Numerical Investigation of the Multiple Dynamic Crack Branching Phenomena

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Summary

In this study a phenomenon of multiple branching of dynamically propagating crack is investigated numerically. The complicated paths of cracks propagating in material are generated by moving finite element method based on Delaunay automatic triangulation, which was extended for such problems. For evaluation of fracture parameters for propagating and branching cracks a switching method of the path independent dynamic J integral was used. Using this techniques the generation phase simulation of multiple dynamic crack branching was performed. Various dynamic fracture parameters, experimental obtaining of which is very difficult, were accurately evaluated.

Introduction

The phenomenon of multiple cracks branching (i.e. when crack branch is bifurcating again) is very often observed in dynamic fracture of brittle materials (Fig.1). However, the detailed mechanism of multiple cracks branching, when some crack branches arrests and some continues propagating and bifurcates again, is not fully elucidated yet. Recently the problem of governing condition of dynamic crack branching was investigated in our experimental studies[1,2,3]. The experiments on dynamic branching phenomena revealed, that the energy flux per unit time into a propagating crack tip or into a fracture process zone governs the dynamic crack branching.

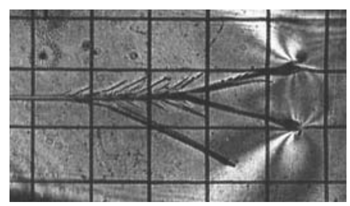

Fig.1 A high speed photograph of multiple crack branching

Over the past few decades, many numerical methods have been proposed to model crack problems. In previous studies[4,5], the authors developed the moving finite element method based on Delaunay automatic triangulation, which allows the simulation of the phenomenon of dynamic crack bifurcation into two cracks. In present work this method is extended for simulation of the multiple crack branching phenomena. The cracks are modeled after the Delaunay mesh generation for each time step. Using this crack modeling technique the moving singularities at the tips of dynamically propagating cracks are treated accurately and even complicated fracture paths are carefully generated.

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In this work, we provide generation phase simulation of multiple dynamic crack branching based on experimental fracture history data. Various dynamic fracture parameters, such as dynamic J integral, dynamic stress intensity factors, energy flux are accurately evaluated even immediately after the crack branching. The same simulations were also performed for simple crack bifurcation into two branches without preceding multiple crack branching. Comparing the calculation results for both cases the influence of multiple crack branching on crack propagation was studied.

Moving Finite Element Method for Dynamic Crack Branching Phenomena

In this study for mesh generation we used the modified Delaunay automatic triangulation[6], which requires only exterior, interior boundary points and specified interior points (if they are necessary). In consideration of the stress singularity each propagating crack tip is always surrounded by the specified interior points.

At the Delaunay automatic mesh generation stage the two surfaces of crack path have common nodal points, and the crack surfaces are described by element boundaries. In order to distinguish both surfaces of crack after Delaunay automatic mesh generation, dual nodes setting on crack path is used, so that, the nodal points with the same coordinates are have different numbers if there are lying on opposite crack surfaces (Fig.2). Therefore, the total number of nodal points increases and the element-nodes relations are changed. During crack propagation, when crack length is increased more than certain value, new nodal points are placed on crack path behind the group of surrounding interior points. Furthermore, only an area of the group of specified interior points with its neighborhood is actually re-meshing during crack propagation, the rest of the mesh pattern is remaining fixed for more accuracy of analysis. For the time integration of the finite element equation of motion the Newmark method is used. The details of time integration procedures are given in [4].

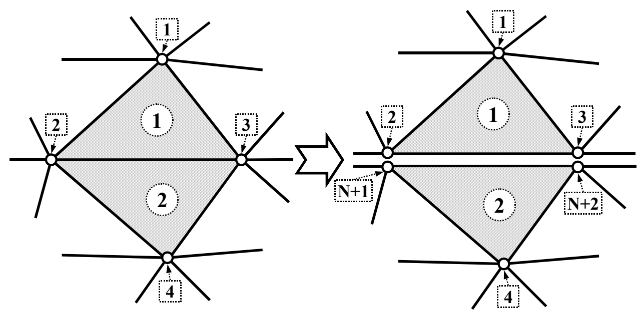


Fig.2 Crack modeling by distinguishing the crack surfaces after mesh generation (*N* is the total number of nodal points after mesh generation)

Evaluation of Dynamic Fracture Mechanics Parameters

In this study, to evaluate various fracture mechanics parameters for a dynamically propagating and branching cracks the path independent dynamic J integral derived by Nishioka and Atluri (1983)[6] is used.

