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Cyclic Elasto-Plastic Constitutive Model and Finite Element Analysis

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Summary

The strain softening and cyclic elasto-plastic constitutive model for geomaterial is applied to a plane strain compression test analysis. The constitutive model is based on experimental findings about inherent and induced anisotropies involved in sand. The finite element method employing dynamic relaxation method combined with the generalized return-mapping algorithm are applied to the analysis.

Introduction

The dynamic relaxation method combined with the generalized return-mapping algorithm is applied to the integration algorithms of elasto-plastic constitutive relations including the effect of the shear band. Both explicit type and implicit-explicit type dynamic relaxation methods are applied to cyclic soil problems.

In order to guarantee a mesh-objective consumption of energy, the strain softening modulus is made a function of element size [1]. This kind of shear banding model can incorporate a characteristics length of shear band in the material modeling based on physical experimental observations of strain localization with a finite size.

Explicit Dynamic Relaxation Method

Solution to systems of nonlinear equations involving the governing non-linear equation is obtained as

$$P - P^{init} = F \quad \text{and} \quad P = \sum_{N} \int_{vol} B^T \varsigma dv \qquad (1)$$

where P is the internal force vector, P^{init} is the nodal forces due to initial stresses, F is the external force vector, B^T is the strain-displacement transformation matrix, N is the number of elements in Finite Element discretization, σ is the stresses at Gauss points in each element, and *vol* is the volume of each element. The solution to the above governing equation can be

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obtained by achieving the steady state response of the following dynamic equation of motion.

$$M_D a + Cv + P - P^{init} = F \tag{2}$$

where M_D is the diagonalized mass matrix, C is the damping matrix, which is a vector for critically damped dynamic relaxation, v is the velocity vector, and a is the acceleration vector.

Then, applying the central difference method to Eq.(2) and replacing the damping by the following relation;

$$C = \alpha M_D \tag{3}$$

the following relaxation equation can be derived.

$$q_{n+1} = \frac{1}{1+0.5\alpha\Delta t}\left[\frac{\Delta t^2}{M_D}(F - P + P^{init})^t + 2q_n - (1-0.5\Delta t)q_{n-1}\right] \tag{4}$$

Here, q_n is the displacement vector at time n, Δt is the time increment and α is the damping ratio which is the most critical value to be determined.

A number of methods can estimate a reasonable value of the critical damping parameter. We employ the Rayleigh's quotient to determine the approximate damping in an adaptive way using the current solution parameters.

Implicit-Explicit Dynamic Relaxation Method

The explicit dynamic relaxation method suffers from the stability problem. The dynamic relaxation method with implicit-explicit type [2] is effective for soils-structures interaction problems. The explicit method without stiffness matrix is applied to parts of soil mass and the implicit method is used to a part of the stiff structures such as retaining wall, therefore two methods are used simultaneously. The algorithms are based on the Newmark scheme and the Skyline solver is applied. In large strain calculations, we use the rotation neutralized strain proposed by Nagtegaal [3].

Constitutive Model for Cyclic Behavior of Soil

A simplified and generalized version of mesh size-dependent softening modulus method is used in this study. A material model for a real granular material (i.e., Toyoura sand) used with the features of nonlinear pre-peak, pressure-sensitivity of the deformation and strength characteristics of sand, non-associated flow characteristics, post-peak strain softening, and strain-localization into a shear band with a specific width.

The yield function (f) and the plastic potential function (Φ) are given by:

$$f = \alpha I_1 + \frac{\bar{\sigma}}{g(\theta_L)} = 0 \tag{5}$$

$$\Phi = \alpha' I_1 + \bar{\sigma} = 0 \tag{6}$$

where

$$\alpha = \frac{2\,sin\,\phi}{\sqrt{3}(3 - sin\,\phi)} \qquad \alpha' = \frac{2\,sin\,\psi}{\sqrt{3}(3 - sin\,\psi)} \tag{7}$$

where I_1 is the first invariant (positive in tension) of deviatoric stresses and $\bar{\sigma}$ is the second invariant of deviatoric stress. With the Mohr-Coulomb model, $g(\theta_L)$ takes the following form:

$$g(\theta_L) = \frac{3 - sin\,\phi}{2\sqrt{3}\,cos\,\theta_L - 2\,sin\,\theta_L\,sin\,\phi} \tag{8}$$

ϕ is the mobilized friction angle and θ_L is the Lode angle. The frictional hardening-softening functions expressed as follows were used:

$$\alpha(\kappa) = \left(\frac{2\sqrt{\kappa \varepsilon_f}}{\kappa + \varepsilon_f} \right)^m \alpha_p \quad (\kappa \le \varepsilon_f) \ \text{: hardening-regime} \tag{9}$$

$$\alpha(\kappa) = \alpha_r + (\alpha_p - \alpha_r)\exp\left\{ -\left(\frac{\kappa - \varepsilon_f}{\varepsilon_r} \right)^2 \right\} (\kappa \ge \varepsilon_f)$$

:softening-regime (10)

where m, ε_f and ε_r are the material constants and α_p and α_r are the values of α at the peak and residual states. The residual friction angle (ϕ_r) and Poisson's ratio (ν) were chosen based on the data from the test of air-dried dense Toyoura sand. The peak friction angle (ϕ_P) was estimated from the empirical relations based on the plane strain compression test on dense Toyoura sand. ψ is dilatancy angle. The introduction of shear banding in the numerical analysis was achieved by introducing a strain localization parameter s in the following additive decomposition of total strain increment as follows:

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + sd\varepsilon_{ij}^{p} \quad , \quad s = F_b / F_e \tag{11}$$

where F_b is the area of a single shear band in each element; and F_e is the area of the element

We propose the jump kinematic hardening model considering the cumulative deformation from cyclic loading. This is a modified and extended soil model of strain-hardening-softening model in order to take into account the cyclic behavior. Within bounding surface, plastic behavior is assumed and hardening modulus is much greater comparing the plastic behavior outside the bounding surface.

