The Compressive Strength of Fiber Composites

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Abstract

The compressive strength of continuous fiber-reinforced composites is significantly less than their tensile strength, a phenomenon which is still not well understood. Dow and Rosen attempted in 1965 to explain this phenomenon by developing a fiber micro-buckling model, in which each fiber was modeled as a beam-on-an-elastic-foundation. The model, however, grossly overestimated the compressive strength of continuous fiber-reinforced composites. Researchers have since focused on introducing other factors that could affect the compressive strength, including initial fiber waviness, fiber properties, matrix properties, interfacial strength, and inter-phase properties. While empirical results have demonstrated the influences of these factors, an analytical model that can comprehensively interpret the failure mechanism and accurately predict the strength is still lacking. The present study treats the compressive strength prediction of continuous fiber-reinforced composites using a different approach. Here, the matrix surrounding the fiber is not treated as homogeneous, but divided into a discrete inter-phase region adjacent to the fiber and a bulk matrix phase beyond the inter-phase. The three-phase model, verified in this study both analytically and numerically using a two-dimensional plane strain finite-element analysis, is shown to provide a more accurate prediction of the compressive strength of typical high-performance composites, such as carbon fiber-reinforced epoxy. The results of this study are validated by the extensive experimental database available in the literature.

The Three-Phase Fiber Microbuckling Model

The modulus of the inter-phase between fiber and matrix can have a significant effect on the compressive strength of continuous fiber reinforced composites. We propose an analytical model that includes the inter-phase whose properties were estimated directly from independent tests. Finite-element analysis was conducted to validate the model, shown in Fig. 1, consisting of a 2-dimensional laminated plate containing layers of fibers in a matrix, each fiber separated from the matrix with an inter-phase.

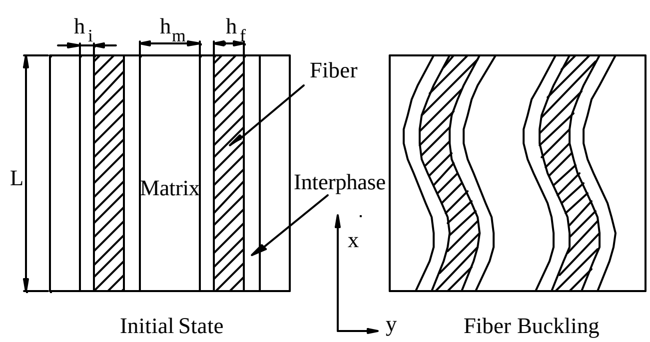

Fig. 1. Three-phase micro-buckling model.

The fiber/inter-phase and inter-phase/matrix boundaries are assumed to be perfectly bonded. Parameters h_f, h_m, and h_i represent the equivalent thickness of fiber, matrix, and inter-phase, respectively. Writing $\frac{h_m}{h_f} \approx \frac{1-v_f}{v_f}$, we derive the composite compressive strength,

$$\sigma_{cu} = \frac{1}{\left(\frac{1-v_f}{G_m v_f} + 2\frac{h_i}{G_i h_f}\right) v_f} = \frac{1}{\frac{1-v_f}{G_m} + 2\frac{h_i}{h_f}\frac{1}{G_i} v_f} = \frac{G_m}{1-v_f + 2\frac{h_i}{h_f}\frac{G_m}{G_i} v_f} \quad (1)$$

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The estimated strength based on Eq. (1) includes the effects of fiber size, the inter-phase size and modulus, the matrix shear modulus, and fiber volume fraction. When h_i/h_f is zero (no inter-phase), the third term in the denominator of Eq. (1) vanishes resulting in Dow and Rosen's solution [1]. When the product $(\frac{G_m}{G_i}) \times (\frac{h_i}{h_f})$ is equal to 1/2, Eq. (1) becomes $\sigma_{cu} = G_m$ which was suggested by Hayashi [2].

We also note that: $$\frac{\text{Interphase Volume}}{\text{Fiber Volume}} = \frac{2h_i}{h_f} = \frac{d_f \pi t_i}{\frac{d_f^2 \pi}{4}} \Rightarrow \frac{h_i}{h_f} = \frac{2t_i}{d_f}$$

where, t_i is the inter-phase thickness and d_f is the fiber diameter. Figs. 2 and 3 show the predicted compressive strength S (in terms of the matrix shear modulus) according to Eq. (1).

