oil with $Pr = 100$, is appraised in Fig. 4. It is clear from the fluid energy equation that a higher Pr has an unfavorable effect on the *LTE* condition. As Pr increases, the fluid temperature θ_f will feel the hot boundary condition less than the solid porous domain feels it, so there is no way that $\theta_f = \theta_s$. Physically, a higher Prandtl number refers to the condition at which a fluid with low thermal diffusivity is being carried through the porous channel at a higher flow rate. This condition makes the fluid less capable of receiving heat from the heated surface of the cylinder and exchanging this heat to the solid phase in an efficient manner. Although with a lower Re_D, the fluid has more time to exchange energy with the solid domain and attain its temperature closer to the solid temperature which leads to satisfy *LTE*, this has no considerable influence on fluids with high Pr, as can be seen in Fig. 4 at $Re_D = 1.0$ and $Pr = 100$.

The results of the *LTNE* against the thermal properties, i.e. solid/fluid thermal conductivity ratio k_r and interfacial heat transfer coefficient represented by Biot number Bi, respectively, of the porous medium are presented in Figs. 5 and 6, respectively, at $Pr = 7.0$, $D_{cy}/d_p = 20$, $\varepsilon = 0.5$, and for different values of $Re_D = 10$, 40, 100 and 250. From these figures, in addition to the increase of Re_D value, one may conclude that the *LTNE* condition can also be triggered by lower values of both k_r and Bi, and the temperature difference between both phases is relatively large. This is attributed to the degrading impact in which they both imply a low energy transport capability by the fluid as compared to its solid counterpart. As a result, the fluid exhibits poor ability to bring the solid phase to its operating temperature. Fig. 5 depicts the unfavorable effect of low values of k_r, at constant $Bi = 1.0$, on the *LTE* condition. It is clear that a lower k_r as opposed to a higher k_r, puts much constraint on the validity of the *LTE*. This is because the fluid conduction dominates the heat transfer process in the porous medium and the solid temperature is much more influenced by the degree of the internal heat exchange between the fluid and solid phases rather than influenced by its own conductivity. However, the increase in k_r reduces the value of the *LTNE* leading towards thermal equilibrium between the solid and fluid phases at high k_r. Higher k_r implies that we have more solid having high thermal conductivity compared with less fluid having low thermal conductivity. This enables the solid to bring the fluid phase to its temperature easier. As can be seen, satisfying the *LTE* condition by increasing k_r depends strongly on the value of Re_D. For example at $Re_D = 10$, this condition can be satisfied for $k_r \geq 60$ while at $Re_D = 100$ it is justified for $k_r \geq 1000$, and the reason is

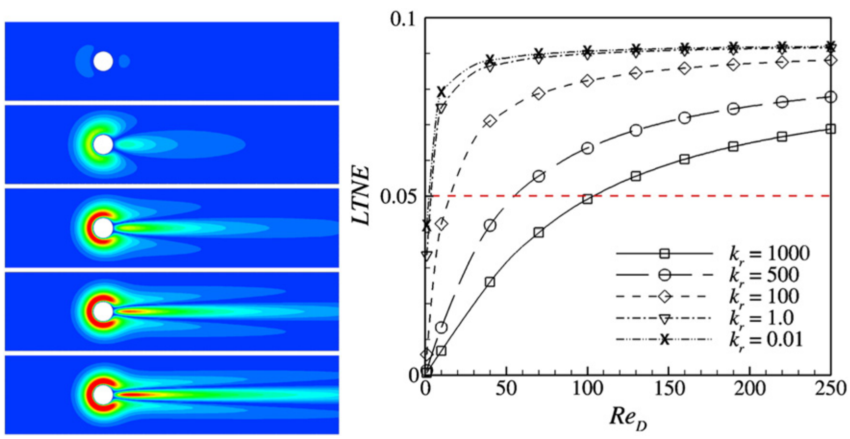

Fig. 3. Effect of Reynolds number on: (left) the local temperature difference between the fluid and solid phases around a heated cylinder embedded in a porous medium, from top to bottom $Re_D = 1, 10, 40, 100$ and 250, at $k_r = 100$, with red (blue) colors have magnitudes of 0.3(0.0), and (right) the *LTNE* parameter versus Re_D for different values of k_r, and at $Bi = 1.0$.

