17th Australasian Fluid Mechanics Conference
Auckland, New Zealand
5-9 December 2010

Frequency selection in time-dependent open flows

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Abstract

This paper studies the process by which open flows select a global frequency of oscillation. Two model systems are presented: the canonical flow past a circular cylinder; and the flow over an open rectangular cavity. The method used is a global linear stability analysis of the time-mean flow. For both the model systems, this method is shown to very accurately capture the saturated global frequency of the flow, even at Reynolds numbers far beyond that at which the flow becomes unsteady. Additionally, the time-mean flow in both systems is shown to be close to marginally stable over a wide range of Reynolds numbers, an important finding in the effort to understand the nonlinear saturation process.

Introduction

This paper analyses the stability characteristics of the time-mean of two periodic flows, the vortex-shedding wake behind a cylinder, and a periodic open cavity flow. A linear stability analysis of the time-mean flow is conducted, to determine a growth rate and a characteristic frequency.

This process can appear circular; to obtain the mean flow, the time-dependent periodic flow needs to be calculated, from which the flow frequency could be measured directly. However, the goal of the analysis is not to come up with a complicated way to measure frequency, but to try to understand the saturation process, the process by which a flow selects a certain frequency. For open flows such as the cylinder wake, this is typically a highly nonlinear process, and its understanding has important ramifications for flow control techniques.

The analysis procedure used is a linear stability analysis, treating the time-mean flow as a new steady base state. This analysis returns two important quantities; a characteristic frequency, and a growth rate of any introduced perturbations. If the final, saturated flow is the sum of the base state (in this case, the mean) and a single, linear mode, the characteristic and saturated frequencies should be similar. If the base state (in this case, the mean) is marginally stable, the growth rate should have unit magnitude.

Previous researchers have studied this problem. In particular, [2] performed the linear stability analysis of the time-mean cylinder wake for $Re \leqslant 180$, where the flow remains two-dimensional. This study found that the mean flow over this range of Reynolds numbers remained very close to marginally stable, and the characteristic frequency was very close to the measured saturated frequency.

[5] then performed a weakly nonlinear analysis of this same flow, strictly valid only in the vicinity of the bifurcation to periodic flow (which occurs around $Re = 47$ [3]). This study found that the mean flow of the circular cylinder wake would remain close to marginally stable, and the characteristic frequency close to the saturated frequency. However, this study also produced an apparent counter-example, in the form of the flow over an open rectangular cavity. Here, the asymptotic analysis predicted the mean flow would not remain marginally stable, invalidating its use as a new base state.

However, it should be noted that the asymptotic analysis is only strictly valid in the vicinity of the bifurcation, and the results gained require some interpretation. The interpretation of [5] may have been stricter than some.

Because of this, the current study has taken two courses: first, the range of the analysis of the cylinder wake mean flow, to Reynolds numbers where the flow is strongly three-dimensional, has been conducted; second, extending and testing the findings of [5], the mean flow over an open rectangular cavity has been analysed. In both cases, it is found that the mean flow remains quite close to marginally stable, and the characteristic frequency tracks the saturated frequency very closely.

The success of the linear analysis over such a wide range of Reynolds numbers, and for different open flows, indicates that in general, the mean flow plays the role of a new base state. The mean flow is corrected by the fluctuation to the point where it reaches marginal stability.

Methodology

Computational Method

All the flows for this study were produced using direct numerical simulations (DNS). The DNS were conducted using a highly-accurate spectral-element method for the spatial discretisation. The geometry was divided into a series of macro quadrilateral elements, that were free to have curved surfaces (to accurately model the cylinder). The macro elements were then further discretised, by using high-order (anywhere from 7th-11th order) tensor-product Lagrange polynomials as shape functions, associated with Gauss-Lobatto-Legendre points. This configuration was used to solve the weak form of the incompressible Navier-Stokes equations. All the flows were resolved using a three-way time-splitting method. This solver has been extensively validated and successfully utilised in many previous studies [7, 4].

Two- and three-dimensional simulations were conducted for the cylinder flow. For the three-dimensional case, the code employed the same spectral-element method in planes perpendicular to the cylinder axis, and a Fourier expansion in the spanwise direction. Steady flows for the cylinder and cavity problems were obtained using the same timestepper, employing the selective frequency damping (SFD) method of [1], allowing steady solutions to be obtained at values of Re well above that at the natural onset of unsteady flow.

Exact definitions of the geometry used can be found in [4] for the cylinder, and [5] for the cavity.