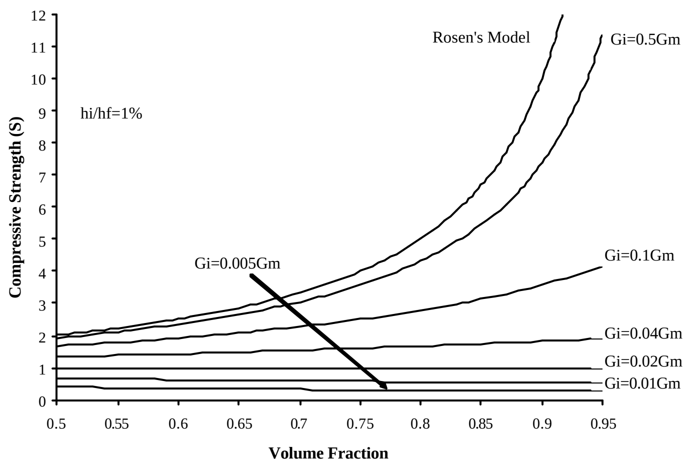

Fig. 2. Predicted compressive strength for various inter-phase moduli.

In Fig. 2, the equivalent inter-phase thickness is fixed at 1% of the fiber thickness, and the compressive strength (S) versus volume fraction (V_f) for different inter-phase moduli is shown. When the inter-phase modulus is close to that of the matrix, the predicted curve is close to Rosen's model. For a very soft inter-phase ($G_i < 0.02\ G_m$), increasing the fiber volume fraction results in decreasing compressive strength, which was observed experimentally in glass/polyester by Piggott [3]. One should note that the compressive strength is also more sensitive to the inter-phase modulus as the fiber volume fraction increases.

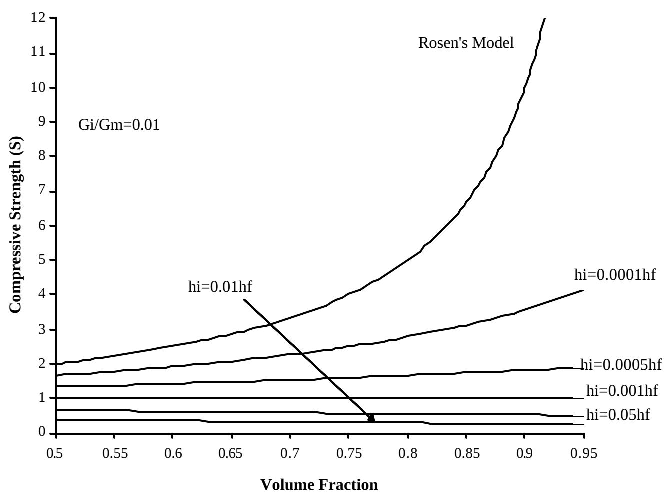

Fig. 3. Predicted compressive strength as a function of inter-phase thickness.

In Fig. 3, the inter-phase shear modulus is fixed at 1% of the matrix modulus, and the predicted compressive strength (S) versus volume fraction (V_f) for different h_i/h_f is shown. When the inter-phase thickness is very small or the fiber diameter large, the predicted curve is close to Rosen's model. If the inter-phase properties (thickness and modulus) are held constant, composites with larger fibers are less sensitive to the inter-phase and have a larger compressive strength. This is due to the smaller volume of inter-phase for large fibers for a given fiber volume fraction. This also explains why boron/epoxy (h_i/h_f is about 0.2%) has a larger compressive strength than HS carbon/epoxy (h_i/h_f is about 3%), and composites with T300 carbon fiber (diameter of 7 μm) are slightly stronger than composites with T700 carbon fiber (diameter of 5 μm) [4,5]. When h_i/h_f is small, the predicted compressive strength is close to Rosen's model. Model composites have large fibers (1-2 mm) relative to the inter-phase (100 nm). Hence, their compressive strengths [6] can be well predicted by the Dow and Rosen model.

