Effective Range of V-line Extrapolation Method to Calculate SIF of a Three Dimensional Crack Near to a Surface

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Summary

In this paper, the effective ranges of the extrapolation by the stress methods were investigated on a K_I mode-type 3-D crack model in a rectangle pillar under tensile load with varying the thickness. The more the thickness goes thinner, the more the range decreases in the conventional method. On the other hand, the range of the V-line extrapolation method is found to be independent of the thickness by using BEM code BEASY.

Introduction

The stress intensity factors in 3-D crack problems have been addressed in Ref. [1]. In general, the commercial codes such as NASTRAN, MARC, ANSYS and BEASY, are predominantly used for the industrial applications. It is convenient but the two disadvantages are pointed out; 1) It is not easy to built in an original element to obtain much higher accuracy. 2) It is not easy to verify the precision of SIF of 3-D crack under real complicated boundary conditions such as the problems that need a couple of million degrees of freedoms. Therefore, one of the most desirable methods to calculate SIF precisely in a short time is a devised short-cut method using the above mentioned commercial codes.

From the points of view mentioned above, the V-line stress extrapolation method [2, 3] is actually applied in this paper to the embedded 3-D crack problems and its usefulness is examined.

V-line stress extrapolation method

In the conventional stress extrapolation method[4], a stress σ_y in the extension direction of the crack is used to calculate SIF. However, this method is not suitable to apply for a crack near to a surface. On the other hand, the V-line stress extrapolation method applies the parameter as Eq.(1) with the stress $\sigma_{y\theta}, \sigma_{y-\theta}$ on the two lines symmetric with respect to the crack to eliminate the influence of K_{II}.

$$\sigma^{+}_{y\theta} \equiv \left(\sigma_{y\theta} + \sigma_{y-\theta}\right)/ 2 \qquad (1)$$

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Finally, a stress intensity factor KI is determined as follows.

$$K_I = \lim_{r\to 0}\left\{\frac{\sqrt{2\pi r}}{f^A_{II}(\theta)}\sigma^+_{y\theta}\right\} \equiv \lim_{r\to 0} K^A_I(r,\theta) \tag{2}$$

Here, KI(r, θ) is an analytical SIF curve which can be expressed as follows.

$$K^A_I(r,\theta) \equiv K_I + \frac{f^A_{I3}(\theta)}{f^A_{I1}(\theta)}a_3 r + \frac{f^A_{I5}(\theta)}{f^A_{I1}(\theta)}a_5 r^2 \tag{3}$$

where $a_n = \sqrt{2\pi} A_{In}$, and

$$f^A_{In}(\theta) = \frac{n}{2}\left\{\left(2-(-1)^n-\frac{n}{2}\right)\cos\left(\frac{n}{2}-1\right)\theta + \left(\frac{n}{2}-1\right)\cos\left(\frac{n}{2}-3\right)\theta\right\}$$

Hereafter, the characteristics of the angle coefficients in Eq.(3) are investigated to obtain good accuracies of SIF KI. In Figure 2, we show the variation of the angle coefficients of the r and r^2 terms in K_I (r, θ). The former angle coefficient decreases as the angle increases from θ =0 which is used in the conventional method. This means that the slope of the SIF curve becomes smaller. The latter angle coefficient decreases as the angle increases. This means that the analytical extrapolation curve approaches to a straight line. Therefore, two of three disadvantages of the conventional method can be overcome using V-line method. Furthermore, V-line method can be applied to solve the crack problems near the boundary.

