Adiabatic Shear Bands in Functionally Graded Particulate Composites

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Summary

The initiation and propagation of adiabatic shear bands (ASBs) in functionally graded materials (FGMs) deformed at high strain rates in plane strain tension have been studied by the finite element method. An ASB is a narrow region, usually a few microns wide, of intense plastic deformation that forms after softening of the material due to heating and damage evolution has overcome its hardening due to strain- and strain-rate effects. The FGM is comprised of tungsten and NiFe with continuously varying volume fraction. Each constituent and the composite are modeled as isotropic microporous strain- and strain-rate hardening and thermally softening materials with effective properties of the composite derived by the rule of mixtures. Each phase and the homogenized composite obey the Johnson-Cook thermoviscoplastic relation, Gurson's type flow potential, the associated flow rule, and a hyperbolic heat equation. A Galerkin approximation of governing equations written in the Lagrangian description of motion results in a set of coupled ordinary differential equations (ODEs) which are integrated with respect to time by using the Livermore solver for ODEs. No initial defect is introduced in the specimen of square cross-section. The volume fraction of each phase is assumed to vary radially till a boundary point is reached and then stay constant. It is found that ASBs, always aligned along the direction of the maximum shear stress, form sooner in an FGM than in either of the two constituent materials with their location, orientation, pattern and speed depending upon the compositional profile.

Introduction

Tresca [1] observed hot lines now called ASBs during the hot forging of a platinum bar. Even though heat conduction determines the width of an ASB, the adjective adiabatic is used to signify that they form rapidly in a body deformed at a high strain rate and there is not enough time for the heat to be conducted away. Their study is important because they precede ductile fracture. The research activity in ASBs increased subsequent to their being observed in [2] during the punching of a hole in a steel plate. They postulated that a material point becomes unstable when thermal softening equals the combined hardening due to strain- and strain-rate effects. Since then, several investigators have used this criterion to determine the average strain at the time of initiation of an ASB and rank materials; a material in which an ASB forms sooner is more susceptible to shear banding. Bai [3] studied simple shearing deformations of a homogeneous thermoviscoplastic body, perturbed the homogeneous solution of the governing equations, and hypothesized that an ASB forms when superimposed infinitesimal perturbations grow. Batra and Chen [4] have shown that this approach gives the same result as that proposed by Zener and Hollomon [2]. Experimental observations of Marchand and Duffy [5] on the torsion of thin-walled

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Proceedings of the 2004 International Conference on
Computational & Experimental Engineering & Science
26-29 July, 2004, Madeira, Portugal

tubes have revealed that an ASB forms much later than when the shear stress peaks and is accompanied by a rapid drop in the torque needed to deform the tube. Numerical solutions of simple shearing, plane strain and axisymmetric problems [6] have confirmed that the load carrying capacity of a member drops significantly upon the formation of an ASB. Previous studies have focused on analyzing the initiation and propagation of ASBs in a homogenous body. Here we analyze the problem for a FGM comprised of two constituents with the compositional profile and hence material properties varying continuously only in a cross-section. We delineate the time of initiation, the direction of propagation, the material point from where an ASB forms relative to that in a homogeneous body comprised of only one constituent. It is found that these are affected noticeably by the compositional profile.

Formulation of the Problem

The balance laws for mass, linear momentum, moment of momentum, and the internal energy, are written in the Lagrangian or the referential description of motion. Elastic deformations, heat conduction and stresses due to thermal expansion are considered. Following assumptions are made: (i) the strain rate tensor has additive decomposition into an elastic part, a plastic part and a thermal part; (ii) the Jaumann rate of the Cauchy stress tensor is a linear function of the elastic part of the strain rate tensor; (iii) Young's modulus and the shear modulus decrease affinely to zero when the porosity approaches one; (iv) the porosity represents damage induced in the body; (v) the specific heat and the thermal conductivity are affine functions of porosity; (vi) a material point deforms plastically when the stress state satisfies Gurson's flow potential modified for its dependence upon the porosity [7] and the dependence of the flow stress upon the plastic strain, plastic strain rate and temperature; the latter is assumed to be given by the Johnson-Cook [8] relation; (vii) the associative plasticity rule holds; (viii) Chu and Needleman's [9] expression for the evolution of porosity applies; (ix) the rate of change of internal energy is a linear function of the first and the second time derivatives of temperature thereby resulting in a hyperbolic heat equation; (x) each constituent and the composite are isotropic. Thus, both mechanical and thermal disturbances propagate at a finite speed. A complete set of equations is given in [10] where a parabolic rather than a hyperbolic heat equation is used.