Inter-phase Properties

The presence of a softer inter-phase between fiber and matrix has been established by several investigators [7-13]. Micro-cracks usually initiate and propagate in this region [14-16]. The thickness of the inter-phase was estimated to be about 100-250 nm for high strength (HS) carbon fiber/epoxy [17] and 100-200 nm for glass fiber/epoxy [18]. Tsai, Arocho, and Gause [19] estimated the inter-phase modulus using an analytical model and a finite element model to correlate test data from Mandell et al. [20] and Williams et al. [17,21]. Tsai et al. [19] developed an elastic shear lag model and correlated the results with the test data from Mandell et al. [20] from fiber de-bonding tests. Carbon/epoxy with an inter-phase shear strength of 27 MPa, fiber diameter 7 μm and thickness of inter-phase of 100 nm was found to have a G_m/G_i value about 74. S-glass/epoxy with an inter-phase shear strength 39 MPa, fiber diameter 10 μm, and thickness of inter-phase 100 nm has a G_m/G_i value of about 92.

Comparison between the Analytical Predictions and Experimental Results

The composite compressive strength for failure in the fiber micro-buckling mode can be estimated using the three-phase fiber micro-buckling model, Eq. (1). All the parameters can be determined from independent tests and measurements. One should note that the non-linearity of the matrix should be incorporated in Eq. (1). In order to obtain the matrix tangent shear modulus, the experimental data [22] for a typical epoxy (3501-6) tested at room temperature is used. For high strength carbon/epoxy and boron/epoxy composites, the failure strain is about 1% and the non-linearity effect from the matrix on the compressive strength of composites is small. For glass/epoxy composites, the failure strain is about 2 %, at which the tangent modulus of matrix is about 88 % of original modulus.

Compressive Strength of High Strength Carbon Fiber/Epoxy

The compressive strength of high strength carbon fiber reinforced composite is dominated by the fiber micro-buckling mechanism. Typical properties of high strength carbon fiber/epoxy composites are listed in Table 1.

Table 1 Properties of high strength carbon fiber/epoxy composite

Diameter d_f(um)	Thickness of the inter-phase t_i (nm)	Effective ratio $h_i/h_f = 2\, t_i/d_f$
7	100	0.029

The predictions from the analytical model and experimental data for AS/3501-6 and T300/5208 with volume fraction around 0.6 are shown in Fig. 4. In Fig. 4, the predicted compressive strengths from the analytical model using the inter-phase property estimated by Tsai [19] is presented with experimentally measured compressive strengths. The predicted strengths (G_i/G_m= 0.04) match the experimental results very well. The curve with the best data fit (G_i/G_m= 0.05) is also presented in Fig. 4 for comparison. This proves the validity of the analytical model in predicting the compressive strength of high strength carbon fiber-reinforced composites.

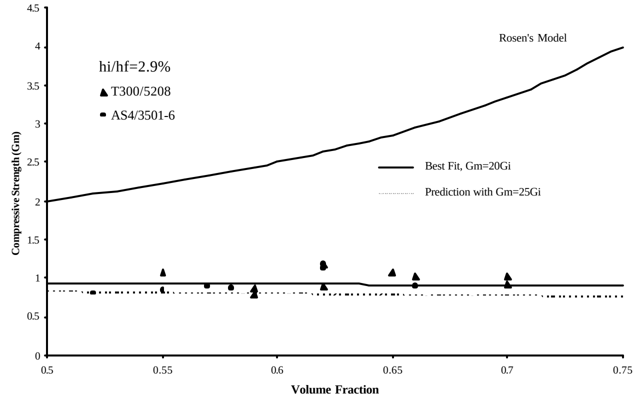

Fig. 4. Compressive strength of high strength carbon fiber composites (data from [23, 24]).

Compressive Strength of Glass Fiber/Epoxy

The compressive strength of glass/epoxy is dominated by micro-buckling. The shear modulus of the inter-phase is assumed to have the same non-linearity as the matrix modulus as there is no direct empirical data available for the inter-phase modulus of glass/epoxy. The inter-phase modulus in the glass/epoxy composite is assumed to be the same as in the high strength carbon fiber/epoxy composite. The properties of glass/epoxy are shown in Table 2.