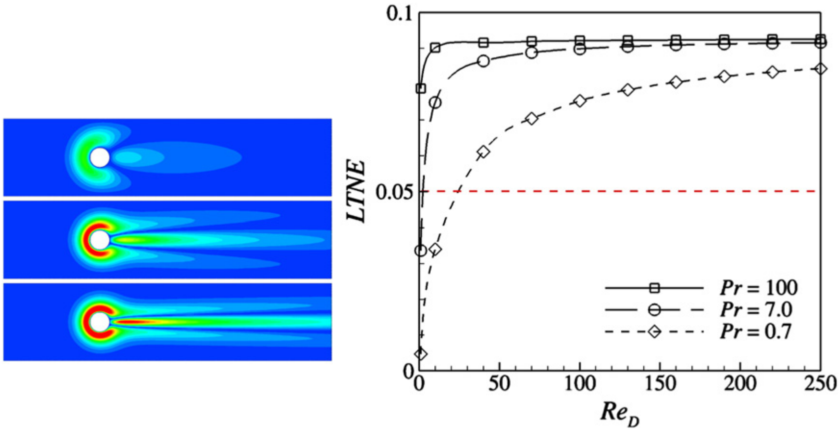

Fig. 4. The influence of using different fluids (Prandtl number) on: (left) the local *LTNE* condition, from top to bottom $Pr=0.7, 7.0$ and 100, at $Re_D=10$, with red (blue) colors have magnitudes of 0.3(0.0), and (Right) the *LTNE* parameter versus Re_D for three types of fluids, at $k_r=1.0$ and $Bi=1.0$.

mentioned above. High k_r values are common in most engineering applications for heat transfer enhancement given that the solid thermal conductivity is typically several folds larger than its fluid counterpart.

Furthermore, a lower *Bi* value implies that the thermal resistance associated with the interstitial convective heat exchange between the two phases is larger than the effective conductive resistance in the solid domain. On the contrary, the increase of *Bi* value, which implies an increased specific area for heat exchange, improves the energy communication between the two considered phases. This causes θ_f to approach θ_s at a faster pace and satisfy the thermal equilibrium condition in the porous system. It is interesting to note that the influence of the parameter *Bi* on the equilibrium condition which is found to be secured at moderate values of $1.0 \le Bi \le 5.0$, and on the *LTNE* status which is observed to be very strong at smaller values of $Bi \le 0.1$, as illustrated in Fig. 6 at $k_r=10$ for the range of $Re_D=10$–250, is much more significant than that of Re_D and k_r shown in previous Figs. 3 and 5, respectively.

Figs. 7 and 8 show the effect of the structural properties, i.e. particle diameter represented by cylinder/particle diameter ratio D_{cy}/d_p and porosity ε, respectively, of the porous medium on the local temperature difference between the porous phases and the validity of the *LTE* assumption, at $k_r=100$ and $Bi=1.0$, and for different $Re_D=$ 10, 40, 100 and 250. Once again, as the thermal properties act, the increase in particle diameter or porosity tends to decrease the discordancy between the phase thermal fields, and consequently, leads to a reduction in the value of *LTNE*. For a given high k_r and moderate *Bi*, increasing the particle diameter of the porous medium causes an increase in the inter-phase surface area and decreases the particles contact area. This results in an increase in the interfacial heat transfer between the fluid and solid phases and reduces the conductive heat transfer within the solid particles. Consequently, this enables the fluid temperatures closer to the hardly changing solid temperatures. Although there is a considerable reduction in the value of *LTNE* by decreasing the ratio of D_{cy}/d_p from 100 to 10, the *LTE* condition is not secured within this range. In addition, the positive impact of the porosity ε displayed in Fig. 8 is due to the removal of solid material from the porous channel. As ε increases, the channel and in particular the area close to the cylinder contains less solid material, and as a result, the amount of heat transferred from the hot surface of the cylinder to the fluid phase becomes much higher than that moved throughout the solid material. This brings the fluid temperature to be closer to the solid temperature and to attain the local thermal equilibrium between them at $\varepsilon \ge 0.7$ for the whole selected range of Re_D. It is worth to notice from these figures that the influence of ε on the phase temperature differential and the validity of *LTE* assumption is more pronounced than that of particle diameter.

