Output only modal identification of a laboratory model

F. Magalhães[1], E. Caetano[1], A. Cunha[1]

Summary

This work concerns the experimental modal identification of a small physical model of a building structure, measuring the structural response to an unknown ambient excitation. Different system identification methods, in frequency and in time domain, are applied and the results are compared. The model has the particularity of having two modes with close frequencies, which is an interesting challenge for the identification methods.

Introduction

Civil engineering structures, because of size, are very difficult to be artificially excited. So the best approach, to identify their modes shapes and natural frequencies, is to apply methods that use only the response (output) of the structure under ambient excitation. These methods are called output-only methods. The most commonly used output-only method in civil engineering applications is the Peak-Picking (PP). However, this method has some limitations, as it is shown in the paper. In the last years, some more powerful methods, like the Frequency Domain Decomposition (FDD) and the Stochastic Subspace Identification (SSI), have been developed and applied with success [1].

In this paper, the implementation of the PP and FDD methods was done in MatLab [2] and their limitations and potentialities are shown using an application to a small laboratory model. The SSI method was applied using a toolbox for MatLab developed at the University of Leuven (MACEC) [3].

Description of the model and tests

The analyzed model, represented in Figure 1, is formed by three slabs connected by four columns. The slabs are made of steel and have a square shape with a width of 15 centimeters and a thickness of 1 centimeter. The columns are aluminum blades with a height of 17 centimeters between floors.

The accelerations were measured using 6 piezoelectric accelerometers in 2 setups as represented in Figure 2 (the arrows represent the measured direction). In each setup, acceleration time series of 140 seconds were collected with a sampling frequency of 256 Hz. The three accelerometers in the top slab (reference sensors) were kept in the same

[1] Faculty of Engineering of University of Porto (FEUP), Portugal

position during the two setups, while the other 3 (moving sensors) were located at the second floor, in the first setup, and were moved to the first floor, in the second setup.

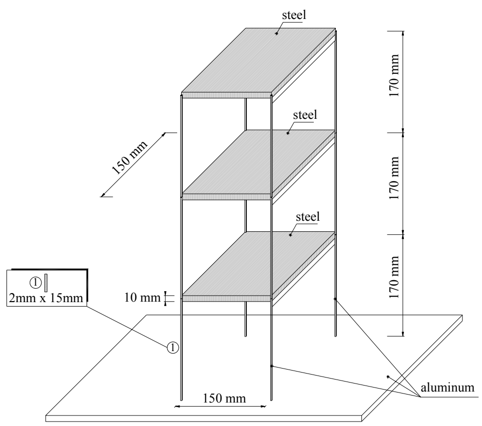

Figure 1 – Perspective of the model

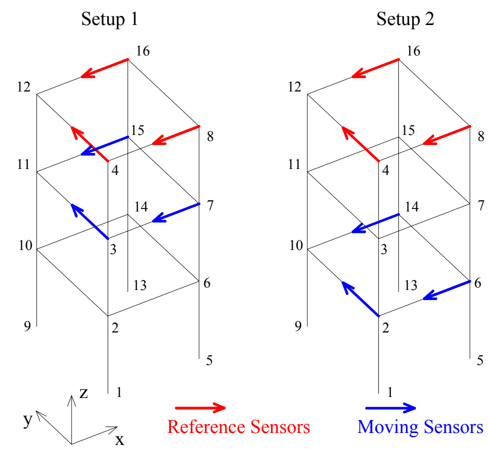

Figure 2 – Placement of accelerometers in each setup

Output-only modal identification methods

Output-only modal identification methods can be classified in two groups: the ones that work in the frequency domain, as the PP and FDD, and those that work in the time domain, as the SSI methods.

For frequency domain methods the first step consists in the estimation of the spectral density functions of the structural response. Those estimates can be obtained using the Welch procedure [2], which involves: (i) division of the response records in several segments (eventually with some overlap), (ii) application of a signal processing window to the data segments in order to reduce leakage (usually a Hanning window), (iii) computation of the discrete Fourier transform (DFT) of the windowed segments, (iv) computation of the auto and cross spectra using the DFT of the windowed segments and finally average the spectra of the segments of the same records.

In the PP method, all the auto-spectra are normalized and averaged in order to obtain an averaged normalized power spectral density function (ANPSD), which shows all the resonance peaks of the structure. If the excitation doesn't contain any particular periodic contribution, the damping is small and the natural frequencies are well separated, these peaks are in correspondence with the frequencies of the dynamic system. The mode shapes are identified looking at the amplitude and phase of the transfer function between the responses which are calculated using the auto and cross spectra [4]. In fact, doing this, instead of obtaining mode shapes, we obtain operational deflection shapes, which for closely spaced modes are the superposition of multiple modes.