We first analyzed transient deformations of a representative volume element (RVE) to evaluate effective properties of the composite as a function of the volume fraction of constituents. Values of elastic parameters so determined were found to match well with those given by the Mori-Tanaka [11] scheme. However, values of parameters characterizing the plastic deformation could not be satisfactorily determined from deformations of the RVE. We thus use the rule of mixtures to evaluate values of all material parameters from those of the constituents and their volume fractions. It gives exact values of the mass density and the heat capacity, is simple to use, and often gives an upper bound for values of the composite.

Assuming that the body is prismatic having a uniform square cross-section, and initial and boundary conditions are independent of the axial coordinate, a plane strain state of deformation is taken to prevail in the body. Furthermore, thermomechanical deformations are assumed to be symmetric about the two centroidal axes. Thus deformations of only a quarter of the cross-section in the first quadrant are analyzed with boundary conditions arising from the symmetry of deformations imposed at points on the centroidal axes. The other vertical surface is taken to be traction free and thermally insulated. Normal velocity, null tangential tractions and zero heat flux are prescribed on the top horizontal surface. The prescribed normal velocity increases linearly with time to its steady state value in 1 μs and is then held there. The body is initially at rest, at a uniform temperature and has zero initial porosity. Consistent with experimental observations of Marchand and Duffy [5] we use Batra and Kim's [12] criterion for the initiation of an ASB. That is, an ASB is assumed to initiate at a point when the maximum shear stress there has dropped to 80% of its peak value at that point and the material point is deforming plastically.

Weak Formulation of the Problem

The constitutive assumptions identically satisfy the balance of moment of momentum. The balance of mass is used to compute the present value of the mass density after the deformation field has been computed from the remaining equations. Galerkin's method is used to derive a weak form of the balance of linear momentum, the balance of internal energy, equations expressing the Jaumann rate of the Cauchy stress tensor in terms of the elastic part of the strain rate tensor, and evolution equations for the effective plastic strain rate and the porosity, and of the velocity equaling the time rate of change of the present position. The Johnson-Cook relation is rewritten to express the effective plastic strain rate in terms of the effective stress, the effective plastic strain and the temperature. Also, an auxiliary variable equal to the time rate of change of temperature is introduced. Thus at each node there are twelve unknowns, namely two components of the position vector, two components of velocity, four components of Cauchy's stress tensor, porosity, temperature, its rate of change, and the effective plastic strain. Galerkin's approximation incorporates natural boundary conditions and results in a system of coupled nonlinear ODEs for the unknowns. These ODEs are integrated by using the subroutine LSODE. During this integration process, essential boundary conditions are imposed. The subroutine adjusts the time step adaptively to compute the solution within the prescribed accuracy.

Computation and Discussion of Results

Results have been computed for tungsten (W) particles interspread in an NiFe matrix with the volume fraction, $\nu_{f,NiFe}$, of NiFe given by one of the following two expressions.

$$\text{Type I:}\quad \nu_{f,NiFe} = \begin{cases} c\frac{r}{H}, & r < H \\ c, & r \geq H \end{cases} \qquad \text{Type II:}\quad \nu_{f,NiFe} = \begin{cases} c\left(1-\frac{r}{H}\right), & r < H \\ 0, & r \geq H \end{cases} \tag{1}$$

Here r is the radial distance from the specimen centroid, and $2H = 10$ mm equals the length of a side of the square cross-section. In both type-I and type-II FGMs, material properties in the region $r > H$ are constants. In type-I FGMs, the volume fraction of NiFe varies from zero at the specimen centroid to c at $r = H$, and in type-II FGMs, it varies from c at the specimen centroid to zero at $r = H$. The steady value of the normal velocity prescribed on the top surface equaled 25 m/s thereby giving an average strain rate of 5000/s for $t > 1\mu s$. Literature values of material parameters for the W and NiFe were used, and the thermal relaxation time for both equaled $10^{-8}s$.

The computer code was validated by comparing computed results with the published ones and also by using the method of so-called manufactured solutions in which body forces and sources of internal energy density can be found for any assumed deformation and temperature fields. These and initial and boundary conditions corresponding to the assumed solution are input into the code. The computed solution should match the presumed analytical solution of the problem; e.g. see Batra and Liang [13].

