Extension of New Local Error Estimators for Panel Methods in 2-D Potential Flow Problems

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Summary

Two error estimator approaches, originally derived for direct boundary element methods for potential problems, are extended in this work for panel methods applied to 2-D inviscid incompressible flows. The first approach is based on the external problem formulation, and the second approach uses the gradient recovery approach. Both formulations are post-processing procedures and measure the local error. For comparison purposes, an analytical method using conformal transformation is used for evaluating the local error to compare with the error estimates. Numerical results were obtained for several airfoils with varying mesh refinement and vortex distributions.

Introduction

Boundary element methods (BEM) have been used successfully for many years in the solution of incompressible inviscid flows around complex configurations [1]. In the aerodynamics and hydrodynamic fields, these methods, also known as panel methods, have matured to a level that allows their use as design tools on a routine basis. An entry into the panel method literature is available through a recent review by Hess [2], a survey by Erickson [3], and a book by Katz and Plotkin [4].

For efficient airfoils, the viscous effects should be small at normal operating conditions. In the actual case, drag arises from skin friction effects, further additional drag is formed due to the small change of pressure on the body due to the boundary layer (which primarily prevents full pressure recovery at the trailing edge), and drag due to increasing viscous effects with increasing angle of attack. A well-designed airfoil will have a drag value very nearly equal to the skin friction and nearly invariant with incidence until the maximum lift coefficient is approached. The incompressible inviscid flow field model is in agreement with experimental data for well-designed subsonic airfoil. Agreement is good at low angles of attack, where the flow is fully attached. The agreement deteriorates as the angle of attack increases, and viscous effects start to show up as a reduction in lift with increasing angle of attack, until, finally, the airfoil stalls. The inviscid solutions cannot capture this part of the physics. To assume an incompressible flowfield, the local flow must be at such low speed everywhere. In panel methods, there are many ways to tackle the problem

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varying the singularities, distributions of the singularity strength over each panel, and geometry panel. The influence of these variations could be captured with local error estimators, allowing for a reduction in the computational effort.

In the present work, two post-processing error estimators described by Jorge *et al* [5] are extended to panel methods. The calculation of the potential flow is performed for Joukowski, van de Vooren and Karman-Trefftz airfoils, for which an analytical solution is known. The first error estimator approach is associated with an external problem formulation and gives both a local and a global measure of the error, depending on the location of the external point where the equation is applied. The potential should vanish at points in the exterior of the domain due to the fact that the swept angle appearing in the boundary integral formulation is zero. Non-zero values indicate errors. By collocating the external point near the boundary, the influence of nearby elements on the error estimate will be dominant, so that the error information can be considered to be local. The second approach is a local error estimator based on a gradient recovery procedure, wherein the smoothed or recovered functions are rates of change of the boundary variables in the local tangential direction. The potential is continuous throughout the boundary, while the flux and the tangential derivative of the potential may be discontinuous, such as where the boundary is not smooth. Any recovery procedure of the tangential derivatives must allow for these inter-element discontinuities.

Analytical solutions

Any analytic function of a complex variable satisfies the Laplace's equation for incompressible inviscid flow ($\nabla^2 F = 0$). We can, therefore, relate one flow field to another by setting $F(Z) = F(\zeta)$ where Z is related to ζ by an analytic function. The idea behind airfoil analysis by conformal mapping is to relate the flow field around one shape, which is already known to the flow field around an airfoil. Most often a circle is used as the first shape. The problem is to find an analytic function that relates every point on the circle to a corresponding point on the airfoil.

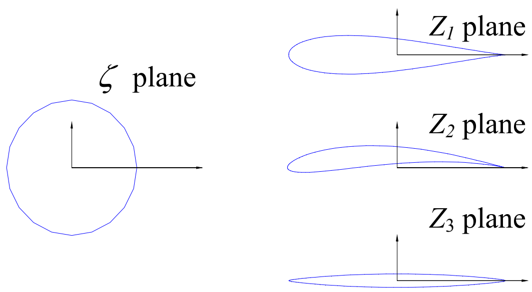

Figure 1 – Most well known transformations

The equation of circle (ζ plane) is

$$\zeta = r_o e^{i\theta} + \xi_c + i\eta_c \tag{1}$$

Three of the most well known transformations used to construct foils that possess analytical solutions are the Van de Vooren (Z_1 plane), Joukowski (Z_2 plane) and Karman-Trefftz (Z_3 plane) transformations. These transformations are shown in Figure 1 and detailed below.