Stability analysis

Global linear stability analysis was carried out, using the time-mean of the periodic flow, as the base flow. Perturbation equations were formed by decomposing the flow variables into base and perturbation components, cancelling the base components, and linearising the remaining terms. The equations for the perturbation quantities were then solved using the same spectral-element method as the base flow. To determine characteristic frequencies, and growth rates, eigenvalues were determined using an Arnoldi decomposition of saved snapshots of the perturbation field at regular intervals (typically around 1000 timesteps). This allowed the full complex eigenvalues of the fastest-growing mode, , to be resolved, the magnitude of which determine stability, and the phase of which determine frequency.

Results and Discussion

The cylinder flow

The flow of interest to this study was the unsteady flow behind a fixed circular cylinder. The flow is characterised by a single dimensionless parameter, the Reynolds number $Re = UD/\nu$, where U is the freestream velocity, D is the cylinder diameter, and ν is the kinematic viscosity. At $Re > 47$, this flow is periodic, and two-dimensional, and is characterised by the alternate shedding of vortices into the wake from each side of the cylinder, forming the von Kámán vortex street. This is clearly illustrated in figure 1.

At $Re > 190$, the two-dimensional flow becomes unstable to a three-dimensional instability known as mode A, and with further increases in *Re*, to a second instability known as mode B [8]. However, while these three-dimensional modes alter the flow geometry, the dynamics of the flow are still dominated by the periodic vortex shedding. Because of this, a single dominant frequency is chosen by the flow, over a wide range of *Re*. It is the process of the selection of this single global frequency by the flow that is the subject of this study.

The cavity flow

The cavity in question is square, and the length of the side of this square, L, is taken as the relevant length scale. Again, this flow is characterised solely by the Reynolds number $Re = UL/\nu$. For this setup used in this study, this two-dimensional flow first becomes unsteady at $Re \simeq 4350$. This unsteady flow is characterised by vortices being shed from the rear edge of the cavity. The basic mechanism appears to be that the free shear layer develops some instability at this rear edge, which is then fed back to the start of the shear layer through the recirculating flow in the cavity. This feedback loop leads to a global instability, and the flow again settles on a single global frequency. An example of the flow produced in shown in figure 1 There are two instability modes that can influence the flow, and there is a discontinuity in the curve of frequency against *Re* as the two modes exchange dominance.

Frequency prediction using linear stability analysis

Here the results of the linear stability analysis of the mean flow are presented. This technique is slightly unconventional, and therefore perhaps needs some explaining. In a traditional stability analysis, the stability of a steady solution is studied. Perturbation equations are formed by linearising around this steady solution. An eigenvalue problem is formed for the perturbation velocity field. The eigenvectors of this system are the linear instability "modes", and the eigenvalues indicate stability. If an eigenvalue, μ, is such that $|\mu| > 1$, is is predicted that the linear perturbations grow in time. If this is the case, the original steady flow is said to be *linearly unstable*. The eigenvalue also contains frequency information, as it is, in general, complex, and therefore results in the instability mode oscillating.

For this analysis, however, a steady solution is not analysed. Instead, the mean flow, extracted from the fully saturated time-dependent flow, is used. In this case, the stability analysis can be interpreted as investigating the effect of linear disturbances on the mean flow, as long as the forcing term in the time-averaged Navier-Stokes equations due to the fluctuation is unperturbed at linear order (see [2] for further explanation).

If the fluctuation is unperturbed at linear order, it therefore might be expected that the flow is well described simply by a linear combination of the mean flow, and a single global mode. If this is the case, this single global mode should have a frequency similar to that of the original flow. Further, since we know the original, time-dependent, flow is fully saturated (that is, not growing or decaying in time), it should be expected that this linear mode does not decay or grow in time, and therefore has an eigenvalue $|\mu| \simeq 1$. This therefore leads to an hypothesis that there exists a class of time-dependent flows, where saturation occurs exactly where the mean flow reaches marginal stability, with $|\mu| = 1$.

The frequency behaviour, and the growth rates, of the linear global modes growing on the mean cylinder and cavity flows are presented in figure 2.

Figure 2a shows the frequency data for the circular cylinder. The crosses show the frequency of the saturated flow measured directly from the DNS simulations. For $Re < 190$, these simulations were two-dimensional, whereas for $Re > 190$, fully three-dimensional DNS simulations were conducted. For validation of the numerical simulations, the solid line in figure 2 shows the frequency measured from laboratory experiments [9]. Finally, the open circles show the predicted frequency from the eigenvalue of the linear mode growing on the mean flow. The match between all three is excellent, over the entire range of *Re*. Note that the mean flow for the three-dimensional simulation was the time-mean of the spanwise spatial mean. The match of the data strongly supports the hypothesis that, for the cylinder flow, the flow is comprised of a single, linear global mode combined with the mean flow.

