# MICROSTRUCTURAL VISCOPLASTIC CONTTINUUM MODEL FOR ASPHALT MIXES

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## **Summary**

Permanent deformation is one of the major distresses that cause severe damage in asphalt mixes. It is caused by high traffic loads associated with high field temperatures. This study is concerned with the development of the viscoplastic constitutive model to describe the permanent deformation of asphalt mixes. The model is based on Perzyna's formulation with Drucker-Prager yield function modified to account for the material anisotropy, the direction of loading, and microstructure damage. The model's parameters were determined using microstructure analysis and a series of triaxial experiments at different strain rates. The relationship between the model results and experimental measurements are presented in this paper.

## Introduction

Rutting in asphalt mixes develops gradually as the number of load applications increases. It is caused by a combination of densification (decrease in volume, and hence increase in density) and shear deformation. Tashman et al. [1] attributed permanent deformation to energy dissipation in three mechanisms:

- Overcoming the friction between the aggregates coated with binder,
- Overcoming interlocking between the aggregates, which is responsible for the material dilation, and
- Overcoming the bonding within the binder elements (cohesion) and between the binder and aggregates (adhesion).

Even though asphalt concrete consists of multi-components and interacted discrete particles, the concept of a representative continuum has been notionally accepted and used in describing the response to external loads and temperature change [2]. Researchers have for long evidenced the presence of elastic, viscoelastic, viscoplastic, and plastic components of mix response, where the presence of each is mainly affected by temperature, and loading rate. Asphalt mix behavior varies from elastic and linear viscoelastic at low temperatures and/or fast loading rates to, viscoplastic and plastic at high temperatures and/or slow loading rate. The model presented in this paper is formulated within the framework of theory of viscoplasticity since permanent deformation is associated

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with high temperature and slow loading rate. The model builds on the formulation developed by Tashman et al. [1]. However, the procedure for including anisotropy has been simplified, and the model is modified to account for the influence of stress path direction.

#### **Model Parameters Definition and Development**

The proposed constitutive model is part of an effort to relate the mechanisms discussed previously or their manifestations to permanent deformation. Hence, the following yield function is proposed:

$$f = F(I_1, J_2, J_3, d, \Delta, \xi) - \kappa = 0$$
(1)

where  $I_1$ ,  $J_2$  &  $J_3$  are the first stress invariant, second deviatoric stress invariant, and third deviatoric stress invariant, respectively. These invariants account for the effect of confinement, the dominant shear stress causing the viscoplastic deformation, and the direction of stress. D is a parameter that reflects the influence of the stress path direction.  $\Delta$  is an internal parameter that accounts for the effect of the material anisotropy.  $\xi$  is an internal parameter that accounts for the effect of damage in terms of cracks and air voids.  $\kappa$  is a hardening parameter that describes the growth of the viscoplastic yield surface.

The viscoplastic strain rate is defined using Perzyna's viscoplastic model and non -associative flow rule as follows:

$$\boldsymbol{\mathscr{E}}^{p} = \Gamma \cdot \langle \phi(f) \rangle \cdot \frac{\partial g}{\partial \sigma}$$

$$\tag{2}$$

where  $\Gamma$  is the fluidity parameter, which establishes the relative rate of viscoplastic straining, g is the plastic potential function, and  $\frac{\partial g}{\partial \sigma}$  is a deviatoric vector in stress space which defines the direction of the viscoplastic flow.  $\phi(f)$  is taken as a power law function of the viscous flow [1,3]. The Macauley brackets, <>, are used to indicate the following:

$$<\phi(f)>=\begin{cases} 0, & \phi(f) \le 0\\ \phi(f)=f^{N}, & \phi(f)>0 \end{cases}$$
(3)

where N is a parameter characterizing the material rate-sensitivity. As discussed earlier, a modified formulation of the Drucker-Praher yield function is adopted here with the following form:

$$f = \overline{\tau}^e - \alpha \overline{I}_1^e - \kappa = 0 \tag{4}$$

$$\overline{\tau}^{e} = \frac{\sqrt{\overline{J}_{2}^{e}}}{2(1-\xi)} \left[ 1 + \frac{1}{d} - \left(1 - \frac{1}{d}\right) \frac{\overline{J}_{3}^{e}}{\left(\overline{J}_{2}^{e}\right)^{3/2}} \right]$$
(5)

Based on the work of Tobita [4], the modified stress tensor ( $\overline{\sigma}_{ij}$ ) is expressed as a function of stress tensor  $\sigma_{ij}$  and fabric tensor  $F_{ij}$  as shown in Eq. (6):

$$\overline{\sigma}_{ij} = \frac{3}{2} \Big[ \sigma_{ik} F_{kj} + F_{ik} \sigma_{kj} \Big]$$
(6)