Table 2 Properties of glass fiber/epoxy composites

Diameter d_f(um)	Thickness of the inter-phase t_i (nm)	Effective ratio $h_i/h_f = 2\ t_i/d_f$	Failure Strain	Effective G_m (% of the initial modulus)
10	100	0.02	2 %	88 %

The experimental data for E-glass/epoxy and E-glass/polyester with fiber volume fraction of 0.6 are shown in Fig. 5.

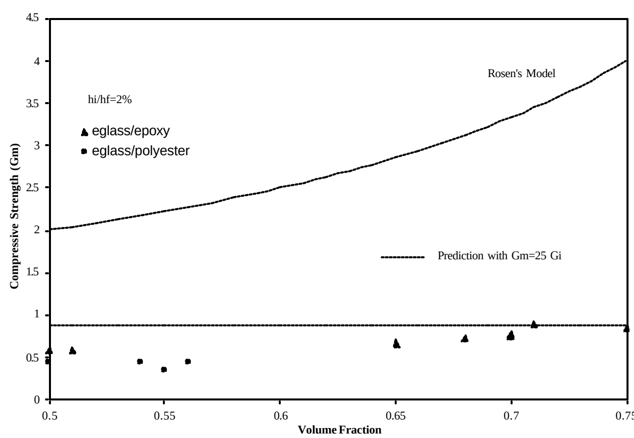

Fig. 5 Compressive strength of E-glass fiber composites (data from [23, 24]).

Fig. 5 shows that the model's prediction (for G_i/G_m= 0.04) matches the experimental results very well for E-glass/epoxy. The compressive strength data shown for E-glass/polyester is somewhat less than the predicted value, which is due to the softer inter-phase in polyester composites relative to epoxy composites

Compressive Strength of Boron Fiber/Epoxy

The diameter of boron fiber is about 100-150 μm; boron fiber/epoxy composite properties are given in Table 3.

Table 3 Properties of boron fiber/epoxy composites

h_i/h_f	G_i/G_m	V_f	Prediction (MPa)	Experiment (MPa)
0.13%	0.04	60 %	2314	2520

As boron fiber has a large diameter, the inter-phase has a much smaller influence. Hence, boron fiber composites have higher compressive strengths than high strength carbon fiber/epoxy composites. The analytical model also shows good agreement with the experimental data for this case.

Concluding Remarks

The conclusions from this study can be summarized as follows.

1. The composite compressive strength resulting from fiber micro-buckling depends on matrix modules, fiber modulus, inter-phase modulus, inter-phase volume, and fiber volume fraction. A composite with high inter-phase and matrix moduli has a higher compressive strength. The non-linearity in the moduli of the inter-phase and matrix play an important role in the composite compressive strength. A stiffer fiber like high strength carbon fiber will result in a stronger composite than a compliant fiber like glass fiber due to the smaller failure strain when the inter-phase and matrix moduli are higher. Composites reinforced with a larger fiber such as boron fiber have a higher compressive strength due to the smaller volume of inter-phase at the same fiber volume fraction. Composites with a higher fiber volume fraction contain more reinforcements that will increase the composite strength, but also increase the inter-phase volume that can have the opposite effect of reducing the compressive strength. Combining both effects, the compressive strength appears to be insensitive to the volume fraction for composites with fiber volume fractions ranging from 0.50 to 0.70.

2. Initial waviness of small magnitude and random arrangement is found to exist in composites. While not explicitly addressed in this paper, fiber waviness is small and cannot explain the compressive strength that is observed. Compressive failure occurs when the applied load reaches the buckling capacity of the composite when fiber micro-buckling is initiated and fibers self-adjust into a uniform sine wave (shear mode) leading to the formation of kink bands.

3. An inclined kink band is observed in the composites exhibiting the fiber buckling failure mode. The formation of this kink band is a complicated post-failure mechanism that can be modeled using non-linear finite element analysis.

4. The compressive strength of composites reinforced with some fibers increases with increasing matrix modulus. When the matrix modulus reaches a critical value, the failure mode changes from fiber micro-buckling into fiber failure. Composites reinforced with Kevlar fiber, ultra-high modulus (UHM) carbon fiber and high modulus (HM) carbon fiber all fail in the fiber failure mode, while lower modulus composites of glass fiber, boron fiber and high strength (HS) carbon fiber fail by fiber micro-buckling. Accounting for a softer inter-phase adjacent to each fiber provides a consistent prediction of composite compressive strength.

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