Further confirmation is drawn from the growth rates of this linear mode. These growth rates are presented in figure 2b. For all of the simulation where the flow is two-dimensional ($Re < 200$), the growth rates remain extremely close to unity. Even when the flow is three-dimensional, the growth rates never exceed $|\mu| = 1.05$. These findings combined suggest that even at Reynolds numbers far beyond the onset of vortex shedding, the cylinder wake can be characterised by its mean and a single mode, and that the flow saturates when the mean flow is "corrected" to the point that it becomes marginally stable.

Figure 2c presents the frequency data for the flow over the open cavity. The range of *Re* spanned sees two separate linear global modes controlling the dynamics, and hence there is a discontinuity in the frequency curve, and begins just after the onset of periodic vortex shedding from the cavity. The measured frequency taken directly from the simulation are presented with open squares; the frequency predicted by the linear stability analysis is presented with filled squares. Over the whole range of *Re*, the match is again excellent,

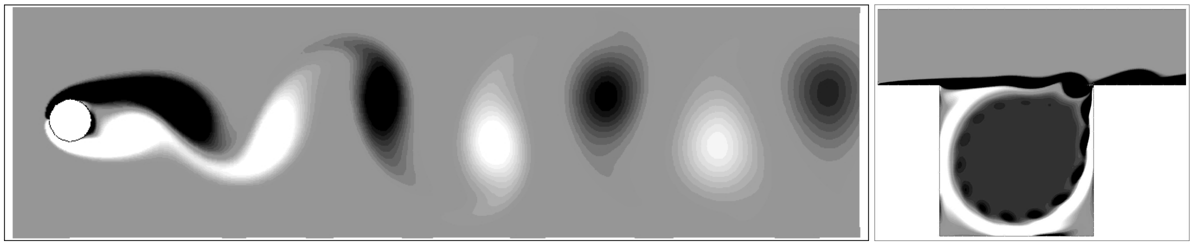

Figure 1: Examples of the model flows, illustrated with contours of vorticity. Left is the cylinder wake at $Re = 100$, showing periodic vortex shedding. Right is the open cavity flow at $Re = 6000$, showing the periodic ejection of vortices from the rear edge of the cavity. Flow is from left to right in both images

