Boundary Element Method for Convective Heat Transfer in a Tall Porous Cavity using Forcheimer Model

R. Jecl[1], L. Škerget[2]

Summary

The main purpose of this work is to present the use of the boundary element method (BEM) for analyzing the convective flow in a tall porous enclosure heated from the side by utilizing the Brinkman-Forchheimer momentum equation (Forcheimer model) in order to investigate the effect of the Forchheimer inertia term on the global heat transfer through the cavity. Namely the effects that this term have upon the engineering parameters of interest for the case of a fluid saturated porous media, is known to be minimal but was not confirmed yet with the use of the BEM or any of its extension. The numerical solution of the problem (velocity, vorticity, temperature and rates of heat transfer) are obtained for different Rayleigh and Darcy numbers and results are compared with the Brinkman extended Darcy momentum equation (Brinkman model).

Introduction

Buoyancy induced convection in a fluid saturated porous media is of considerable interest, due to its numerous applications in energy-related engineering problems. Studies have been reported dealing with different geometries and variety of heating conditions. For example, a vertical cavity in which a horizontal temperature gradient is induced by side walls maintained at different temperatures has been analyzed by Lauriat & Prasad [1], Vasseur *et al*. [2] and Jecl *et al*. [3] using Brinkman-extended Darcy formulation. The significance of Forcheimer modification is given for the same type of the problem by Lauriat & Prasad [4] and for the natural convection in porous layer heated from below by Lage [5]. The numerical methods used for the solution of governing equations, these in most cases written in vorticity-stream function formulation, are finite difference method (FDM) and finite element method (FEM) while in the present contribution the boundary domain integral method (BDIM) is used.

Governing equations

Consider a two dimensional vertical porous cavity heated from the left side at a constant temperature and isothermally cooled at the right side; the top and the bottom walls are adiabatic. In the porous media of nondeformable solid matrix, the saturating fluid is considered to be a Newtonian. The thermophysical properties of the fluid and the solid phases of the porous media are taken to be constant except for the density variation, which is handled according to the Boussinesq approximation. Furthermore, the solid particles and the fluid are assumed to be in local thermodynamic equilibrium, while the porosity and permeability of the

[1] University of Maribor, Faculty of Civil Engineering, Maribor, Slovenia
[2] University of Maribor, Faculty of Mechanical Engineering, Maribor, Slovenia

media are assumed to be uniform throughout the system. The general set of equations for conservation of mass, momentum, and energy based on the Brinkman-Forcheimer extended Darcy momentum equation are known as a Forcheimer model consist of continuity, momentum and energy equation

$$\frac{\partial v_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{1}{\phi}\frac{\partial v_i}{\partial t} + \frac{1}{\phi^2}\frac{\partial v_j v_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + F g_i - \frac{\nu_f}{K} v_i + \nu_e \frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{c_F}{\sqrt{K}} v\, v_i \tag{2}$$

$$\frac{\partial}{\partial t}\left[\phi\left(\rho c_f\right) + (1-\phi)(\rho_s c_s)\right]T + \left(\rho c_f\right)\frac{\partial v_j T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\lambda_e \frac{\partial T}{\partial x_j}\right) \tag{3}$$

In equations (1), (2) and (3) v_i is volume-averaged velocity, x_i the i-th coordinate, ϕ is porosity, ν_f the fluid kinematic viscosity, ν_e the effective kinematic viscosity, K permeability of porous media, $\partial P/\partial x_i$ pressure gradient in the flow direction, ρ the fluid density, g_i gravity. The normalized density-temperature variation function F is taken as $F = (\rho - \rho_0)/\rho_0 = -\beta_T(T - T_0)$, with ρ_0 denoting the reference fluid mass density at temperature T_0 and β_T being the thermal volume expansion coefficient of the fluid. The permeability K is independent of the nature of the fluid but it depends on the geometry of the media. The coefficient c_F as a dimensionless form-drag constant varying with the nature of the porous media. Using the Ergun model, as described in Nield and Bejan [6], it is possible to calculate the coefficients as $K = d^2\,\phi^3/150\,(1-\phi)^2$ and $c_F = 1.75/\left(150\,\phi^3\right)^{1/2}$, where d_p is the solid particle diameter. The effective viscosity ν_e depends on the geometry of the porous media. It may have a different value than the fluid viscosity ν_f, therefore parameter Λ denoting viscosity ratio, is introduced. Since Λ depends on the geometry of the media, its value is approximated by $\Lambda = 1/\phi$. Furthermore ρ_s and ρ are the solid and fluid densities respectively, c_s and c_f the solid and the fluid specific heats at constant pressure respectively, T stands for temperature, and λ_e represents the effective thermal conductivity of the saturated porous media, $\lambda_e = \phi\,\lambda_f + (1-\phi)\lambda_s$.