For most numerical analyses, considering dynamically propagating crack in an elastic solid, the global-axis components of the dynamic J integral (J') can be evaluated by the following expression:

$$J'_k = \int_{\Gamma+\Gamma_c} [(W+K)n_k - t_i u_{i,k}]dS + \int_{V_\Gamma} [(\rho \ddot{u}_i - f_i)u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}]dV \tag{1}$$

where u_i, t_i, f_i, n_k and ρ denote the displacement, traction, body force, outward direction cosine, and mass density, respectively. W and K are the strain and kinetic energy densities, respectively, and $(\;)_{,k} = \partial(\;)/\partial X_k$. The integral paths $\Gamma_\varepsilon, \Gamma$ and the Γ_c denote a near-field path, far-field path and crack surface path, respectively. V_Γ is the region surrounded by Γ, while V_ε is the region surrounded by Γ_ε.

The crack-axis components of the dynamic J integral can be evaluated by the following coordinate transformation:

$$J'^0_l = \alpha_{lk}(\theta_0) J'_k \quad , \tag{2}$$

where α_{lk} is the coordinate transformation tensor and θ_0 is the angle between the global X_1 and the crack axis x_1^0 . where is the coordinate transformation tensor and is the angle between the global and the crack axis. The tangential component of the dynamic J integral J'^0_1 corresponds to the rate of change in the potential energy per unit crack extension, namely, the dynamic energy release rate G.

To accurately evaluate the inplane mixed-mode stress intensity factors from the dynamic J integral values, we used the component separation method[8] which can be expressed as:

$$K_I = \delta_I \left\{ \frac{2\mu J'^0_1 \beta_2}{A_I(\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{1/2} = \delta_I \left\{ \frac{2\mu G \beta_2}{A_I(\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{1/2} \tag{3}$$

$$K_{II} = \delta_{II} \left\{ \frac{2\mu J'^0_1 \beta_2}{A_{II}(\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{1/2} = \delta_{II} \left\{ \frac{2\mu G \beta_2}{A_{II}(\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{1/2} \tag{4}$$

where μ is the shear modulus, β_1, β_2 are the crack velocity parameters normalized by the dilatational and shear wave velocities, and $A_I(C), A_{II}(C)$ are functions of crack velocity[4].

Some of the features of the component separation method are: (i) mixed-mode stress intensity factors can be evaluated by ordinary non-singular elements, and (ii) the signs of K_I and K_{II} are automatically determined by the signs of δ_I and δ_{II}, respectively.

Because of difficulty in setting far-field integral path separately for each just bifurcated crack tip, a switching method of the path independent dynamic J integral [4] was proposed:

$$J'_k = \int_{\Gamma+\Gamma_C} \left[(W+K)n_k - t_i u_{i,k}\right] s dS + \int_{V_\Gamma} [\{(\rho \ddot{u}_i - f) u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}\} s dV \;\; + \sigma_{ij} u_{i,k} s_{,j} - (W+K) s_{,k}] dV \qquad (5)$$

where Γ is a far-field integral path, that encloses all branched crack tips and s is a continuous function defined in V_Γ.

For calculation of the dynamic J integral for certain crack tip the s function is set as s=1 for the point at that crack tip and for the points in whole domain and s=0 for the points at the others crack tips. Equation (5) made possible accurate evaluation of the dynamic J integral components for interacting branched crack tips.

Simulation Results

Basing on the experimental data and the history of dynamic crack branching the generation phase simulation was carried out using the moving finite element method. Considering the stress singularity each propagating crack tip is always surrounded by the specified interior points, placed regularly around the crack tip by 28 points in the radial direction and by 6° increment in the circumferential direction. Therefore, although the initial number of elements and nodes were 6756 and 3500 respectively, due to large number of crack tips propagating, the whole number of elements and nodes increased exceedingly and was 22516 and 11668 at the stage shown in Fig.3. The time increment of $\Delta t = 1\mu s$ was used. Furthermore, the generation phase simulation of two cracks bifurcation without multiple branching of central crack based on the same experimental data was performed and simulation results for two cases were compared.

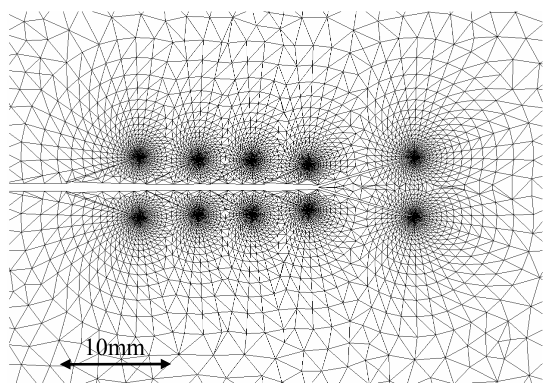
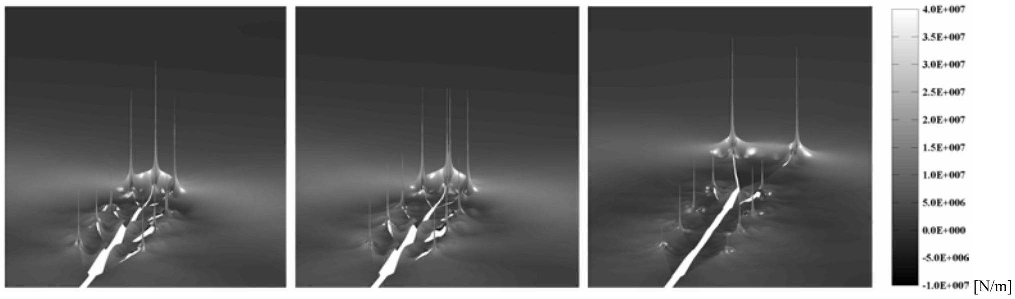