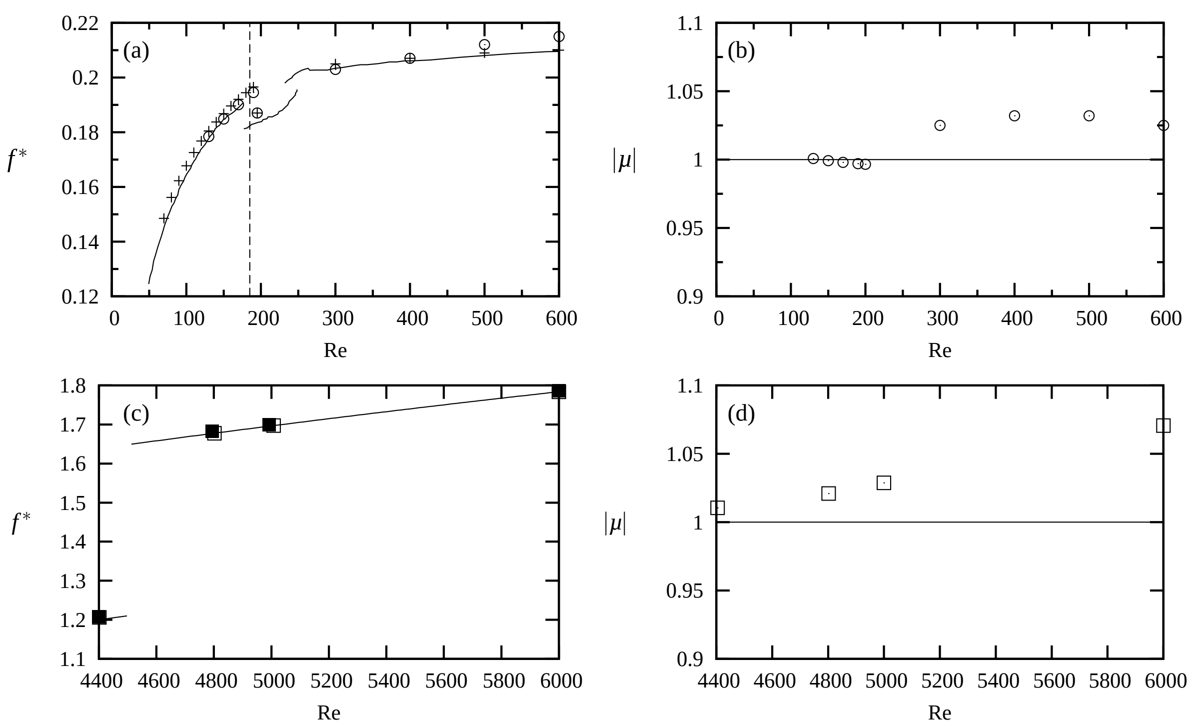

Figure 2: A comparison of frequencies measured directly from the saturated flow, with the frequency predicted by the linear stability analysis, along with growth rates from the linear stability analysis. For both flows, the measured and predicted frequencies are in excellent agreement, and the growth rate of the linear modes remain close to the marginally stable value of 1. (a) Frequencies for the cylinder flow: measured saturated frequency ($+$); measured frequency from experiments (-); predicted frequency from stability analysis ($\circ$). The vertical dashed line indicates where the flow becomes three-dimensional (b) Growth rate of the linear mode growing on the mean flow for the cylinder flow. (c) Frequencies for the cavity flow: measured saturated frequency ($\square$); predicted frequency from stability analysis ($\blacksquare$). (d) Growth rate of the linear mode growing on the mean flow for the cavity flow.

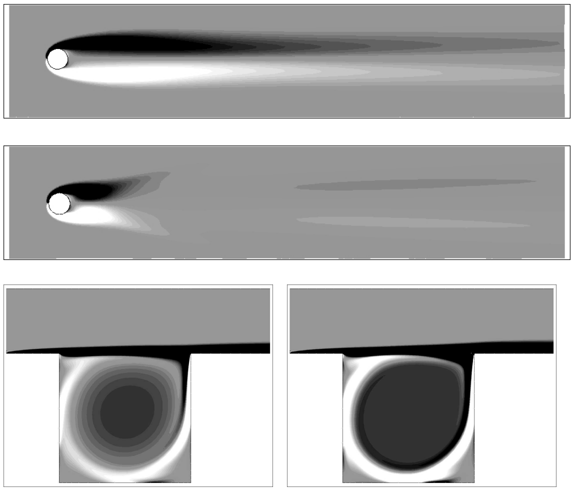

Figure 3: Mean and steady flows for the two model flows, represented with vorticity contours. Steady (top) and mean (bottom) cylinder flows at $Re = 100$. Steady (left) and mean (right) cavity flows at $Re = 6000$. It is clear from these images that the nonlinearity of the flow induces a strong mean-flow "correction" in both cases.

even well beyond the initial onset of periodic flow.

Further evidence for the efficacy of this method is presented in figure 2d. Here the growth rates of the linear modes growing on the mean cavity flow are presented. Again, the are shown to only slightly deviate from the marginally stable value of unity. This suggests that, again, this flow is well-characterised by its mean and a single linear mode, and that the flow saturates at the point where the mean becomes marginally stable.

The positive result may at first seem to be in contradiction to the result of [5], who presented this flow as a counter-example to this theory of saturation at marginal stability. The nonlinear analysis of [5] was certainly carefully conducted, and we are not suggesting that the results presented therein are erroneous. However, the conclusion drawn in that study relied on interpretation of results from a complex analysis. The results of the current study indicate that the original interpretation was perhaps slightly too strict, and that this theory can be used to analyse this cavity flow.

Correction of the mean flow

The success of the this linear analysis of the mean flow perhaps suggests that nonlinear effects are not so important in these flows. However, this is not the case, as the mean flow in both problems is significantly different to equivalent steady flow. This "correction" of the mean flow [6] is due to the Reynolds stress induced by the fluctuating flow field.

The extent of this correction is clearly seen in figure 3. Here mean and steady flows are presented for the cylinder flow at $Re = 100$, and the cavity flow at $Re = 6000$. In both cases, the topology of the flow is clearly different. For the cylinder flow, the recirculation region is far shorter. For the cavity flow, the circulation in the cavity is far more intense in the mean flow than in the steady flow. These images show that nonlinear effects are highly important, however, they can essentially be lumped together and considered simply in a mean sense when considering the saturation process.

Conclusions

Linear stability analysis of the mean flow of two open flows has shown that the saturation process, at least for these two examples, can be understood as a process of mean flow correction. In essence, these flows select a frequency such that the eventual saturated flow has a mean that is marginally stable, and this mean plays the role of a new base state, with a single linear global mode growing on this mean. The success of the method in analysing two disparate flows, with different physical instability mechanisms, suggests that a wider class of open flows exists for which this hypothesis holds. It remains an open research question to identify this class. Its resolution may lead to the development of control strategies and analysis techniques of open flows far more complicated than those presented herein.

Acknowledgements

The authors acknowledge the support of the National Computational Infrastructure National Facility through the Merit Allocation Scheme (grant N67) for the provision of the computational resources used for the simulations of this study. The authors acknowledge the financial support of the Australian Research Council through the Discovery Grants program (grants DP0877327,DP0878304 and LX0668992).

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