Also, Darcy number *Da* is a dimensionless parameter representing the permeability *K* of the porous medium. For a packed bed of spheres, *K* is a strong function of d_p and ε, see Eq. (6). It is clear that *K* increases as d_p or ε increases, which leads to an increase in *Da*. Therefore, it can be well established that increasing *Da* has a negative impact on *LTNE* parameter, and might drive to the *LTE* validity condition. Moreover,

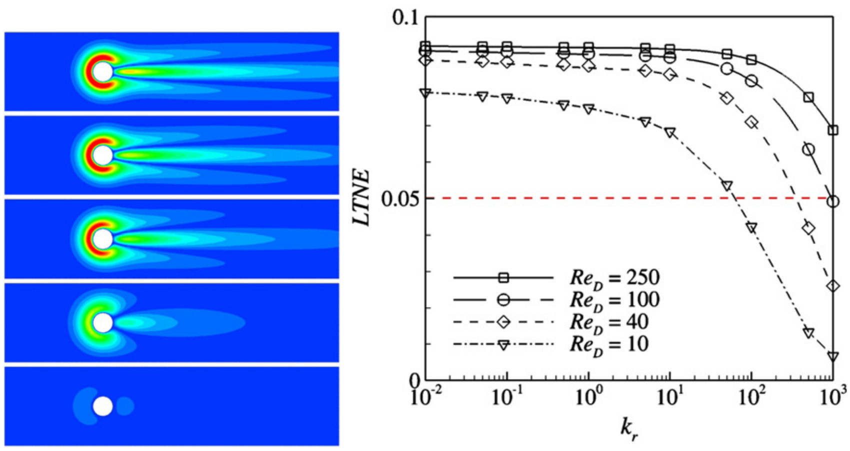

Fig. 5. (Left) Contours of the local temperature difference between the two phases around a heated cylinder embedded in a porous medium at different solid/fluid thermal conductivity ratio, from top to bottom $k_r=0.01, 1.0, 10, 100$ and 1000 at $Re_D=10$. (Right) Variation of the *LTNE* parameter with k_r, for $Re_D=10, 40, 100$ and 250, and at $Bi=1.0$.

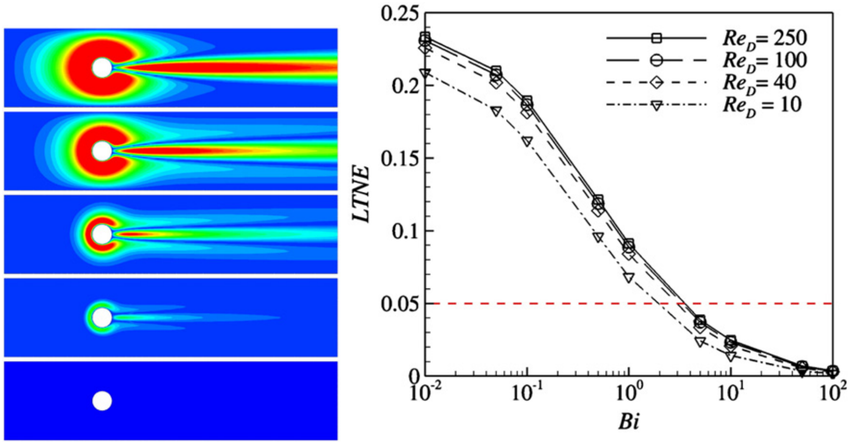

Fig. 6. (Left) Isotherms of the local *LTNE* for different *Bi*, from top to bottom $Bi = 0.01, 0.1, 1.0, 10$ and 100, at $Re_D = 100$. (Right) Variation of the *LTNE* parameter with *Bi* for $Re_D = 10, 40, 100$ and 250, and at $k_r = 10$.

one can be also concluded through varying ε is the influence of the inertial effect in the Forchheimer term in Eq. (2). As can be seen, the increase in this effect which is caused by increasing ε translates into a relatively slower motion due to the increase in the macroscopic and microscopic frictional offered impedances. This will subsequently provide ample heat transfer communication between the two phases, which increases the *LTE* region at the expense of the *LTNE* one.

