# Orbiting Cylinder at Low Reynolds Numbers 

László BARANYI<br>Department of Fluid and Heat Engineering, University of Miskolc, H-3515 Miskolc-Egyetemváros, Hungary,arambl@uni-miskolc.hu


#### Abstract

Sudden changes found in the time-mean and rms values of force coefficients of a circular cylinder in forced orbital motion placed in a uniform stream when plotted against ellipticity of the orbital path suggest that two solutions (states) exist. For a better understanding of the changes in state, some new factors are considered: the torque coefficient; computations for Reynolds numbers up to 300; the investigation of lock-in domain for $\mathrm{Re}=160$.


Key words: orbiting cylinder, lift, drag, torque, lock-in, 2D flow, low Reynolds number flow.

## 1. Introduction

While investigating the flow around an orbiting cylinder placed in an otherwise uniform flow at low Reynolds numbers, a rather peculiar phenomenon appeared. When plotting the time-mean or root-mean-square (rms) values of lift, drag, and base pressure coefficients against ellipticity of the orbital path, sudden changes were found in the values in all coefficients investigated at certain ellipticity values [2]. Later energy transfer between cylinder and fluid was investigated by investigating limit cycles, time histories, phase angles and flow patterns confirming the phenomenon, [4], [8]. Here the dimensionless torque coefficient is added which also shows sudden jumps, and computations are repeated for different forced frequency ratios and for Reynolds numbers up to 300.

The incompressible flow around an orbiting cylinder was simulated by a 2D code developed by the author [1], and is based on Finite Difference Method. Boundary-fitted coordinates are used and both the computational domain and the governing equations are transformed into a computational plane [1].

## 2. Results for an orbiting cylinder

The flow arrangement can be seen in Figure 1. The orbital motion of the cylinder is created by a superposition of two forced harmonic oscillations with identical frequencies $f$ [2]. In Fig. $1 U$ is the free stream velocity $A_{x}$ and $A_{y}$ are dimensionless oscillation amplitudes in $x$ and $y$ directions, respectively. Nonzero $A_{x}$ and $A_{y}$ values give an ellipse, shown in the dotted line in Figure 1. $A_{x}$ alone yields pure in-line oscillation, and then as $A_{y}$ is increased, the ellipticity $e=A_{y} / A_{x}$ increases to yield a full circle at $e=1$.


Figure 1. Layout of a cylinder in orbital motion
During each set of computations the Reynolds number $\operatorname{Re}(\operatorname{Re}=U d / v$ where $v$ is the kinematic viscosity) and $A_{x}$ are fixed and $f$ is kept constant at some percentage of Strouhal number $\mathrm{St}_{0}$ (dimensionless vortex shedding frequency from a stationary cylinder at that Re), and amplitude of transverse oscillation $A_{y}$ is varied to produce varying ellipticity $e=A_{y} / A_{x}$. Only cases were considered where lock-in prevailed, even at zero ellipticity (in-line motion). In this study this was between $70-105 \%$ to ensure lock-in at moderate oscillation amplitudes. Here results will be shown only for a cylinder in clockwise direction of orbit.

An interesting phenomenon was observed when looking at the time-mean value (TMV) and rms values of lift, drag and base pressure coefficients for an orbiting cylinder in a uniform flow. Abrupt jumps were found when these values were plotted against ellipticity $e$ with Re and $A_{x}$ kept constant [2], [3]. A typical example for the TMV of lift coefficient $C_{\text {Lmean }}$ is shown in Figure 2a for $\mathrm{Re}=140$, $A_{x}=0.4, f=0.9 \mathrm{St}_{0}=0.16389$. Note that there are two envelope or state curves, which are roughly parallel with each other and of positive slope, and values jump between these two curves.

The TMV and rms of drag and base pressure, as well as the rms of lift, behaved differently from $C_{\text {Lmean }}$, characterised by two state curves which are not parallel but intersect each other at $e=0$. A typical example is shown in Figure 2 b . The main parameters ( $\mathrm{Re}, A_{x}$ and $f$ ) are the same as in Fig. 2a. The number and location of jumps are identical in these two figures. In this study investigations are extended to include torque coefficient $t_{q}$ (measured clockwise), which is determined from the moment of the shear stress on the cylinder. Omitting details:

$$
t_{q}=-\frac{1}{4} \int_{0}^{2 \pi} \tau_{0}(\varphi) \mathrm{d} \varphi=\frac{1}{4 \operatorname{Re}} \int_{0}^{2 \pi} \omega_{0}(\varphi) \mathrm{d} \varphi
$$

where $\varphi$ is the polar angle, $\tau_{0}$ and $\omega_{0}$ are dimensionless wall shear stress and vorticity, respectively. A similar torque coefficient is defined in [7].


Figure 2a,b. Time-mean value of lift (a) and $r m s$ value of drag (b) versus ellipticity
Figure 3a shows the TMV of $t_{q}$ for three frequency ratios of $f / \mathrm{St}_{0}=0.75 ; 0.8 ; 0.85$ versus ellipticity $e$ for $\mathrm{Re}=160$ and $A_{x}=0.4$. The TMV of torque coefficient $t_{\text {qmean }}$ shows a similar pattern to $C_{\text {Lmean }}$ (see Fig. 2a), though the lines have negative slopes. It can be seen in Figure 3a that the distance between lower and upper envelope curves increase with $f / \mathrm{St}_{0}$.



