Law of void evolution for porous materials

B. Chen[1], Z. H. Gao[1], X. H. Peng[1], J. H. Fang[1],[2]

Summary

In this paper, a microscopic cylinder cell including a cylindrical void is considered. The microscopic velocity and strain fields of the cell are analyzed by assuming that the matrix is a mixed isotropic-kinematic hardening media. From the relation of the stress-stain of the matrix material a rate equation of void growth is obtained as a function of stress triaxiality and the void volume fraction.

Introduction

Ductile rupture mainly results from the initiation, growth and coalescence of microscopic voids in engineering materials. As a model of voids, long and roughly cylindrical voids are important because they were often observed at the neck of tensile specimens after large deformation. It might also result from long cylindrical inclusions (e.g., sulfides in steels or silicones in aluminum alloys) that decohere from the matrix after straining. In addition, although the cylindrical geometry is less general than the spherical one, its simplicity allows deriving solutions almost in simple and closed forms. It, therefore, states almost explicitly the influence of the parameters such as void volume fraction, stress triaxiality and the rate sensitivity of the matrix material [1]. These advantages account for its application in several theoretical analyses. McClintock's [2] and Rice and Tracey's [3] pioneering works put forward the exponential dependence of void growth-rate on the triaxiality ratio of remote stresses in a rigid-plastic material. Budiansky, Hutchinson and Slutsky [4] suggested that this dependence be described with a polynomial for a power law matrix material. Licht and Suquet [1] presented a solution, with a closed form, of the growth of a cylindrical void in a finite shell of a nonlinear power law viscous matrix. Tracey [5] derived upper and lower bounds for the growth rate of cylindrical voids in a finite volume of a strain-hardening matrix. Needleman [6] studied the growth of cylindrical voids in a viscoplastic matrix material subjected to plan-strains. Gurson [7] investigated two different deformation modes of a representative volume element, which is fully plastic, and proposed a form of the yield criterion for a porous material with cylindrical void. Pan and Huang [8] considered effects of void growth on constitutive relations for viscoplastic materials containing circular-cylindrical voids. Pan [9] studied cylindrical void growth in shear bands for nonlinear power-law viscous solids. Most of models and theories mentioned above were derived based on viscous or isotropic matrix materials which do not strictly adapt to many metallic materials, and the influences of void evolution on constitutive equation is not clear

[1] Department of Engineering Mechanics, Chongqing University, Chongqing 400044, P. R. China
[2] Division of Mechanical Engineering, Alfred University, Alfred, N Y 14802, USA

enough. The present work is devoted to the research on porous metallic materials, such as cast aluminum alloys, steel alloys, sintered powder and metallic foam and so on, which are widely used in engineering. The research on the porous materials is significant and needs combined micro-macroscopic analysis. The work is the establishment of the evolution law of void which is obtained through the analysis of microscopic speed and strain/stress fields of cylinder void and the conversion from microscope to macroscope.

Cylindrical void model and void evolution law

A long thick-walled cylinder cell containing a long cylindrical void under axial and radial tension, which aligned in the axial direction and with a circular cross-section, is shown in Fig. 1. The radius of the void and the cell are *a* and *b*, respectively, and the radius of an arbitrary point of the matrix is *r*. The volumes of the void and the matrix are V_v and V_m, respectively. The volume of the cell is then $V = V_v + V_m$. In the following, upper-case letters and lower-case letters denote the macroscopic and microscopic quantities, respectively. For example, Σ_{ij} and E_{ij} are macroscopic stress and strain, σ_{ij} and ε_{ij} are the corresponding microscopic stress and strain. The matrix is assumed homogeneous, incompressible, rigid-plastic and mixed isotropic-kinematic hardening. The boundary condition at $r = b$ and the matrix incompressibility condition can be expressed as:

$$v_i = \dot{\mathrm{E}}_{ij} x_j \,, \tag{1}$$

$$\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33} = 0 \,. \tag{2}$$

Assuming x_1, x_2, x_3, are orthogonal principal local axes, and an axisymimetic motion with symmetric axis x_3 is considered, one has

$$\dot{\mathrm{E}}_{11} = \dot{\mathrm{E}}_{22} \neq 0, \quad \dot{\mathrm{E}}_{ij} = 0 \ (i \neq j) \,. \tag{3}$$

The contraction in x_3-direction or the value of $d\mathrm{E}_{11}/d\mathrm{E}_{33}$ is important, which determines the deformation in x_3 direction and relates the effect of stress triaxiality on void growth. A parameter, *w*, is therefore introduced:

$$\dot{\mathrm{E}}_{33} = -w\dot{\mathrm{E}}_{11} \,, \tag{4}$$

and the volumetric strain can be expressed as:

$$\frac{dV}{V} = d\mathrm{E}_{kk} = Bd\mathrm{E}_{11} \geq 0, \text{ with } B = 2 - \omega \,, \tag{5}$$

where *B* can be regarded as a strain restriction function which determines the effects of stress triaxiality on void evolution.