Figs. 9 and 10 present the effects of conductivity ratio k_r, Prandtl number *Pr*, particle diameter D_{cy}/d_p and porosity ε on the variation of the *LTNE* parameter at three different, low, moderate and high, values of Biot number $Bi = 0.01$, 1.0 and 100, respectively, and for two values of $Re_D = 40$ and 250. In these figures, the effects of *Pr*, D_{cy}/d_p and ε, are calculated at $k_r = 1.0$. The figures demonstrate that the variable *Bi* is the main controller for satisfying the thermal equilibrium between both fluid and solid phases in the porous system. It is clearly seen that its influence is much more significant than other variables, i.e. k_r, *Pr*, D_{cy}/d_p, ε and Re_D. Where at low $Bi = 0.01$, the system is entirely in non-equilibrium condition regardless of the impact of these variables. However, at high $Bi = 100$, the *LTE* is perfectly secured within the porous domain also without considerable effect from other variables. Only at moderate value of $Bi = 1.0$, that the influence of these variables emerged. Therefore, one can deduce that the variable *Bi* which represents the interfacial heat transfer coefficient is the direct responsible and the key phenomenon on satisfying the *LTE* in porous media. In fact, the interstitial heat transfer coefficient h_{sf} is not an independent parameter, hence it is a function of fluid and flow conditions such as *Pr* and Reynolds number in the pore scale Re_p, as well as the structural and material properties of the solid phase such as d_p and k_r. Therefore, it can be generally announced that the *LTE* assumption can become valid at the fluid, flow and medium circumstances that satisfy higher values of h_{sf}.

In addition, it is interesting to note that when the low *Bi* value is considered, the hardness of bringing the solid temperatures closer to the fluid temperatures by increasing k_r appears. Also, it can be seen that decreasing *Bi* to a low value 0.01 changes the trend of the porosity effect on the *LTNE* variation. It is obvious that removing solid material from the porous channel at low *Bi* increases instead of decreases as previously found the *LTNE*, but to a certain high value of $\varepsilon = 0.85$, so afterward the *LTNE* decreases.

4.2. Pulsatile flow

The above-stated steady and stable flow field can be destabilized by including pulsating, which may lead to a better flow mixing and further enhanced thermal interphase transport and consequently decrease the temperature difference between the two phases and then satisfy the *LTE* condition in the porous channel. The influence of external forced pulsating on the validity of the *LTE* assumption is now investigated. Fig. 11 portraits the variation of the *LTNE* parameter versus the dimensionless pulsating flow amplitude $A = 0.2 - 3.0$ and the dimensionless pulsating frequency represented by Strouhal number $St = 0.1 - 2.0$ at fixed values of thermal and structural properties of the porous medium

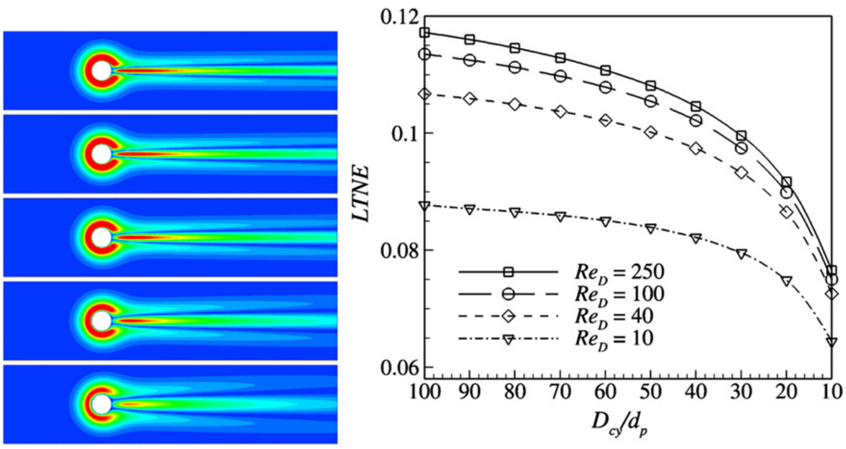

Fig. 7. Influence of particle diameter on: (left) the local temperature difference between both fluid and solid phases around a heated cylinder embedded in a porous medium, from top to bottom $D_{cy}/d_p = 100, 60, 40, 20$ and 10, at $Re_D = 100$, and (right) the *LTNE* parameter, for different Re_D, and at $k_r = 100$ and $Bi = 1.0$.