Fig. 3 Deformed mesh pattern at the crack branching area (150μs)

The distributions of the equivalent stress at different time steps are shown in Fig.4. Due to the crack tip singularity, a large stress concentration can be seen around the crack tips. The stress in the vicinity of central crack tip is much larger then around others cracks before branching (a), but become almost equal right after the bifurcation (b). After

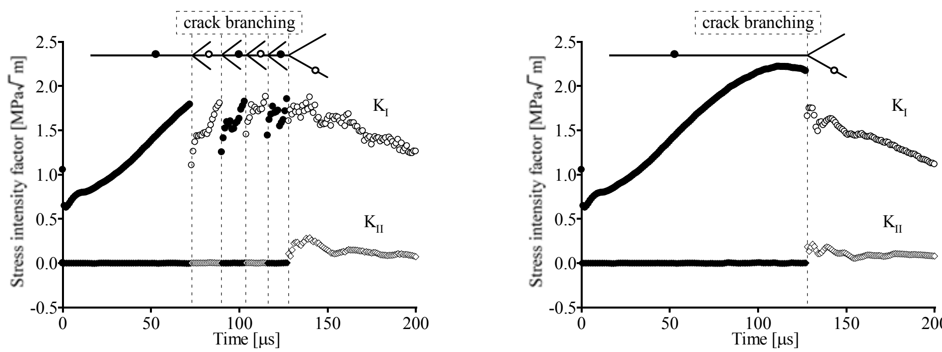


(a) 127ms (b) 128ms (c) 150ms

Fig.4 Equivalent stress distribution at the crack tips neighborhood

the last bifurcation, the stress concentrations around the two main propagating cracks become much larger, than stress around arrested branches (c).

The dynamic stress intensity factors for central main crack tip and one of the cracks after last bifurcation in multiple crack branching case (a) and simple two cracks branching case (b) are plotted in Fig.5. The K_I factor for single straight crack (b) is much larger then K_I for straight multiply branching crack (a) and has a maximum around 110μs. However it can be noticed, that values of stress intensity factors for last two branches are

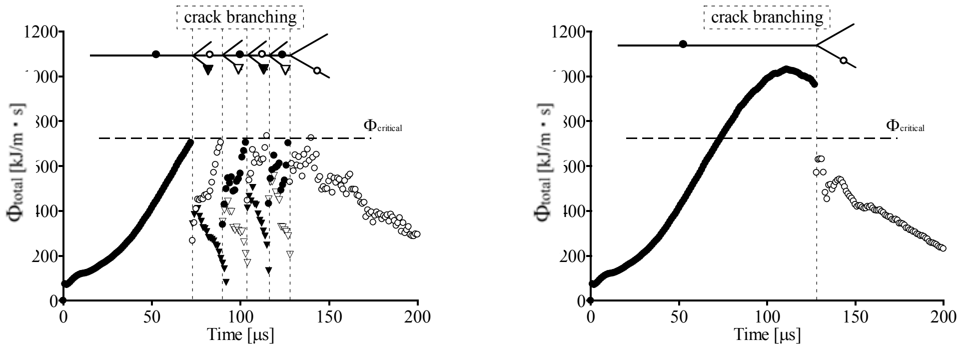


(a) (b)

Fig. 5 Variations of K factors for main crack tip (a) multiple crack branching (b) two cracks branching

very similar in both cases.

The energy flux to the propagating crack tip per unit time was calculated as $\Phi_{total} = J' \cdot C$ and plotted in Fig.7. In case of multiple cracks branching, observed in the experiment, the bifurcation of central crack occurs when energy flux to the crack tip reaches some critical value, represented by dashed line in Fig.7. It can be seen that energy flux for side crack much less than for energy flux for central crack in case of crack trifurcation (Fig.7a).



(a) (b)

Fig. 7 Variations of energy flux (a) multiple crack branching (b) two crack branching

Conclusions

In this study, the moving finite element method based on Delaunay automatic triangulation extended for simulation of complicated crack branching problems, such as multiple cracks branching phenomenon was developed. Experimentally observed phenomenon of multiple cracks branching was successfully reproduced by the generation phase simulation. The mechanism of multiple crack branching was modeled and various fracture mechanics parameters were accurately obtained. The simulation results confirmed the idea, that the energy flux per unit time into a propagating crack tip governs the dynamic crack branching.

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