Figure 3a,b. Time-mean and $r m s$ values of torque coefficient versus $e\left(\operatorname{Re}=160 ; A_{x}=0.4\right)$
Figure 3b shows the rms values of torque coefficient for the three cases shown in Fig. 3a. As can be seen in the figure, the number and location of jumps at the corresponding curves are identical with each other and the pattern of envelope curves in Fig. 3b is similar to that in Fig. 2b. By increasing the frequency of oscillation $f$ the envelope curves shift upward.

In earlier papers [2], [3] only results below $\mathrm{Re}=190$ have been shown. At this Reynolds number three-dimensional effects start to appear for stationary cylinders [5]. Experimental evidence from [6] for oscillating cylinders shows that lock-in increases the span-wise correlation of signals and the two-dimensionality
of the flow compared to flow around stationary cylinders. Poncet [9] shows how the 3D wake behind a circular cylinder can be made 2D by using lock-in triggered by rotary oscillation of the cylinder. For this reason a 2D code is suitable even at higher Reynolds numbers than 190. In [8] a 2D code was used for simulation of flow around an oscillating cylinder at $\mathrm{Re}=500$. The effect of Re on TMS and rms curves for lower Re values was investigated in [3]. Time-mean and $r m s$ results are shown in Figures 4a and 4 b for orbiting cylinders at $\mathrm{Re}=200$; 250; 300. As seen in Fig. 4a, the distance between the two state curves decreases with Re. Figure 4b shows the rms of drag coefficient vs. $e$ for the same three Re numbers. The location and number of jumps in the corresponding curves in Figs. 4a and 4b are identical. Fig. 4b shows that state curves shift upward with Re.


Figure 4a,b. Time-mean and rms of lift coefficient versus ellipticity for $\mathrm{Re}=200 ; 250 ; 300$
Finally, the domains of [0.7,1.05] and [0.1,0.5] in $f / \mathrm{St}_{0}$ and $A_{x}$ (see Figure 5) was investigated using the CFD code mentioned for $\mathrm{Re}=160$ in order to determine where lock-in prevails over the whole $e=0-1.2$ interval investigated. In Figure 5 filled squares show full lock-in, empty diamonds represent cases where not a single point in the $e$ interval is locked-in. Empty squares show cases where lock-in cease to exist within the $e$ interval.


Figure 5. Range of lock-in for $\mathrm{Re}=160$

## 3. Conclusions

To better understand the changes in state, some new factors are considered:

- torque coefficient, which has similar features to those of the lift coefficient, (identical number and location of jumps). The distance between upper and lower state curves for the TMV of the torque coefficient increases and the $r m s$ state curves shift upward with frequency of cylinder oscillation $f$.
- investigating Reynolds numbers up to 300. The distance between upper and lower state curves for $C_{L r m s}$ decreases with Re.
- systematic investigation of lock-in domain for Re=160 over the ellipticity interval of $e=0-1.2$. All computational results show similar patterns.
The results of these investigations confirm the previous findings in this field and strengthen the case for the existence of a phenomenon behind these jumps.


## Acknowledgements

The support provided by the Hungarian Research Foundation (OTKA, Project No. T 042961) is gratefully acknowledged.

## References

1. Baranyi, L., Computation of unsteady momentum and heat transfer from a fixed circular cylinder in laminar flow. J. Computational and Applied Mechanics 4(1) (2003) 13-25.
2. Baranyi, L., Sudden jumps in time-mean values of lift coefficient for a circular cylinder in orbital motion in a uniform flow. In: $8^{\text {th }}$ International Conference on Flow-Induced Vibration, Eds. E. de Langre and F. Axisa, Paris, Vol. II, (2004) 93-98.
3. Baranyi, L.: Abrupt changes in the root-mean-square values of force coefficients for an orbiting cylinder in uniform stream. In: $4^{\text {th }}$ Symposium on Bluff Body Wakes and VortexInduced Vibrations, Eds. T. Leweke and C.H.K. Williamson, Santorini, Greece, (2005) 5558.
4. Baranyi, L., Energy transfer between an orbiting cylinder and moving fluid. In: PVP2006-ICPVT-11 ASME Pressure Vessels and Piping Division Conference, Vancouver (2006), on CD ROM, 1-10, ISBN 0-7918-3782-3.
5. Barkley, D., Henderson, R.D., Three-dimensional Floquet stability analysis of the wake of a circular cylinder. Journal of Fluid Mechanics 322 (1996) 215-241.
6. Bearman, P.W. and Obasaju, E.D., An experimental study of pressure fluctuations on fixed and oscillating square-section cylinders, Journal of Fluid Mechanics 119 (1982) 297-321.
7. Chen, M.M., Dalton, C. and Zhuang, L.X., Force on a circular cylinder in an elliptical orbital flow at low Keulegan-Carpenter numbers. Journal of Fluids and Structures 9 (1995) 617-638.
8. Lu, X.Y., Dalton, C., Calculation of the timing of vortex formation from an oscillating cylinder. Journal of Fluids and Structures 10 (1996) 527-541.
9. Poncet, P., Vanishing of B mode in the wake behind a rotationally oscillating cylinder. Physics of Fluids 14(6) (2002) 2021-2023.