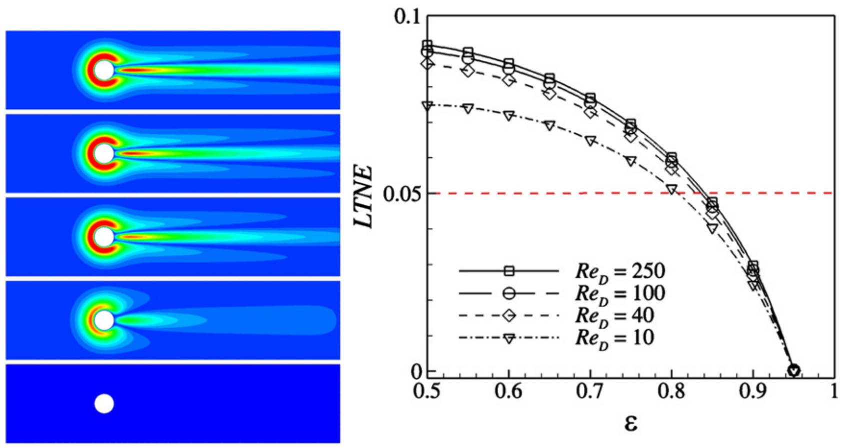

Fig. 8. Influence of porosity on: (left) the local temperature difference between both fluid and solid phases around a heated cylinder embedded in a porous medium, from top to bottom $\varepsilon = 0.5, 0.6, 0.7, 0.8$ and 0.95, at $Re_D = 100$, and (right) the *LTNE* parameter, for different Re_D, and at $k_r = 100$ and $Bi = 1.0$.

used, i.e. $k_r = 1.0$, $Bi = 1.0$, $D_{cy}/d_p = 20$ and $\varepsilon = 0.5$, for different values of $Re_D = 130 - 250$, at $Pr = 7.0$. It can be seen that the degree of non-equilibrium is slightly decreased by increasing A at constant $St = 1.0$, but for a particular value $A = 2.8$, where afterward there is no change in the *LTNE*. Based on the inlet pulsating velocity in Eq. (11), the larger the pulsating A, the higher the flow deceleration becomes during the flow pulsating reversal (phase angle $= \pi$ to 2π). This leads to a better communication between the phases and decreases the deviation between θ_f and θ_s. In contrast, at constant $A = 1.0$ the variation of the *LTNE* against St has an opposite trend. It is gradually increased with increasing St from 0.2 to 0.6. After this optimal St for the non-equilibrium condition, the *LTNE* parameter has an asymptotic behavior, where it remains unaffected by changing St. This is due to that increasing St causes heat penetrating distance into the fluid phase and the oscillation of this depth during a cycle to decrease. This decreases the oscillating interaction and prevents convecting more thermal energy between the phases. In general, for the selected range of $Re_D = 130 - 250$, the oscillatory flows appear to have slight effect on the phase temperature difference, e.g. much less significant effect than that in the steady case. Also, the changing A and St throughout their wide chosen ranges does not satisfy the *LTE* status in the porous channel.

5. Conclusion

The problem of forced convective steady and pulsatile flows over an isothermally heated circular cylinder immersed in a horizontal porous channel is investigated numerically using a spectral element method. A Darcy–Brinkman–Forchheimer model is adopted to describe the