The anisotropic tensor  $F_{ij}$  is a function of  $\Delta$  which is measured using image analysis of two dimensional vertical sections of asphalt mix specimens. The  $\Delta$ value is equal to unity when all aggregates are oriented in the same direction, and is equal to zero for isotropic distribution. The  $\Delta$  value was found to vary between zero and 0.5 for asphalt mixes [1,2]. The effective stress in Eq. (6) is used to evaluate the invariants  $\overline{I}_1^e, \overline{J}_2^e \ll \overline{J}_3^e$ :

$$\overline{I}_{l}^{e} = \frac{1}{1 - \xi} \cdot \sigma_{ik} F_{ki} \tag{7}$$

$$\overline{J}_{2}^{e} = \frac{3}{2} \cdot \frac{1}{\left(1 - \xi\right)^{2}} \cdot \overline{S}_{ij} \overline{S}_{ji}$$

$$\tag{8}$$

$$\overline{J}_{3}^{e} = \frac{9}{2} \cdot \frac{1}{\left(1 - \xi\right)^{3}} \cdot \overline{S}_{ij} \overline{S}_{jk} \overline{S}_{ki} \tag{9}$$

 $\sigma_{ij}$  and  $S_{ij}$  are the stress tensor and the corresponding deviatoric tensor, respectively, and they are related as

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \tag{10}$$

 $\delta_{ij}$  is kronecker delta, where its components are 1 if i=j and 0 if  $i\neq j$ . Effective stress theory is used here to account for the effect of damage by dividing all stresses by the percentage of intact area of the material which is equal to  $1 - \xi$ , where  $\xi$  is the damage parameter or area of air voids and cracks [5], and it is measured using X-ray Computed Tomography of asphalt mix specimens loaded

to different strain levels [2]. d is the ratio of tensile yield stress to compressive yield stress and its value is selected such that the yield surface convexity condition is maintained.  $\alpha$  is a parameter that reflects the material frictional properties.  $\kappa$  is a hardening parameter that reflects the combined effect of the cohesion and frictional properties of the material. The evolution law for  $\kappa$  is postulated based on the experimental measurements presented by Masad et al. [2], and motivated by the work of Dafalias [6]:

$$\kappa = \kappa_o + \kappa_1 \overline{\varepsilon}_{vp}^{\varepsilon_2} \left[ 1 - exp\left( -\kappa_3 \overline{\varepsilon}_{vp} \right) \right]$$
(11)

where  $\overline{\varepsilon}_{vp}$  is the effective viscoplastic strain,  $\kappa_o$  defines the initial yield surface  $\kappa_1$  and  $\kappa_2$  are hardening coefficients to account for the effect of strain rate on the material work-hardening, and  $\kappa_3$  is a dynamic recovery coefficient.  $\overline{\varepsilon}_{vp}$ , and  $\mathcal{E}_{vp}$  are the equivalent viscoplastic strain and strain rate, respectively. The plastic potential function, g, is assumed to have the same form as the yield function but with a slope of  $\beta$  which influences the proportions of the volumetric and deviatoric strains. The evolution of  $\beta$  has the following form:

$$\beta = \beta_1 - \beta_2 e^{(-\beta_3 \cdot \bar{\varepsilon}_{vp})} \tag{12}$$

where  $\beta_1$ ,  $\beta_2$ , and  $\beta_3$  are positive coefficients Figure 1 shows the projection of the yield surface on the  $\pi$  plane at d = 0.8. It can be seen that the yield stress in the axial direction (direction 1) increases as the material anisotropy increases. The opposite happens for the radial direction (directions 2 and 3). This behavior can be explained by micromechanics analysis as discussed by Tobita [4] and Masad et al. [7].

## **Experiment Description and Results**

Four-inch diameter asphalt specimens of granite, limestone, and gravel mixes were compacted to a target air void content of 7.0 percent. The specimens were deformed at strain rates of 0.0660%/min, 0.318%/min, 1.60%/min, 8.03%/min, and 46.4%/min at confining pressures of 0 psi, 15 psi, and 30 psi. Two replicates of each mix were tested, and axial and radial stresses and strains were recorded throughout testing. All specimens were loaded up to an axial strain of 8 percent or until failure, whichever occurred first. Examples of experimental and fitted stress-strain relationships are shown in Figure 2.

The model parameters were able to distinguish between the three mixes in terms of their aggregate shape properties. Masad et al. [2] have reported, based

on image analysis of these three aggregates, that limestone and granite had comparable texture level which was higher than gravel. Granite exhibited about 20% more angularity than limestone, and about 50% more angularity than gravel. On the other hand, gravel was the most spherical, and limestone was the least spherical (most flat/elongated). The experimental results and the viscoplastic model showed that the gravel mix had the highest potential for permanent deformation, and had the lowest anisotropy level due to high aggregate particles sphericity. In addition, the gravel mix was found to have the least work hardening capability. This behavior of the gravel mix is attributed mainly to the low angularity and texture of aggregates. The granite mix had the highest  $\kappa$  and  $\alpha$  values reflecting the high adhesion between binder and aggregates, and the high aggregate friction due to the granite texture. The influence of aggregate angularity was manifested in dilation where the experimental results and model parameters showed granite to have the highest dilation followed by limestone. The gravel mix experienced both dilation and contraction behaviors depending on strain rate and confinement.

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Figure (1): Projections of the Yield Surface on the  $\pi$  Plane at Different Anisotropy Levels.



Figure (2). Experimental Results and Model Fitting to the Data at 15-Psi Confinement a) Granite b) Limestone c) Gravel.