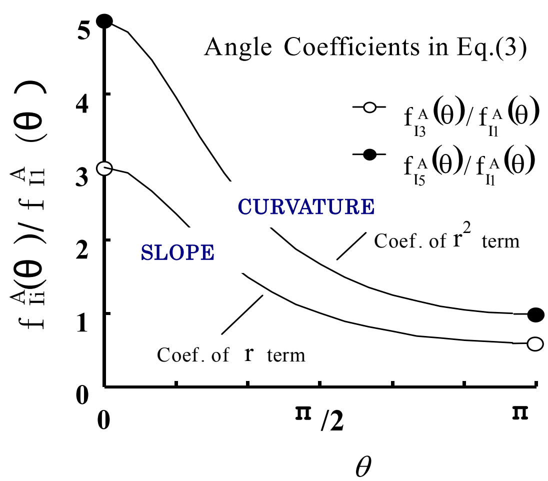


Fig.1 Angle coefficients in $K^A_I(r,\theta)$

Table 1 V-line method and conventional method

Conv. Method	$K_I(r) = K_I + 3a_3 r$
V-line Stress Method	$K_I(r,\theta) = K_I + 3\cdot\frac{3\cos\left(\frac{1}{2}\theta\right)+\cos\left(-\frac{3}{2}\theta\right)}{5\cos\left(-\frac{1}{2}\theta\right)-\cos\left(-\frac{5}{2}\theta\right)}a_3\cdot r + 5\cdot\frac{\cos\left(\frac{3}{2}\theta\right)+3\cos\left(-\frac{1}{2}\theta\right)}{5\cos\left(-\frac{1}{2}\theta\right)-\cos\left(-\frac{5}{2}\theta\right)}a_5\cdot r^2$

Calculation Model and Result

The calculation models used in this study are the rectangle pillars under tensile load with varying the thickness, and the cross section is illustrated in Fig. 2(a). Commercial BEM code BEASY ver. 8.1 is used to calculate the stresses on the internal points near the crack. The surfaces of the model (height:20,width:20) are divided by 8-nodes element. Ratio of the crack size b to the depth d varies with 0.1, 0.3, 0.5, 0.6 , 0.7 to investigate the effective range to extrapolate. The long radius of the crack a is set to be 1 for the circle crack and 0.5 for the elliptical crack. Normalized stress intensity factors of the point closest to the surface are shown in Table 2. Internal points are selected properly near the point E. The exact SIFs can be obtained by Ref.[5].

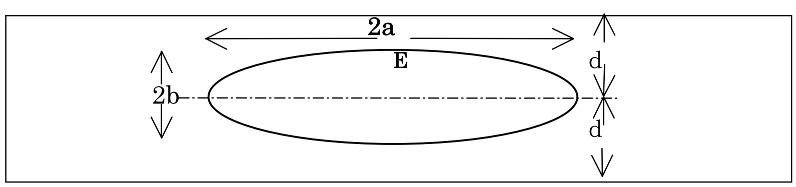


Fig.2 Crack shape and size in cross section

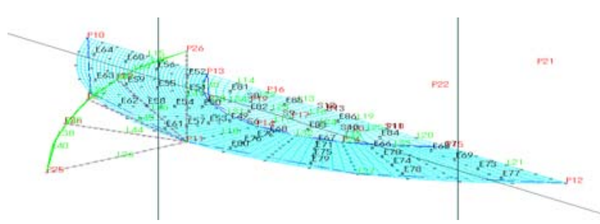
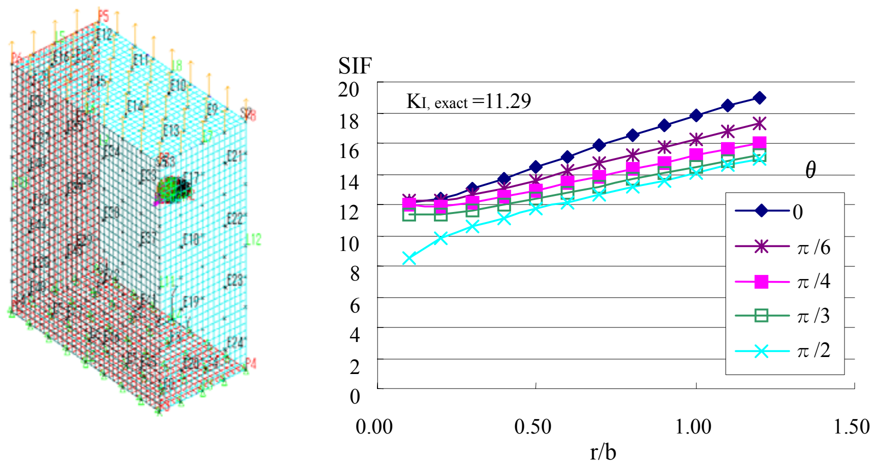
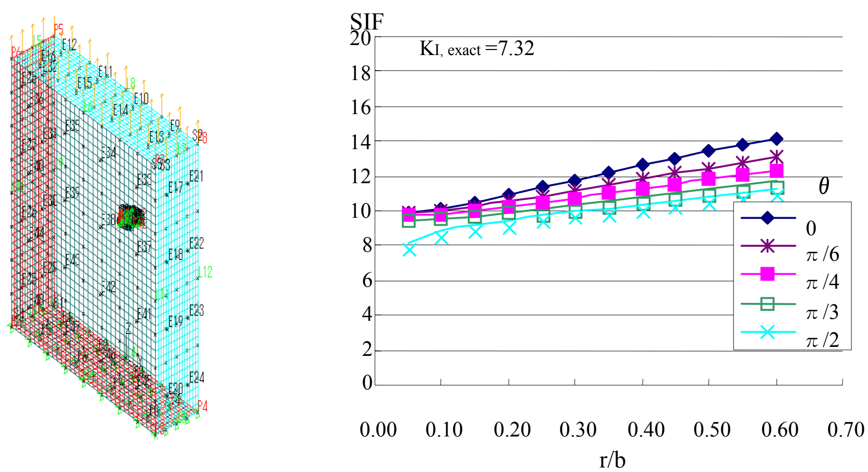