In the FDD method, a spectral matrix is constructed, in which the number of lines is equal to the number of measured points and with as much columns as the measured reference points. Each column contains the cross spectrums between the structure response in all the measured points and the structure response in one of the reference sensors. This matrix is decomposed, at each frequency, in singular values and vectors using the SVD algorithm. The singular values of the spectral matrix, under some assumption (white noise excitation, low damping and orthogonal mode shapes for close modes), are auto spectral density functions corresponding to single degree of freedom systems that are associated with the different modes of the structure [5]. As it is stated in reference [5], the frequencies of the structure are identified looking at the peaks of the singular values of the spectrum matrix and the mode shapes are estimated by the singular vector of the spectrum matrix calculated at the resonance frequency and associated to the singular value where the peak is present.

An interesting alternative to these frequency domain methods is the stochastic subspace identification method. This method relies on a stochastic state space model that is represented by the following equations [6], assuming the excitation as a white noise:

$$\begin{aligned} x_{k+1} &= A \cdot x_k + w_k \\ y_k &= C \cdot x_k + v_k \end{aligned} \tag{1}$$

where y_k is a column vector, with l lines (number of measured outputs), that characterizes the output of the system at the instant of time k , x_k is the state vector, which has n lines (dimension of the state space model), w_k represents the noise used to simulate the ambient excitation and the model inaccuracies, and v_k is the noise that simulates the error introduced by the measurement system and also the ambient excitation. The matrix A ($n \times n$) is the state transition matrix and completely characterizes the dynamics of the structure, the matrix C ($l \times n$) is the output matrix and specifies how the internal states are transformed in outputs.

The identification of the state space model, by determination of matrices A and C, can be done using the correlations of the outputs or using the time series. Both procedures (SSI-COV and SSI-DATA, respectively) are described in reference [6]. In the present application, the SSI-DATA, implemented in MACEC, was used.

After the identification of the state space model, the modal parameters are obtained from the matrices A and C. The eigenvalue decomposition of A gives:

$$A = \Psi \cdot \Lambda_d \cdot \Psi^{-1} \tag{2}$$

where Ψ ($n \times n$) is the eigenvector matrix and Λ_d ($n \times n$) is a diagonal matrix containing the eigenvalues, μ_i, of the discrete state space model.

The eigenfrequencies, ω_i, and the damping ratios, ξ_i, are found from:

$$\mu_i = e^{\lambda_i \Delta t} \qquad \lambda_i, \lambda_i^* = -\xi_i \omega_i \pm j\sqrt{1-\xi_i^2}\,\omega_i \tag{3}$$

where λ_i are the eigenvalues of the continuous sate space model and Δt is sampling time. The mode shapes $V\,(\,l \times n\,)$ are calculated with the following formula:

$$V = C \cdot \Psi \tag{4}$$

Application

The time series of acceleration measured at the instrumented points were used as inputs for the different identification methods, after trend removal (elimination of DC-component).

The application of the PP method gives the ANPSD represented in Figure 3. From the peaks of this plot, it is possible to identify 7 natural frequencies.

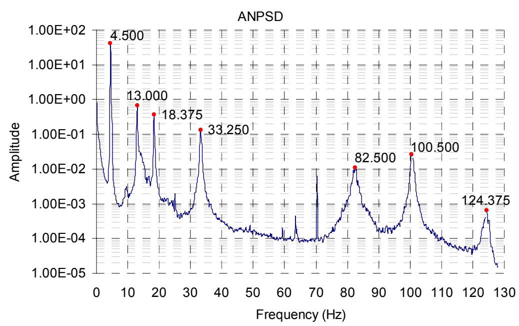

Figure 3 – Averaged normalized power spectral density function

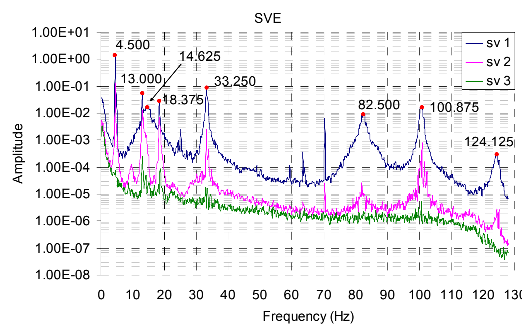

Figure 4 – Singular values spectra

From the transfer functions between the outputs, it is also possible to obtain the operational deflection shapes, in correspondence with the identified natural frequencies, represented in Figure 5. It is clear that the second operational deflection shape has the contribution of two mode shapes: the second bending mode in *x* direction and the first bending mode in *y* direction.