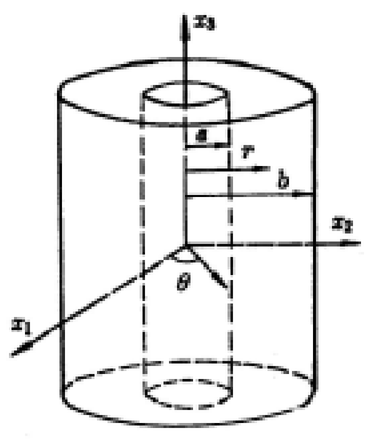

Fig. 1. Long cylindrical void model

When $w=0$, corresponds to the case of plane-strain, in which the deformation restriction in x_3-direction is very strong and B takes the maximum value of 2. The void growth also takes the maximum value under the same applied strain $d\mathrm{E}$. However, if the deformation restriction in x_3-direction is very weak, w will take a large value and B is smaller, as a result, the void growth should also be smaller. If $w=2$, B vanishes, it means no volume dilatation will take place. The range of B may vary between 0 and 2. The condition of continuity for matrix can be derived as follows:

$$df = (1-f)d\mathrm{E}_{kk}, \tag{6}$$

where $f = V_v / V$ denoted the current void volume fraction. From Eq. (1), the following microscopic velocity field under cylindrical coordinates (r, θ, x_3) can be obtained:

$$v_1\big|_b = \dot{\mathrm{E}}_{11} b\cos\theta,\ v_2\big|_b = \dot{\mathrm{E}}_{22} b\sin\theta,\ v_3\big|_b = \dot{\mathrm{E}}_{33} x_3. \tag{7}$$

In a cylindrical coordinate system as shown in Fig. 1, the microscopic strain rate $\dot{\varepsilon}_{ij}$ in the matrix is given by:

$$\dot{\varepsilon}_r = dv_r/dr,\ \dot{\varepsilon}_\theta = v_r/r,\ \dot{\varepsilon}_3 = dv_3/dx_3 \tag{8}$$

where v_r and v_3 are the components of the velocity field $\boldsymbol{v}$. Following these relations and noticing the condition of matrix incompressibility, Eq. (2), we obtain

$$\dot{\varepsilon}_r = \frac{1}{2}[(2-B) - \frac{b^2}{r^2}B]\dot{\mathrm{E}}_{11}, \dot{\varepsilon}_\theta = \frac{1}{2}[(2-B) + \frac{b^2}{r^2}B]\dot{\mathrm{E}}_{11},\ \dot{\varepsilon}_z = (B-2)\dot{\mathrm{E}}_{11}. \tag{9}$$

The increments of intrinsic time measures $d\zeta$ [10] is defined as follows, with the assumptions of rigid-plasticity and incompressibility of the matrix,

$$d\zeta = (d\varepsilon_{ij} d\varepsilon_{ij})^{\frac{1}{2}}. \tag{10}$$

Then, from Eq. (9) one obtains

$$d\zeta = \frac{1}{\sqrt{2}r^2}\sqrt{b^4B^2 + 3(2-B)^2 r^4}\, d\mathrm{E}_{11} = \frac{1}{\sqrt{2}\xi}\sqrt{B^2 + 3(2-B)^2\xi^2}\, d\mathrm{E}_{11}, \tag{11}$$

where $\xi = r^2/b^2$. From Eqs. (9) and (11), we obtain

$$\frac{d\varepsilon_r}{d\zeta} = \frac{1}{\sqrt{2}}\frac{(2-B)\xi - B}{\sqrt{B^2 + 3(2-B)^2\xi^2}}, \tag{12a}$$

$$\frac{d\varepsilon_\theta}{d\zeta} = \frac{1}{\sqrt{2}}\frac{(2-B)\xi + B}{\sqrt{B^2 + 3(2-B)^2\xi^2}}, \tag{12b}$$

$$\frac{d\varepsilon_z}{d\zeta} = \frac{1}{\sqrt{2}}\frac{2(B-2)\xi}{\sqrt{B^2 + 3(2-B)^2\xi^2}}. \tag{12c}$$

The endochronic constitutive equation [10] can take into account mixed isotropic-kinematic hardening, and will be used to derive the required evolution law. The constitutive equation can be expressed as follows:

$$s_{ij} = s_{ij}^0 \frac{d\varepsilon_{ij}}{d\tau} + \int_0^z \rho(\tau - \tau') \frac{d\varepsilon_{ij}}{d\tau'} d\tau', \tag{13}$$

where s_{ij} and s_{ij}^0 denotes the microscopic deviatoric stress and the microscopic initial yield stress, respectively. $d\tau = d\zeta / F(\zeta)$ is microscopic intrinsic time [10].

