# Influence of Dilatancy and Elastic Modulus on Simulation of Undrained Behavior

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# **Summary**

The purpose of this paper is to investigate the influence of dilatancy and elastic modulus in order to accomplish a sufficient prediction for undrained behavior. We could obtain the following conclusions. 1) The initial dilatancy parameter depends on the mean stress. 2) The elastic shear modulus changes according to the stress ratio. 3) The initial dilatancy parameter plays an important role for the simulation of undrained condition.

#### Introduction

There are a lot of constitutive models especially for sand. However, it is not so easy to predict the undrained behavior using these constitutive models from the viewpoint of the effective stress. The behavior of element tests under undrained condition can be simulated based on the relationships among stress-strain, dilatancy and elastic modulus.

The purpose of this paper is to investigate the influence of dilatancy and elastic modulus for accomplishing a sufficient prediction of undrained behavior, although it is restricted within the limits of triaxial compression stress condition.

## **Experimental Data**

The tests were carried out under constant mean effective stresses, 49, 98 and 196 kPa. The material is Toyoura sand with a relative density of 70 %. The definitions of stress and strain quantities are defined as follows.

$$p = \frac{1}{3}\sigma_{ii}, \quad q = \sqrt{\frac{3}{2}\left(\sigma_{ij} - p\delta_{ij}\right)\left(\sigma_{ij} - p\delta_{ij}\right)}$$

$$\varepsilon_{vol}^{p} = \varepsilon_{ii}^{p}, \quad \overline{\varepsilon}^{p} = \sum d\overline{\varepsilon}^{p}$$

$$d\overline{\varepsilon}^{p} = \sqrt{\frac{2}{3}\left(d\varepsilon_{ij}^{p} - \frac{1}{3}d\varepsilon_{vol}^{p}\delta_{ij}\right)\left(d\varepsilon_{ij}^{p} - \frac{1}{3}d\varepsilon_{vol}^{p}\delta_{ij}\right)}$$

$$(1)$$

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The relationship between the stress ratio and the equivalent plastic strain is shown in Figure 1. The stress ratio-equivalent plastic strain curves are almost same for the cases of 98 and 196 kPa. It is desirable to have a unique relationship including the case of 49 kPa from the viewpoint of constitutive model. Therefore, further research is needed. The relationship between the plastic volumetric strain and the equivalent plastic strain is also shown in Figure 2. The contractive volumetric strain becomes larger as the mean effective stress is getting larger at the initial stage of loading. The dilatancy can be expressed using the relationship between the normalized plastic work and the equivalent plastic strain. So far, it is regarded that the relationship between normalized plastic work and equivalent plastic strain is almost unique independent of the density, the mean stress etc. [1].

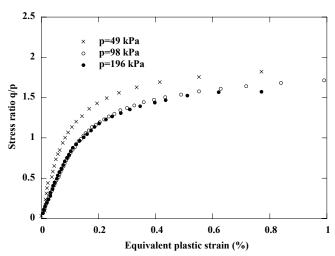


Figure 1 Stress ratio-equivalent plastic strain

#### **Dilatancy Parameter**

Here, the dilatancy is studied carefully, especially for the small strain region. The dilatancy parameter  $\mu$  can be obtained by the following equation derived from the relationship between the normalized plastic work and the equivalent plastic strain.

$$\mu = \frac{q}{p} + \frac{d\varepsilon_{vol}^p}{d\overline{\varepsilon}^p} \tag{2}$$

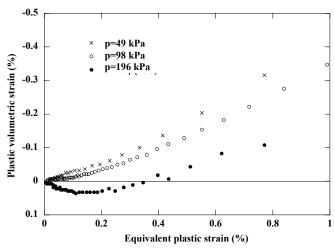


Figure 2 Plastic volumetric strain-equivalent plastic strain

The relationship between the dilatancy parameter and the equivalent plastic strain is shown in Figure 3. It can be found that the dilatancy parameter takes different values for each mean effective stress at small strain region, while they approach a unique value at large strain region. The difference among the initial dilatancy parameter values is caused by the dilatancy index  $d\varepsilon_{vol}^p/d\overline{\varepsilon}^p$ , since the stress ratio is zero at the beginning of loading. The initial dilatancy parameter changes from negative to positive, as the mean effective stress is getting larger.