The SVD of the spectrum matrix at each frequency gives 3 singular values (number of reference sensors) that are plotted in Figure 4. Now, the natural frequencies are identified looking at the peak of the singular values. Looking at Figure 4, it is possible to identify 8 frequencies. Seven of them are very similar to the ones identified using the PP, but a new frequency of 14.625 Hz appears. This frequency wasn't identified in the application of the PP method, because it is relatively close to the frequency of 13 Hz.

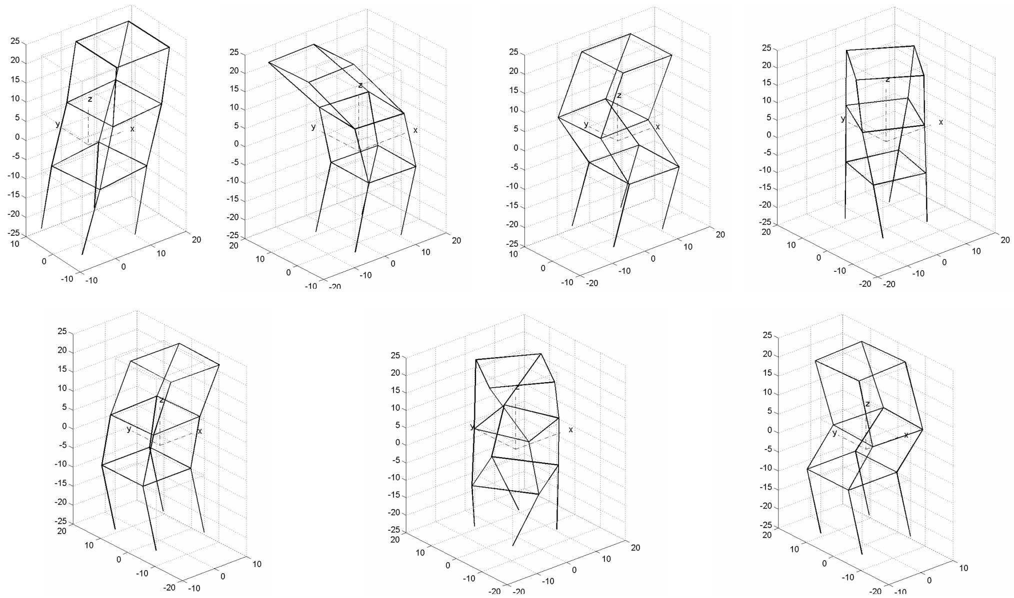

Figure 5 – Operational deflection shapes obtained with PP method

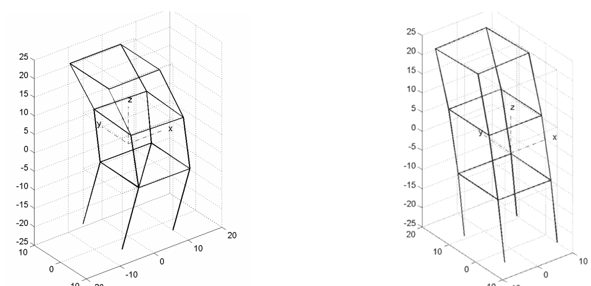

Figure 6 – Mode shapes of closely spaced frequencies identified with FDD method

Table I – Identified frequencies (Hz)

Mode	PP	FFD	SSI-DATA
1	4.500	4.500	4.512
2	13.000	13.000	13.076
3	-	14.625	14.439
4	18.375	18.375	18.406
5	33.250	33.250	33.265
6	82.500	82.500	82.360
7	100.875	100.875	100.747
8	124.375	124.125	124.428

The singular vectors of the spectrum matrix calculated at the closely spaced resonance frequencies (13 Hz and 14.625 Hz), and associated to the first singular value, are represented in Figure 6. Now, the coupled mode shapes are uncoupled. The mode shapes obtained with this method and associated with the other frequencies are similar to the ones obtained with the application of PP.

The application of the SSI-DATA method confirmed the results of the FDD method: the identified frequencies, displayed (in Table I), are similar and the mode shapes are almost equal.

Conclusions

By implementation in MatLab of PP and FDD methods, and its application to a small physical model of a building structure, it was possible to show the limitations and potentialities of each one. It was demonstrated that the FDD method is able to identify modes with close frequencies, overcoming the strongest limitation of the PP method. Beyond that, the comparison between the results of the FDD and SSI-DATA methods shows that they lead to similar results.

Reference

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