

## **An improved procedure for the evaluation of the lateral-torsional buckling resistance of unrestrained steel beams in case of fire**

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### **Summary**

The final draft of the EN version of part 1-1 of Eurocode 3 has introduced significant changes in the evaluation of the lateral-torsional buckling resistance of unrestrained beams at room temperature that reduce the over-conservative approach of ENV 1993-1-1 in the case of non-uniform bending.

In line with the safety format of the lateral-torsional buckling code provisions for cold design, an alternative proposal to fire conditions is presented in this paper that addresses the issue of the influence of the loading type on the resistance of the beam, achieving better agreement with the real behaviour while maintaining safety.

### **Introduction**

Recently, at the occasion of the conversion of Eurocode 3 from ENV to EN status, significant changes were introduced in the evaluation of the lateral-torsional buckling resistance of unrestrained beams at room temperature [1] that reduce the over-conservative approach of ENV 1993-1-1 [2] in the case of non-uniform bending.

Also recently, but for opposite reasons, Vila Real et al [3] proposed an alternative expression for the lateral-torsional buckling resistance of unrestrained beams under fire loading. This change, already adopted by the project team of part 1-2 of EC3 [4], was triggered by the identification of the unconservative nature of the previous expression [5] for the case of a simply-supported beam with fork supports under uniform bending.

It is the objective of the present paper to propose a consistent safety format for the lateral-torsional buckling resistance of beams under fire loading, by adapting the newly proposed methodology for cold design to fire design. This adaptation is subsequently assessed using the specialised finite element code SAFIR [6], which is a finite element code for geometrical and material non-linear analysis, specially developed at the University of Liege for studying structures subjected to fire.

A three-dimensional (3D) beam element has been used, based on the following formulations and hypotheses: Displacement type element in a total co-rotational description; Prismatic element; The displacement of the node line is described by the displacements of the three nodes of the element, two nodes at each end supporting seven degrees of freedom, three translations, three rotations and the warping amplitude, plus

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one node at the mid-length supporting one degree of freedom, namely the non-linear part of the longitudinal displacement; The Bernoulli hypothesis is considered, i.e., in bending, plane sections remain plane and perpendicular to the longitudinal axis and no shear deformation is considered; No local buckling is taken into account, which is the reason why only Class 1 and Class 2 sections can be used; The strains are small (von Kármán hypothesis), i.e.

$$\frac{1}{2} \frac{\partial u}{\partial x} \ll 1 \quad (1)$$

where  $u$  is the longitudinal displacement and  $x$  is the longitudinal co-ordinate; The angles between the deformed longitudinal axis and the undeformed but translated longitudinal axis are small, i. e.  $\sin \varphi \cong \varphi$  and  $\cos \varphi \cong 1$ , where  $\varphi$  is the angle between the arc and the chord of the translated beam finite element; The longitudinal integrations are numerically calculated using Gauss' method; The cross-section is discretised by means of triangular or quadrilateral fibres. At every longitudinal point of integration, all variables, such as temperature, strain, stress, etc., are uniform in each fibre; The tangent stiffness matrix is evaluated at each iteration of the convergence process (pure Newton-Raphson method); Residual stresses are considered by means of initial and constant strains [7]; The material behaviour in case of strain unloading is elastic, with the elastic modulus equal to the Young's modulus at the origin of the stress-strain curve. In the same cross-section, some fibres that have yielded may therefore exhibit a decreased tangent modulus because they are still on the loading branch, whereas, at the same time, some other fibres behave elastically. The plastic strain is presumed not to be affected by a change in temperature [8]; The elastic torsional stiffness at 20°C that is calculated by the code has been adapted in an interactive procedure in order to reflect the decrease of material stiffness at the elevated temperature [9].

### Case Study

A simply supported beam with fork supports was chosen to explore the validity of the beam safety verifications, as shown in figure 1. Regarding the bending moment variation along the member length, three values, (-1, 0, 1), of the  $\psi$  ratio (see fig. 1) have been investigated as well as a uniformly distributed load or a mid span concentrated load. An IPE 220 steel section of grade S 235 was used.

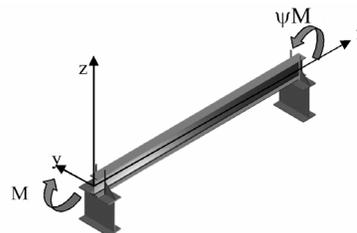


Fig. 1 – Simply supported beam with uniform bending.

Uniform temperature in the cross-section has been used so that comparison between the numerical results and the eurocodes could be made. In this paper the temperatures used were 400, 500, 600 and 700 °C, deemed to adequately represent the majority of practical situations.

A lateral geometric imperfection given by the following expression was considered:

$$y(x) = \frac{l}{1000} \sin\left(\frac{\pi x}{l}\right) \quad (2)$$

An initial rotation around the longitudinal axis with a maximum value of  $l/1000$  rad at mid span was also introduced.

Finally, the residual stresses adopted are constant across the thickness of the web and flanges. A triangular distribution as shown in figure 2, with a maximum value of  $0.3 \times 235$  MPa, has been used [10].