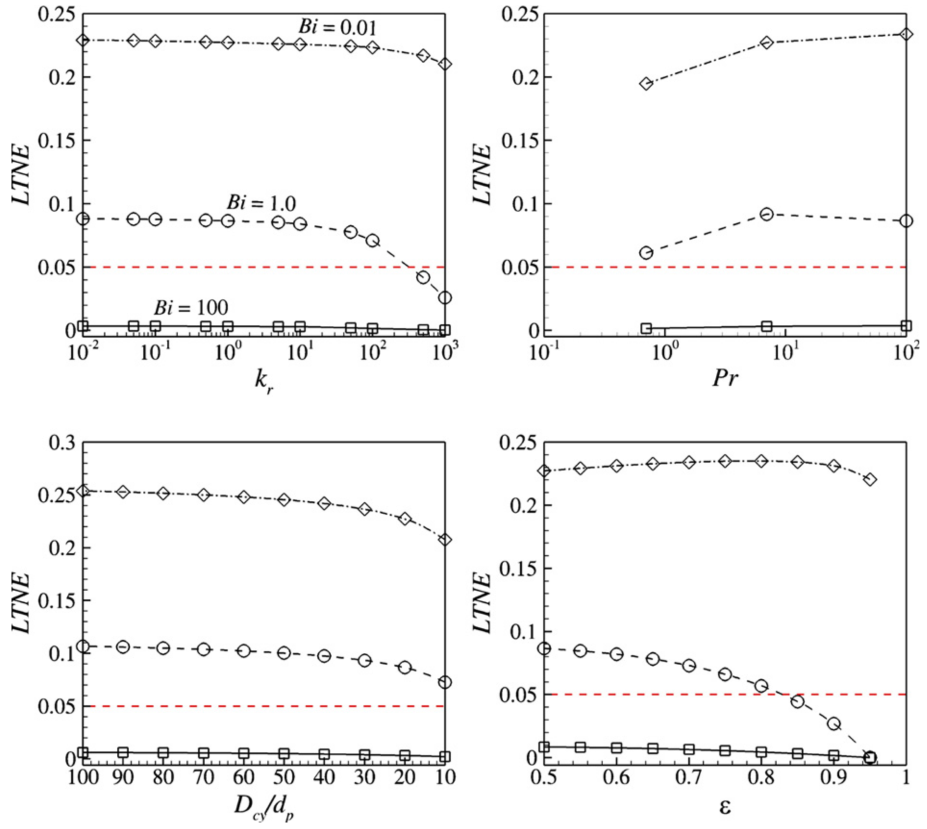

Fig. 9. Variation of the *LTNE* parameter versus k_r, Pr, D_{cy}/d_p and ε, at three different, low, moderate and high, values of Biot number $Bi = 0.01, 1.0$ and 100, respectively, and at $Re_D = 40$.