$$\rho(\tau) = \sum_{r=1}^{3} C_r \exp(-\alpha_r \tau) \tag{14}$$

is the kernel function. If the hardening function $F(\zeta) = 1 + k\zeta$ and assuming proportional loading, one obtains the microscopic stress field of the cylindrical void cell:

$$s_{ij} = [(s_{ij}^0 + \sum_{r=1}^{3} \frac{C_r}{\alpha_r + k}) F(\zeta) - \sum_{r=1}^{3} \frac{C_r}{\alpha_r + k} F(\zeta)^{-\alpha_r / k}] \frac{d\varepsilon_{ij}}{d\zeta}, \tag{15}$$

where C_r, α_r (r=1,2,3) and k are material constants, which can be determined from a $\sigma - \varepsilon^p$ curve by applying Eq. (15) to a simple tension and using nonlinear curve fitting program.

Letting Φ and ϕ be the macroscopic and the microscopic potential functions, respectively, the macroscopic stress can be expressed as:

$$\Sigma_{ij} = \frac{\partial \Phi}{\partial E_{ij}} = \frac{1}{V} \int_{V_m} \frac{\partial \phi}{\partial E_{ij}} dV = \frac{1}{V} \int_{V_m} s_{kl} \frac{\partial \varepsilon_{kl}}{\partial E_{ij}} dV. \tag{16}$$

Substituting (9) and (15) into (16), we can obtain macroscopic stress components Σ_{11}, Σ_{22} and Σ_{33}, which are the functions of *f*, B and material parameters *C*$_r$, α_r *and k*, (r=1,2,3), as following:

$$\Sigma_{11} = \Sigma_{22} = \Gamma_1 \Lambda_1 + \Gamma_2 \Lambda_3, \tag{17a}$$

$$\Sigma_{33} = \Gamma_1 \Pi_1 + \Gamma_2 \Pi_3 - \Gamma_1 \Pi_2 - \Gamma_2 \Pi_4, \tag{17b}$$

then the macroscopic mean stress Σ_m, and the macroscopic equivalent deviatoric stress Σ_e can be expressed as:

$$\Sigma_m = \frac{1}{3}(\Sigma_1 + \Sigma_2 + \Sigma_3) = \Gamma_1 \Pi_1 + \Gamma_2 \Pi_3 - \frac{1}{3}(\Gamma_1 \Pi_2 + \Gamma_2 \Pi_4), \tag{18a}$$

$$\Sigma_e = \Sigma_1 - \Sigma_3 = \Gamma_1 \Pi_2 + \Gamma_2 \Pi_4, \tag{18b}$$

where

$$\Gamma_1 = \Sigma_s + \sqrt{\frac{3}{2}} \sum_{i=2}^{3} \frac{C_i}{\alpha_i}, \qquad \Gamma_1 = \sigma_s k \tag{19a,b}$$

$$\Pi_1 = \frac{1}{2\sqrt{3}} \ln[\frac{\sqrt{B^2 + 3w^2} - B}{\sqrt{B^2 + 3w^2} + B} \cdot \frac{\sqrt{B^2 + 3w^2 f^2} + B}{\sqrt{B^2 + 3w^2 f^2} - B}], \tag{19c}$$

$$\Pi_2 = \frac{1}{\sqrt{3}w}[\sqrt{B^2 + 3w^2} - \sqrt{B^2 + 3wf^2}], \; \Pi_3 = \frac{1}{\sqrt{6}}(\frac{f - f_0}{1 - f_0})(\frac{1 - f}{f}), \tag{19d,e}$$

$$\Pi_4 = \sqrt{\frac{3}{2}} \cdot \frac{w}{B}\left(\frac{f-f_0}{1-f_0}\right)(1-f), \tag{19f}$$

where Σ_s is the macroscopic yield normal stress and f_0 is initial void volume fraction. It is known that the macroscopic stress triaxiality Λ, which plays an important role in void growth, is defined as:

$$\Lambda = \Sigma_m / \Sigma_e . \tag{20}$$

Furthermore, numerical results show that Λ is mainly determined by the strain restriction function B and the initial and currant void volume fraction f_0 and f, and $B = B(f_0, f, \Lambda)$ can be expressed as:

$$\frac{B}{3-B} = (1+\chi_0 f_0)(1+\chi f)\gamma_1 \exp[-\gamma_2(1-\frac{\Lambda}{1+\Lambda})]. \tag{21}$$

where $\chi_0, \chi, \gamma_1, \gamma_2$ is material constants. Some numerical results of the relation are plotted under several void volume f (see Fig. 2).