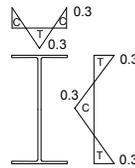


Fig. 2 – Residual stresses: C – compression; T – tension

### Parametric Evaluation of the Lateral-Torsional Buckling Code Provisions of Eurocode 3

In order to provide a basis for the subsequent parametric study, the code provisions for the lateral-torsional buckling of beams at room and high temperatures are described below in detail.

At room temperature, according with the ENV 1993-1-1 and prEN 1993-1-1, beams with cross-sectional classes 1 and 2 subjected to major-axis bending, must generically satisfy the following relation:

$$M_{b,Rd} = \chi_{LT} W_{pl,y} \frac{f_y}{\gamma_{M1}} \quad (3)$$

In both the ENV and EN versions of part 1-1 of Eurocode 3, the reduction factor  $\chi_{LT}$  is formally based on the Rondal-Maquoi formula, detailed derivations being found in [11].

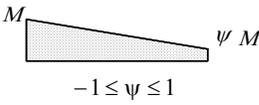
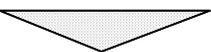
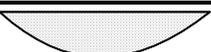
To address the issue of the influence of the bending moment diagram, the use of a modified reduction factor,  $\chi_{LT,mod}$  (4), is allowed, that depends on the moment distribution correction factor,  $k_c$ , illustrated in Table 1 for some common loading cases.

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \quad \text{but} \quad \chi_{LT,mod} \leq 1 \quad (4)$$

with

$$f = \min \left\{ 1 - \frac{1}{2}(1 - k_c) \left[ 1 - 2(\bar{\lambda}_{LT} - 0.8)^2 \right]; 1 \right\} \quad (5)$$

Table 1: Correction factors  $k_c$

	Moment distribution	prEN 1993-1-1	New proposal for elevated temperature
		$k_c$	$k_c$
A	 $M$ $\psi M$ $-1 \leq \psi \leq 1$	$\frac{1}{1.33 - 0.33\psi}$	$0.6 + 0.3\psi + 0.15\psi^2$ but $k_c \leq 1$
B		0.86	0.79
C		0.94	0.91

At high temperatures, according to prEN 1993-1-2, the design buckling resistance moment of a laterally unrestrained beam with a class 1 or 2 cross-section type, is obtained as follows:

$$M_{b,fi,Rd} = \chi_{LT,fi} W_{pl,y} k_{y,\theta,com} f_y \frac{1}{\gamma_{M,fi}} \quad (6)$$

where  $\chi_{LT,fi}$ , is given by

$$\chi_{LT,fi} = \frac{1}{\phi_{LT,\theta,com} + \sqrt{(\phi_{LT,\theta,com})^2 - (\bar{\lambda}_{LT,\theta,com})^2}} \quad (7)$$

with

$$\phi_{LT,\theta,com} = \frac{1}{2} \left[ 1 + \alpha \bar{\lambda}_{LT,\theta,com} + (\bar{\lambda}_{LT,\theta,com})^2 \right] \quad (8)$$

The imperfection factor  $\alpha$  is a function of the steel grade and is given by:

$$\alpha = 0.65 \sqrt{235 / f_y} \quad (9)$$

Figure 3 clearly highlights that there is scope for improvement in the evaluation of the lateral-torsional buckling resistance of beams. Based on the prEN 1993-1-1 version of the Eurocode 3 it seems reasonable to propose, at high temperature, a second method, more accurate and less conservative, that improves the results denoted “prEN 1993-1-2” in Figure 3.

Given that the main factor responsible for the over-conservative nature of the lateral-torsional buckling resistance at high temperatures was linked to the loading type, the new proposal also adopts a modified reduction factor for lateral-torsional buckling,  $\chi_{LT,fi,mod}$ , given by

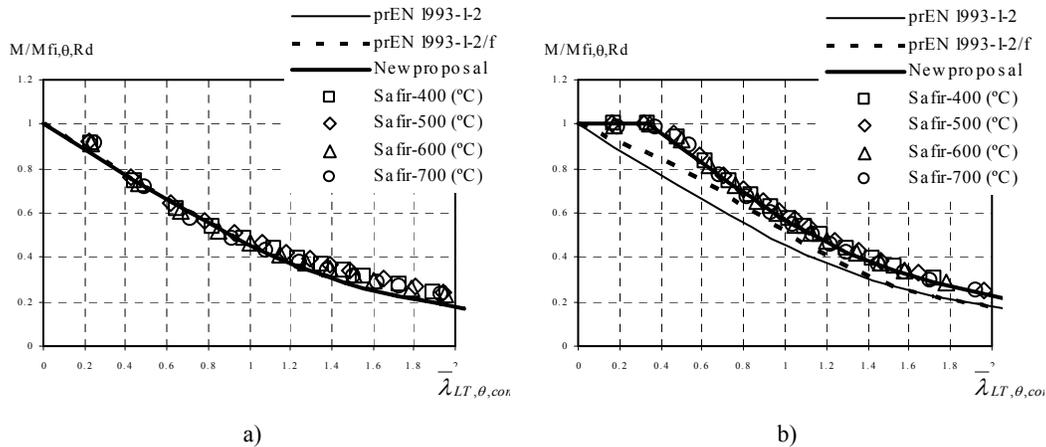
$$\chi_{LT,fi,mod} = \frac{\chi_{LT,fi}}{f} \quad \text{but} \quad \chi_{LT,fi,mod} \leq 1 \quad (10),$$

where  $f$  depends on the loading type.