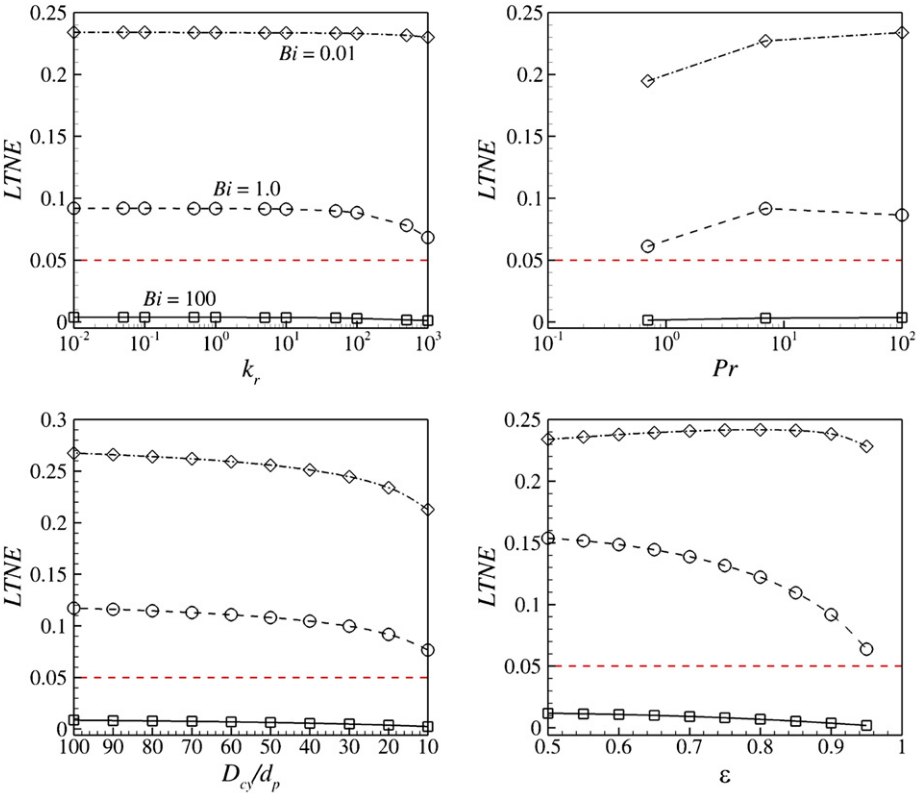

Fig. 10. Variation of the *LTNE* parameter versus k_r, Pr, D_{cy}/d_p and ε, at three different, low, moderate and high, values of Biot number $Bi = 0.01, 1.0$ and 100, respectively, and at $Re_D = 250$.

fluid flow behavior. The thermal behavior is described using a local thermal non-equilibrium *LTNE*, two-phase, model. Numerical solutions obtained over broad ranges of flow, thermal and structural conditions are used to clarify circumstances under which the effect of the local thermal non-equilibrium *LTNE* is important and the *LTE* assumption is, therefore, not valid. The assessment is based on introducing an evaluation parameter *LTNE* used in the literature. The favorable circumstances for the *LTE* assumption to hold are identified. For the steady flow, it is found that higher flow and fluid conditions, i.e. Reynolds and Prandtl numbers, respectively, have the effect of limiting the validity ranges of the *LTE* assumption. However, the circumstances of higher thermal and structural properties of the porous medium used such as solid-to-fluid thermal conductivity ratio, Biot number, cylinder-to-particle diameter ratio and porosity are all found to have a positive impact to satisfy the *LTE* condition. The most significant effective influence in extending the *LTE* validity seems to be from Biot number, however, the particle diameter appears to have the much less effect than other parameters. It is concluded that the Biot number which represents mainly the interstitial heat transfer coefficient is the main governor on satisfying the *LTE* condition in porous media. Thus, this condition is found to be justified at high Biot number, and it is impossible to be secured at low Biot number, for the entire ranges of the pertinent parameters. In reality, this coefficient is a function of these fluid, flow and medium parameters. Therefore, it can be generally said that satisfying the *LTE* condition depends strongly on the circumstances of the above parameters that bring higher values for the interstitial coefficient. Moreover, it is also well established that higher Darcy number representing the permeability of the porous medium and the effects of macroscopic and microscopic frictional resistances has the effect of extending the *LTE*, too. For the pulsatile flow, it is observed that the degree of non-equilibrium decreases with increasing the pulsating flow amplitude or decreasing the pulsating frequency parameter (Strouhal number). However, the effect of oscillatory flow on the validity of the *LTE* is much less significant than that of steady flow. Indeed, the *LTE* is

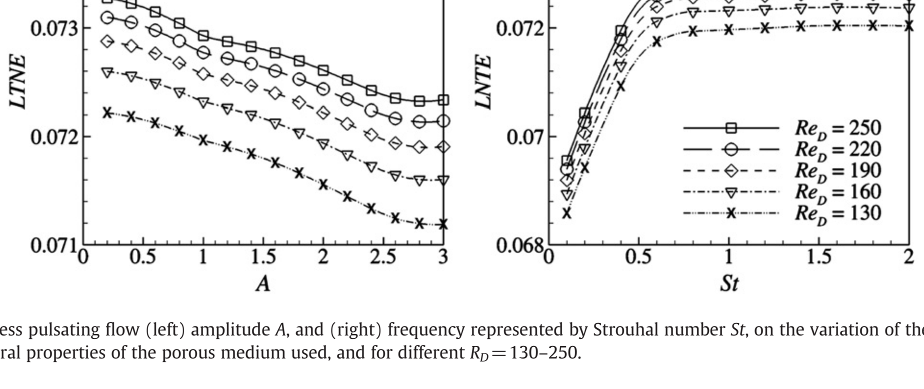

Fig. 11. Effect the dimensionless pulsating flow (left) amplitude A, and (right) frequency represented by Strouhal number St, on the variation of the *LTNE* parameter, at constant values of thermal and structural properties of the porous medium used, and for different $R_D = 130–250$.

shown to be totally not secured for the whole ranges of amplitude and Strouhal number, at a particular selected range of Reynolds number.

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