Fig.3 Half crack shape and internal points



(a) Shape (b/a=1, b/d=0.1) (b) KI (r, θ)

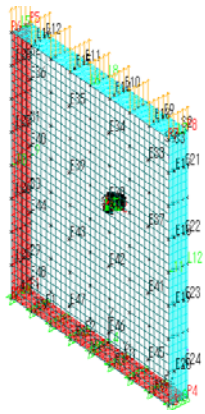
Fig.4 3D model I

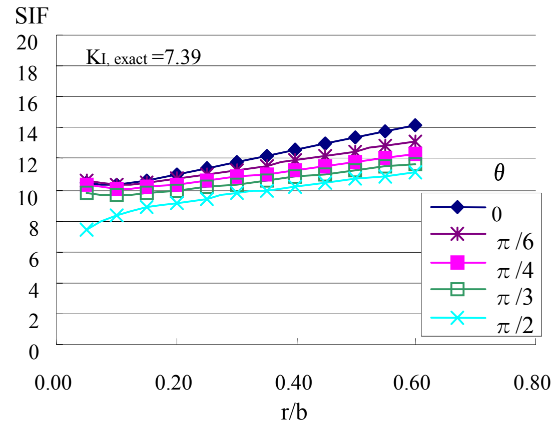


(a) Shape (b/ a =0.5, b/ d =0.1) (b) KI (r, θ)

Fig.5 3D model II -A

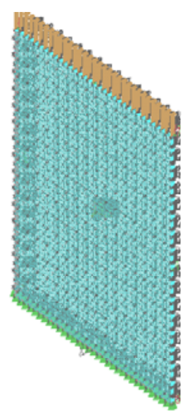
The calculated SIF curve KI(r, θ) of a embedded circle crack(b/ a =1) in a wide pillar (b/ d =0.1) is shown in Fig.4. It can be seen that KI(r, θ) has linearity from θ =0 to $\theta = \pi /3$, but KI(r, π /2) is seemed to be apart from others. It is considered to be occurred due to the rough mesh division of the crack with 8 elements.