The macroscopic intrinsic time and macroscopic intrinsic time measures are denoted with T and Z, respectively, and we have [10]:

$$d\mathrm{T} = d\mathrm{Z}/ f(\mathrm{Z}) . \tag{22}$$

The increments of macroscopic intrinsic time measures is defined as the Euclidean norm of the macroscopic deviatoric plastic strain increment, noticing the assumptions of rigid-plasticity and incompressibility of the matrix, we have

$$d\mathrm{Z} = (d\mathrm{E}_{ij} d\mathrm{E}_{ij})^{\frac{1}{2}} . \tag{23}$$

From Eqs. (4), (5) and (23) we obtain

$$d\mathrm{Z} = \sqrt{\frac{2}{3}}(3-B)d\mathrm{E}_{11} = \sqrt{\frac{2}{3}}\frac{3-B}{B}d\mathrm{E}_{kk} . \tag{24}$$

Combining Eqs. (5), (6) and (24), one can deduce the following relation:

$$\frac{df}{d\mathrm{Z}} = \sqrt{\frac{3}{2}}(1-f)\frac{B}{3-B} . \tag{25}$$

Substituting Eq. (21) into (25), one obtains the equation of growth rate of long cylindrical voids with mixed isotropic-kinematic hardening matrix as following:

$$\dot{f}_{growth} = \frac{df}{d\mathrm{Z}} = \sqrt{\frac{3}{2}}(1-f)(1+\chi_0 f_0)(1+\chi f)\gamma_1 \exp[-\gamma_2(1-\frac{\Lambda}{1+\Lambda})]. \tag{26}$$

Eq. (26) indicates that the rate of void growth is an exponential function of stress triaxiality and related to initial and currant volume fraction of void.

Acknowledgements

This research was supported by the Natural Science Foundation of China (Grant no. 10272120) and Chongqing.

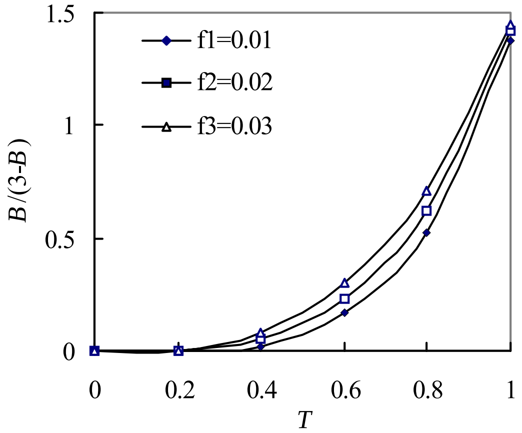

Fig. 2. Relation between $B/(3\text{-}B)$ and T at several values of f

References

[1] Licht, C. and Suquet, P. (1988): "Growth of cylindrical voids in nonlinear viscous material at arbitrary void volume fractions: a simple model", *Arch. Mech.*, Vol. 40, pp. 741-757.

[2] McClintock, F. A. (1968): "A criterion for ductile fracture by the growth of holes", *J. Appl. Mech.* Vol. 90, pp. 363-371.

[3] Rice, J. and Tracey, D. (1969): "On ductile enlargement of voids in triaxial stress field", *J. Mech. Phys. Solids,* Vol. 17, pp. 201-217.

[4] Budiansky, B., Hutchinsin, J. W. and Slutsky, S: (1982) "Void growth and collapse in viscous solids". In: *Mechanics of Solids*, (ed. Hopkins and Sewell). Pergamon Press, Oxford, UK, pp. 13-45.

[5] Tracey, D. M: (1971): Strain hardening and interaction on the growth of voids in ductile fracture, *Engng. Fract. Mech*. Vol. 3, pp. 301-316.

[6] Needleman, A. (1972): "Void growth in an elastic plastic medium", *J. Appl. Mech.*, Vol. 39, pp. 964-970.

[7] Gurson, L. (1977): "Continuum theory of ductile rupture by void nucleation and growth", *J. of Engng. Mater. Tech., Trans.*, ASME, Vol, 99, pp. 2-15.

[8] Pan, K. and Huang, Z. (1994): "Cylindrical void growth model in visco-plastic materials", *Int. J. Damage Mech.* Vol. 3, pp. 87-106.

[9] Pan, K. L. (1994): "Growth of spherical void in shear bands", *Arch. Mech.*, Vol. 46, pp. 343-356.

[10] Valanis, K. C. (1980): "Fundamental consequences on new intrinsic time measure as a limit of the endochronic theory". *Arch. Mech.* Vol. 32, pp. 171-190.