Van de Vooren	Joukowski	Karman-Trefftz
$Z_1(\zeta) = l + \dfrac{(\zeta - a)^k}{(\zeta - a\varepsilon)^{k-1}}$	$Z_2(\zeta) = \zeta + \dfrac{a^2}{\zeta}$	$Z_3(\zeta) = \chi \cdot l \cdot \dfrac{1 + \left(\dfrac{\zeta - l}{\zeta + l}\right)^{\chi}}{1 - \left(\dfrac{\zeta - l}{\zeta + l}\right)^{\chi}}$

The components of velocity on the airfoil surface *W(Z)* are obtained with the aid of transformation and the flow over a cylinder $W(\zeta)$.

$$W(Z) = \frac{dF}{dZ} = \frac{dF}{d\zeta}\frac{d\zeta}{dZ} = W(\zeta)\frac{1}{\dfrac{dZ}{d\zeta}} \tag{2}$$

Numerical solutions

When using panel methods, some choices need to be done, such as surface paneling, type of singularity, singularity distribution and type of boundary conditions. In this work, the behavior of the error estimators is evaluated for the three different airfoils described above. In the three cases, constant and linear vortex distribution in flat panels will be used. Meshes with 30 and 90 panels will be evaluated. The type of boundary condition is an internal zero tangential velocity. This is based in the fact that the potential inside an enclosed body is constant. Thus, the derivatives of the total potential inside the body are zero. Another boundary condition needs to be specified at the trailing edge: the Kutta condition. The Kutta condition states that the flow leaves the sharp trailing edge of an airfoil smoothly, and the velocity at this point is finite. For the current modeling purposes this can be interpreted as the flow leaving the trailing edge along the bisector line at the trailing edge. In most cases involving airfoils, a denser paneling is used near the leading and trailing edges for discretization of geometry. The *full-cosine* method of spacing the panels on the airfoil's surface will be used here.

In the constant strength vortex distribution case, the boundary condition is applied for the collocation points and the Kutta condition is applied at the trailing edge. The *NxN+1* system of equations is solved using Lagrange's variational method. In the linear strength vortex distribution case, the singularity changes linearly along the panel. Consequently, the singularity strength on each panel includes two unknowns and additional conditions need to be formulated. The strength at the beginning of each panel is set equal to the strength of the vortex at the end point of the previous panel, so that a continuous vortex distribution is obtained. In the trailing edge the Kutta condition is applied.

Error Estimators

The external formulation is based in the fact that the potential inside an enclosed body is constant. Consequently, the derivatives of the total potential inside the body are zero. The non-zero speed obtained inside the body from a numerical solution gives an error estimate. For each panel an airfoil's interior point is chosen. This point is located tracing a line perpendicular to the panel in the control point and defining its length. This length is a fraction a the panel length. When the point is near the element, the error measure is local. But, if this point is located too close to the airfoil, near-singular integrals are obtained and the numerical errors increase. Values between 0.01 e 0.25 of the element length are recommended by Jorge *et al* [5] in order to increase the local effect of the error estimator without reaching the near-singular effects. For airfoils, this length value is limited in the trailing edge and cannot be used in a cusped trailing edge, thus limiting the possibilities of use of this error estimator. In the gradient recovery approach, a procedure is used to smooth the gradient, as described in Jorge *et al* [5]. The difference between this refined gradient and the actual gradient in the elements provides a local error measure. This procedure is available for linear of higher order panels.