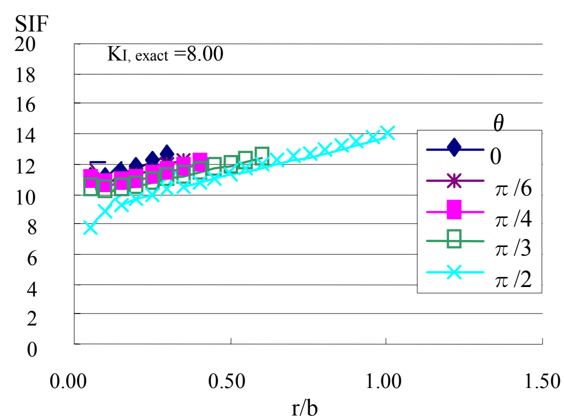




(a) Shape (b/ a =0.5, b/ d =0.3) (b) K_I (r, θ)

Fig.6 3D model Ⅱ-B

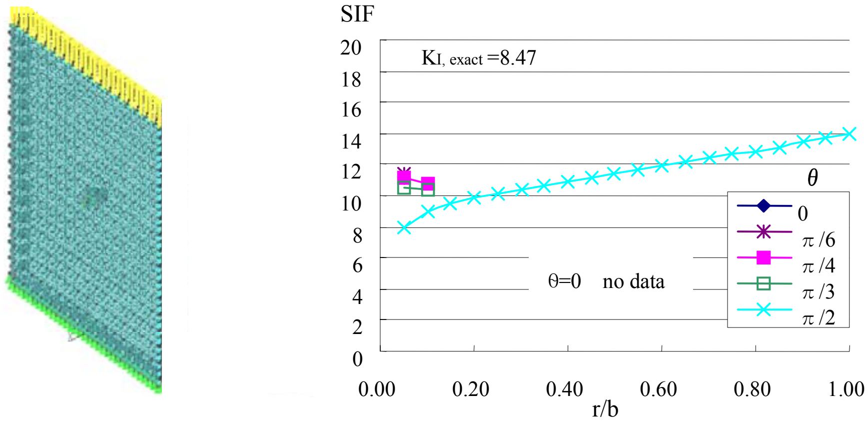




(a) Shape (b/ a =0.5, b/ d =0. 6) (b) K_I (r, θ)

Fig. 7 3D model Ⅱ-C

Figs.5,6,7,8 show the thickness dependency of $K_I(r, \theta)$. In the cases of wide thickness (Figs.5,6), any $K_I(r, \theta)$ has an enough extrapolation range. On the other hand, the thickness becomes thinner (Figs.7,8), $K_I(r, \theta)$ with small θ does not have enough extrapolation range because BEASY does not evaluate the stress value at the internal points near the surface due to the discretization error. Considering these matters, it can be said that the V-line extrapolation method with large θ is suitable to a commercial BEM code or meshless code because not only $K_I(r, \theta)$ has sufficient extrapolation range, but also the accuracy of SIF is confirmed by plotting several $K_I(r, \theta)$ curves.



(a) Shape (b/ a =0.5, b/ d =0. 7) (b) KI (r, θ)

Fig.8 3D model II -D

Conclusions

The following results are mainly obtained.

1. Through the investigation of the relation between the thickness and the effective range of extrapolation, it is found that when the thickness of the pillar becomes thinner, the effective range decreases or disappears in the conventional method using BEM code BEASY.
2. The KI(r, θ) with large θ has linearity even if the crack is near the boundary. This is due to the stress field normal to the crack tip which is almost independent from the influence of neighbor surface. Therefore, the V-line method is suitable to calculate SIF using a commercial BEM code.

References

1. Murakami, Y., Stress Intensity Factors Handbook, Vol.1~5,1987~2001, Pergamom Press etc.
2. Fujisaki, W. and Aliabadi, M.H., Boundary Elements 19, (1997), "Application of modified distance extrapolation method for calculation of stress intensity factors using BEM", pp. 287-296, CMP.
3. Fujisaki, W., Shinohara,S. (2000) : "An Determination Method for Stress Intensity Factor by Extrapolation from an Arbitrary Angle under K_I, K_{II} Mixture Mode" , *JSME International Journal*, Vol.43,A, pp. 203-209.
4. Chan,S.K.,Tuba,I.S.,Wilson,W.K., "On the Finite Element Method in Linear Fracture Mechanics" , Eng. Frac. Mechanics, Vol.2, pp. 1-7, 1970.
5. Ishida,M.,Noguchi,H., *JSME*(in Japanese), Vol. 48,A, pp. 888-895, 1982.