Initially, the adequacy of part 1-1 proposals for  $f$  and  $k_c$  (see table 1) were tested. These results, denoted as “prEN 1993-1-2/ $f$ ” in figure 3, are better and closer to the numerical values but still remain conservative. Consequently, in order to have a better approximation, taking into account the moment distribution between the lateral restraints of members, new coefficients for  $f$  and  $k_c$  were adjusted, given by the following equation

$$f = 1 - 0.5(1 - k_c) \quad (11)$$

where  $k_c$  is a correction factor according to the new proposal of table 1. For others bending diagrams not presented in table 1  $k_c = 1$  should be adopted.



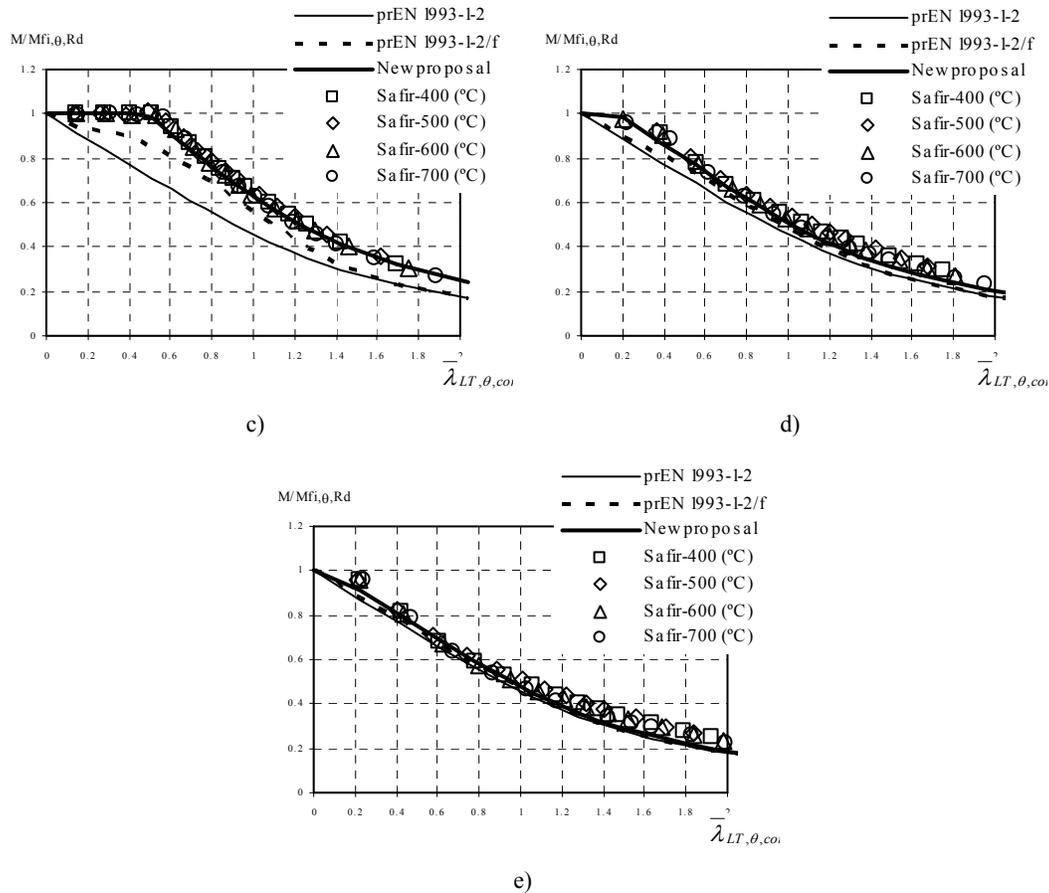


Fig. 3 – Beam design curves. a)  $\psi = 1$ ; b)  $\psi = 0$ ; c)  $\psi = -1$ ; d) Case B; e) Case C

As it can be seen in figure 3, this new proposal shows a very good agreement with the numerical results. This figure illustrates the results for the chosen values of ratio  $\psi$  of case A as well as cases B and C.

### Conclusions

A new proposal for the lateral-torsional buckling resistance of beams under fire loading has been proposed. It was adapted from the newly proposed methodology for cold design from the later version of prEN 1993-1-1 [1]. The proposed method approximates more closely the numerical results of unrestrained steel beams under fire conditions, while still remaining on the safe side.

It is worth noting that experimental confirmation resulting from well instrumented and carefully carried out experimental tests to verify whether the present proposal can

actually reproduce the real behaviour would be welcome. It is nevertheless noted that there is a low probability for the two structural imperfections, residual stresses and initial imperfection, occurring simultaneously in a test, with the high amplitude assumed here in the numerical simulations.

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