An exact local error is obtained comparing the numerical solution using the panel method to the analytical solution. The error estimator results are compared with this exact error. The results obtained are shown in figures 2, 3 and 4. The error estimators indicate correctly the region with higher errors for all cases evaluated, and also indicate correctly the error behavior with variation of strength vortex (fig. 2) and number of panels (fig. 4). The two error estimators give similar results for regions with higher errors, although results could vary in regions with smaller errors (fig. 3).

Conclusions

Two error estimators originally derived for boundary element methods were successfully extended for panel methods. The error estimators indicate correctly the regions with higher errors in all cases evaluated, with varying vortex strengths and number of panels. Each error estimator evaluated presented a limitation. The external error estimator is not suited for use in a cusped trailing edge such as in Joukowski airfoils. Also, there is no gradient recovery error estimator available for constant strength vortex.

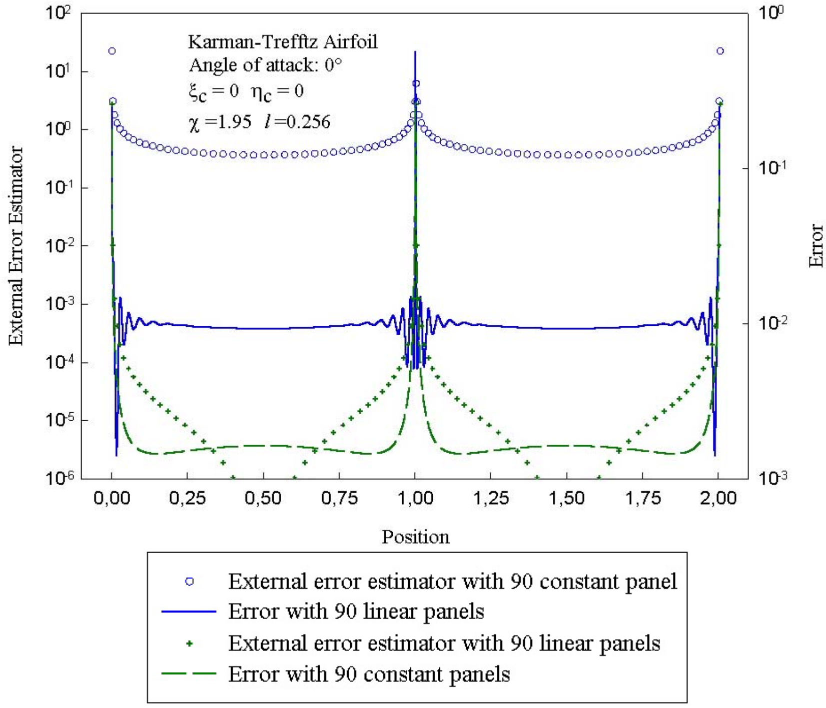

Figure 2 – Influence of strength vortex variation

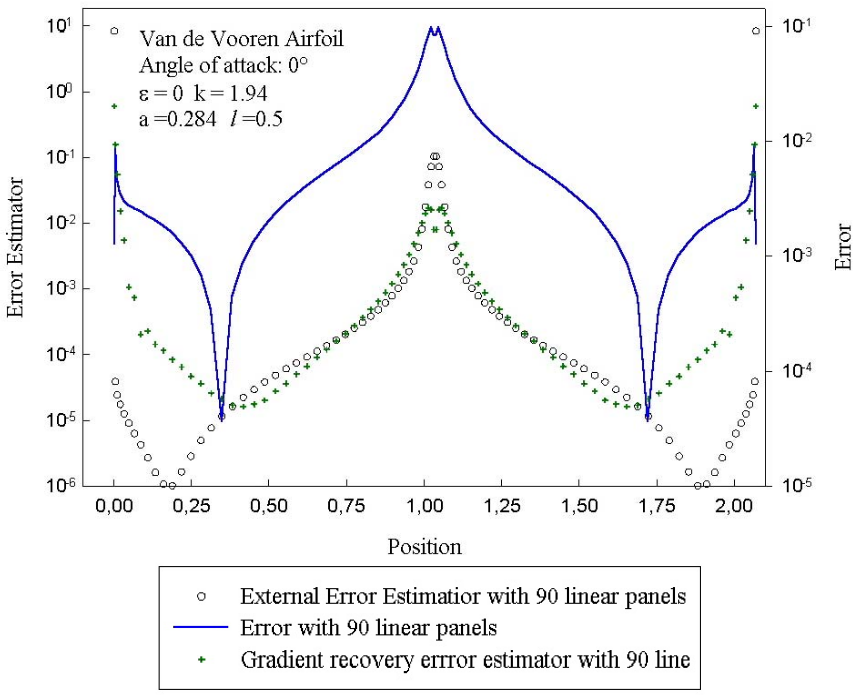

Figure 3 – Comparison between the two error estimations

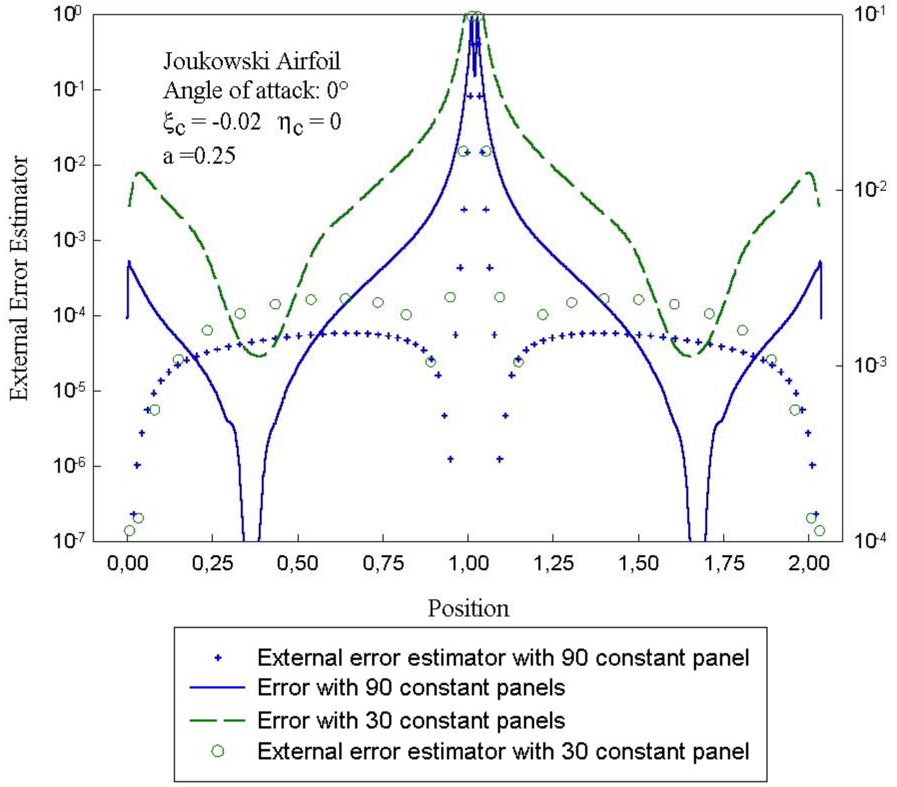

Figure 4 – 30 and 90 constant panels

References

1. Hoymakers, H. (1992): Panel Methods for Aerodynamic Analysis and Design, Technical Report 783, AGARD.
2. Hess, J.L. (1974): "*The Problem of Three-Dimensional Lifting Potential Flow and Its Solution by Means of Surface Singularity Distributions*", Computer Methods in Applied Mechanics and Engineering 4pp.283-319.
3. Erickson, J.L.(1990), "*Panel Methods – An Introduction*", NASA TP-2995.
4. Katz,J., and Plotkin, A. (2001) *Low-Speed Aerodynamics*, Cambridge University Press.
5. Jorge A.B., Ribeiro G. O., Fischer T. S. (2003) "*New Approaches for Error Estimation and Adaptivity for 2-D potential Boundary Element Methods*", International Journal for Numerical Methods en Engineering, 56, pp. 117–144.