Results presented below are with a FE mesh of 1600 4-node quadrilateral elements. Results computed with a finer mesh of 6400 elements changed the ASB initiation time by less than 1%. Furthermore, the solution essentially coincided with that [14] obtained with a FE mesh of triangular elements. For homogeneous materials, the presence of 2% initial porosity at the specimen centroid which decayed exponentially to zero with the distance from the centroid was assumed in [14]. Because of the maximum initial porosity, the centroid acts as a nucleation site for an ASB. However, for a composite, deformations naturally become inhomogeneous because of the spatial variation of material properties and it determines where and when an ASB initiates. For reference and to delineate the effect of the initial porosity distribution, we have listed in Table 1 the ASB initiation times for five materials. It is clear that an ASB initiates considerably sooner in initially porous materials than in those with zero initial porosity. Numerical experiments on type-I and type-II FGMs yielded interesting results summarized in Table 2. In the first subset of tests on type-I FGMs, the maximum volume fraction of NiFe was varied from 0.1 to 0.5. In each case, an ASB initiated at the specimen centroid and propagated along a line inclined at 45° to the horizontal axis in the present configuration. However, the axial strain at the instant of ASB initiation decreased with an increase in the maximum volume fraction of NiFe at the centroid. Contours of the effective plastic strain in type-I FGM are depicted in Fig. 1. The effective plastic strain within the ASB exceeds 1 with a peak value of 1.4 at some points. For type-II FGMs with the maximum volume fraction of NiFe at the centroid, an ASB initiated from a point on the top surface and propagated inwards along the two directions of the maximum shear stress. As shown in Fig.2, there are several narrow regions of large plastic deformation. The point where an ASB initiated varied with the compositional

Table 1. Axial strain at ASB initiation in five homogeneous materials

Material	Axial strain at ASB initiation	
	w/ initial porosity [14]	w/o initial porosity
Tungsten	0.137	0.386
4340 steel	0.315	0.615
Nickel iron	–	0.944
OFHC copper	0.391	1.045
Armco iron	0.433	1.095

Table 2. Axial strain and the point of initiation of ASBs in FGMs

Specimen type and constituents	Max. v_f of NiFe	Axial strain at ASB initiation	Point of ASB initiation (ref. config)	
			X	Y
W/Nife Type I	0.1	0.371	0.000	0.000
	0.2	0.339	0.000	0.000
	0.3	0.327	0.000	0.000
	0.4	0.325	0.000	0.000
	0.5	0.331	0.000	0.000
W/NiFe Type II	0.1	0.340	1.125	5.000
	0.2	0.314	1.250	5.000
	0.3	0.304	1.375	5.000
	0.4	0.298	4.750	2.875
	0.5	0.292	4.875	2.875
W/4340 Type I	0.3	0.364	1.125	5.000
W/NiFe Type I	0.3	0.327	0.000	0.000
W/Cu Type I	0.3	0.218	0.625	5.000
W/Fe Type I	0.3	0.268	1.000	5.000

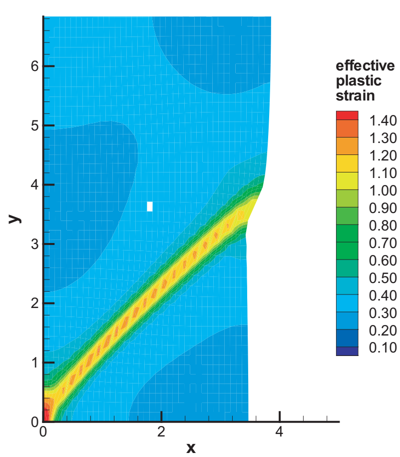

Figure 1. Contours of effective plastic strain for a type-I W/NiFe FGM with $c = 0.3$ at an axial strain of 0.40.

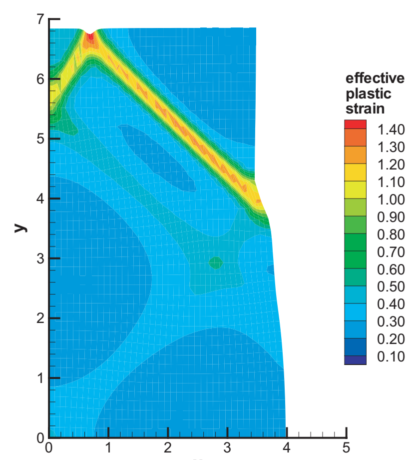

Figure 2. Contours of effective plastic strain for type-II W/NiFe FGM with $c = 0.3$ at an axial strain of 0.3725.

profile. Results for numerical experiments on other type-I FGMs containing different matrix material are also summarized in Table 2. Of the FGMs studied, the type-I with NiFe matrix delays most the onset of an ASB and that with Fe matrix the least. Composites comprised of W in the region $r < 5$ and NiFe elsewhere or vice-a-versa were also analyzed. In these an ASB initiated from different points on the top surface and propagated inwards. For each problem studied, the time step was seen to drop drastically once an ASB had initiated at a point in the body. Additional results are given in [15].

Conclusions

We have developed a Lagrangian finite element code to analyze large plane strain coupled transient thermomechanical deformations of an FGM comprised of two constituents. It is found that an ASB initiates either at the specimen centroid or at a point on the surface where the normal component of velocity is prescribed. In each case, it propagates along the direction of the maximum shear stress. The analysis has been used to optimize the compositional profile to either delay or to enhance the onset of an ASB.

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