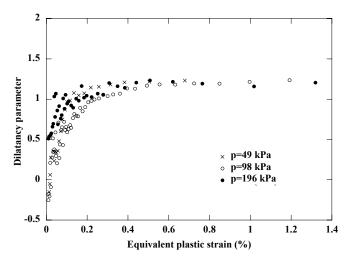


Figure 3 Dilatancy parameter-equivalent plastic strain

# **Dependence of Elastic Shear Modulus on Stress Ratio**

The following equation can be obtained under the condition that the volumetric strain should be zero under undrained condition.

$$\Delta p = -Kd\varepsilon_{vol}^{p} \tag{3}$$

The mean effective stress increment  $\Delta p$  can be calculated using the elastic bulk modulus K and the plastic volumetric strain increment.

The bender elements developed by Shirley and Hampton [2] is used to measure the elastic shear modulus. The bender elements are small electro-mechanical transducers, which either bend as an applied voltage is changed or generate a voltage as they bend. A transmitter element and a receiver element can be fitted in to the top and bottom platens of a variety of soil testing devices such as triaxial test apparatus. As the applied voltage is changed, the transmitter element bends and transmits a shear wave through the sample.

Numerical analyses are carried out on the input and output signals in the bender element test in order to minimize the errors in estimating the travel time of the shear wave. A cross correlation function can be computed as the inverse Fast Fourier Transform of the cross-power spectrum of the two signals [3].

The variation of the elastic shear modulus normalized by the initial elastic shear modulus with the stress ratio is shown in Figure 4. A sweep sine wave was used as an input waveform. The relationship between the elastic modulus and the stress ratio is relatively independent of the mean effective stress. It can be seen that the elastic shear modulus initially increases and then it decreases with the increasing stress ratio.

### **Simulation of Undrained Condition**

The undrained triaxial compression tests were simulated using the relationships among the stress ratio-strain, the dilatancy parameter-strain and the dependence of elastic modulus on stress ratio. Figure 5 shows the comparison between the simulated results and the experimental ones (Kuwano et al. [4]) for the case with an initial confining stress of 98 kPa. Two kinds of dilatancy parameters for 98 and 196 kPa were used for the simulations. The simulated results obtained using the dilatancy parameter of 98 kPa relatively agree with the experimental ones. However, the simulated results obtained using the dilatancy parameter of 196 kPa have a different tendency from others. Therefore, it can be concluded that the initial dilatancy parameter has a significant influence on the simulated undrained behavior. The influence of dependence of elastic modulus on stress ratio was negligible in this case, although these results were not shown.

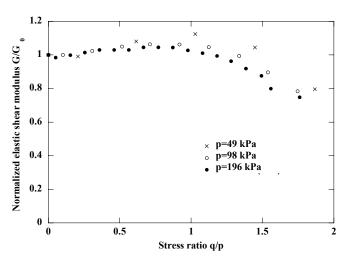


Figure 4 Variation of normalized elastic shear modulus

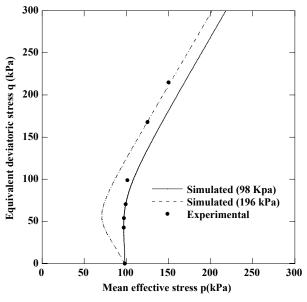


Figure 5 Stress paths

## **Conclusions**

The following conclusions are obtained based on the experiments of drained triaxial compression tests for Toyoura sand with a relative density of 70 % and the

comparison with the undrained experiments, although further study is needed especially for the different stress paths and different relative densities..

- 1) The initial dilatancy parameter depends on the mean effective stress, and the dilatancy parameter finally becomes the same value independent of the mean effective stress.
- 2) The elastic shear modulus depends not only on the mean effective stress and the initial void ratio but also on the stress ratio.
- 3) The initial dilatancy parameter has a significant influence on the simulated results under undrained condition.

#### Reference

- 1 Momen, H. and Ghaboussi, J. (1982): "Stress dilatancy and normalized work for sands", *IUTAM conference on deformation and failure of granular materials*, Delft, pp. 265-274.
- 2 Shirley, D. and Hampton, L. D. (1977): "Shear-wave measurements in laboratory sediments", *J. Acoust. Soc. America*, Vol. 63, No. 2, pp. 607-613.
- 3 Viggiani, G. (1995): "Panelist Discussion: Recent advances in the interpretation of bender element tests", *Proc. of the first international conference on pre-failure deformation characteristics of geomaterials*, Sapporo, Vol. 2, pp. 1099-1104.
- 4 Kuwano, J., Hashizume, H. and Takahara, K. (1994): "Deformation of clayey sand during saturation, consolidation and undrained shear process", *Proc. of the International Symposium on pre-failure deformation characteristics of geomaterials*, Sapporo, Vol. 1, pp. 125-130.