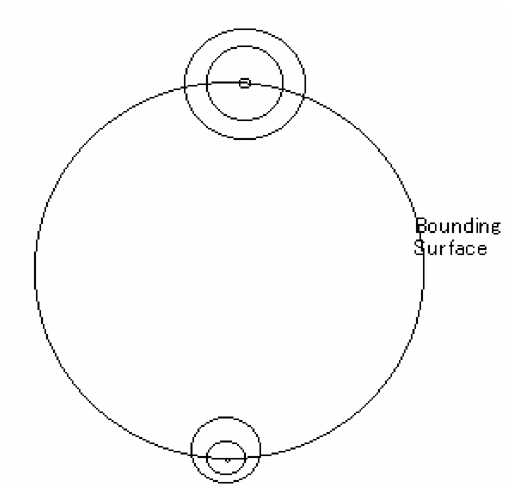

Fig.1 Jump kinematic model on π plane (Mohr-Coulomb model takes pyramid shape)

$$\alpha\ (\kappa) = (\alpha_p - \alpha_0)\left(\frac{2\sqrt{\kappa\varepsilon_f}}{\kappa+\varepsilon_f}\right)^m \text{ or } \alpha'\ (\kappa) = (\alpha_p + \alpha_0)\left(\frac{2\sqrt{\kappa\varepsilon_f}}{\kappa+\varepsilon_f}\right)^m \tag{16}$$

where α_0 indicates the reversing point.

Analysis of Plane Strain Test

The simulation of plane strain tests by the finite element method using one element was carried out. The obtained stress difference-strain relationships are shown in Fig. 1. The material constants of Toyoura sand used for calculation are as follow: D_r = 60%, v = 0.3, φ_r = 34 (deg), ε_r = 0.6, ε_f = 0.1, m = 0.3, shear band thickness = 0.3cm.

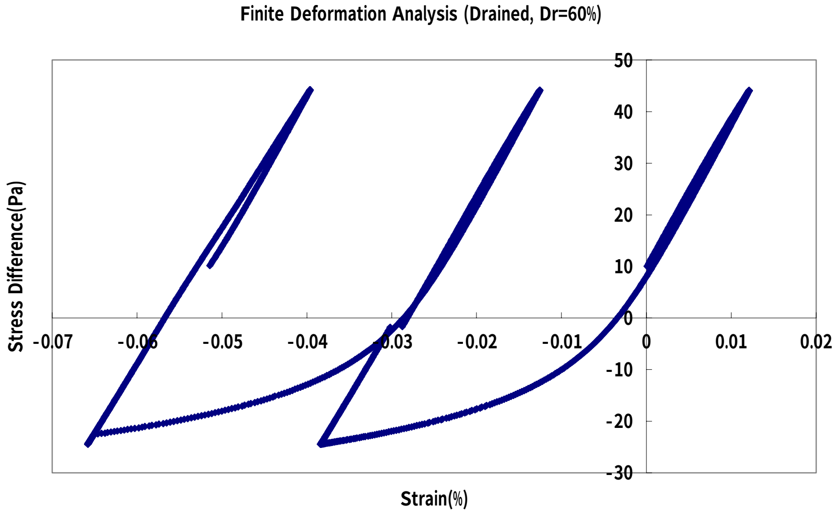

Fig.2 Drained stress-difference-strain relationship

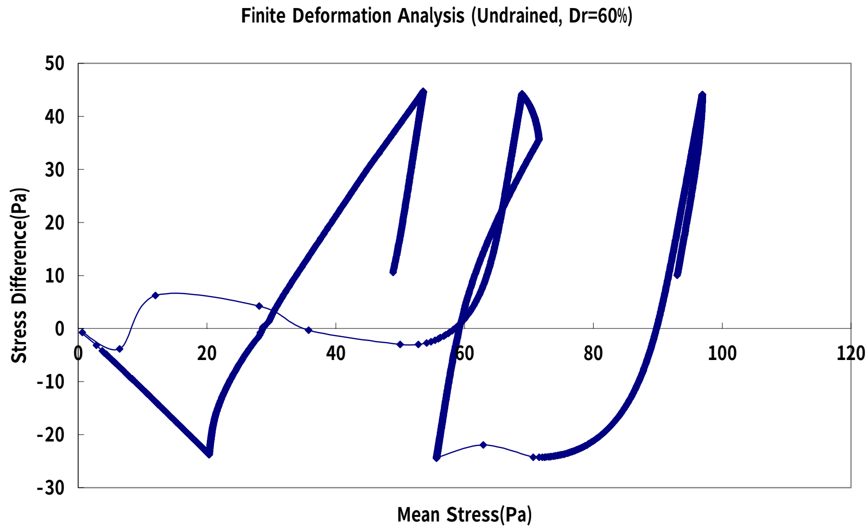

Fig.3 Undrained mean stress-stress difference relationship

Concl

A material model for a real granular material was used with the features of nonlinear pre-peak, pressure-sensitivity of the deformation and strength characteristics of sand, non-associated flow characteristics, post-peak strain softening, and strain-localization into a shear band with a specific width
The jump kinematic hardening model (modified and extended soil model of strain-hardening-softening) is promising for the prediction of cumulative deformation of soil structures and liquefaction problems

Reference

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