The momentum equation (2), commonly known as Brinkman-Forcheimer equation consists of two viscous and two inertia terms. The first viscous term is the usual Darcy term (third on the r.h.s.), and the second is analogous to the Laplacian term that appears in the Navier-Stokes equations for pure fluid (fourth on the r.h.s.). The Laplace term is commonly called Brinkman term or Brinkman extension that expresses the viscous resistance or viscous drag force exerted by the solid phase on the flowing fluid at their contact surfaces. The first inertia term is the convective inertia term represented by the velocity times its divergent (second on the l.h.s.) and the second inertia term is so called Forcheimer inertia term (drag) represented by the velocity times its absolute value (last term on the r.h.s.). In order to examine the effect of Forcheimer inertia term the present results are compared with our previously published results obtained with the Brinkman model [7].

Numerical Method

The numerical method chosen for this investigation is the Boundary Domain Integral Method (BDIM) based on the classical Boundary Element Method (BEM), see Škerget *et al.* [8]. If the viscosity is partitioned into constant and variable parts so that $\nu_f = \bar{\nu}_f + \tilde{\nu}_f$ the Brinkman extension in momentum equation is divided into two parts and the equation (2) is

$$\frac{1}{\phi}\frac{\partial v_i}{\partial t} + \frac{1}{\phi^2}\frac{\partial v_j v_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + F\, g_i - \frac{\nu_f}{K} v_i + \Lambda \bar{\nu}_f \frac{\partial^2 v_i}{\partial x_j \partial x_j} + \Lambda \frac{\partial}{\partial x_j}\left(2\tilde{\nu}_f \&_{ij}\right) - \frac{c_F}{\sqrt{K}} v\, v_i \tag{5}$$

Furthermore, the thermal diffusivity of the porous media a_p, defined with $a_p = \lambda_e / \rho\, c_f$ is similarly as the kinematic viscosity, partitioned into constant and variable parts $a_p = \bar{a}_p + \tilde{a}_p$. Introducing the heat capacity ratio σ with expression $\sigma = \phi\left(\rho\, c_f\right) + (1-\phi)\left(\rho_s\, c_s\right) / \rho\, c_f$, the heat energy equation (3) can be rewritten in the following form

$$\sigma \frac{\partial T}{\partial t} + \frac{\partial v_j T}{\partial x_j} = \bar{a}_p \frac{\partial^2 T}{\partial x_j \partial x_j} + \frac{\partial}{\partial x_j}\left(\tilde{a}_p \frac{\partial T}{\partial x_j}\right) \tag{6}$$

The governing equations (1), (5) and (6) are further transformed with the use of the velocity-vorticity variables formulation. With the vorticity vector ω_i, representing the curl of the velocity field $\omega_i = e_{ijk}\, \partial v_k / \partial x_j$, the computational scheme is partitioned into its kinematic and kinetic part so that the continuity and momentum equations are replaced by the equations of kinematics and kinetics. The formulation has been presented in detail previously by Jecl *et al*. [3] and Škerget *et al*. [8], therefore only the resulting matrix form of the equations for kinematics, heat energy kinetics and vorticity kinetics are presented here as

$$\left([\mathbf{D_x}]_1 + [\mathbf{D_y}]_2\right)\{\omega\}_\Gamma = [\mathbf{H}]\{v_t\} + [\mathbf{H_t}]\{v_n\} - \left([\mathbf{D_x}]_1 + [\mathbf{D}_y]_2\right)\{\omega\}_\Omega \tag{7}$$

$$[\mathbf{H}]\{T\} = \frac{a_p}{\bar{a}_p}[\mathbf{G}]\left\{\frac{\partial T}{\partial n}\right\} - \frac{1}{\bar{a}_p}[\mathbf{G}][v_n]\{T\} + \frac{1}{\bar{a}_p}[\mathbf{D_j}][v_j]\{T\} - \frac{\tilde{a}_p}{\bar{a}_p}[\mathbf{D_j}]\left\{\frac{\partial T}{\partial x_j}\right\} + \frac{1}{\bar{a}_p}\beta\, [\mathbf{B}]\{T\}_{F-1} \tag{8}$$

$$
\begin{aligned}
[\mathbf{H}]\{\omega\} = {} & \frac{\nu_f}{\overline{\nu}_f}[\mathbf{G}]\left\{\frac{\partial \omega}{\partial n}\right\} - \frac{1}{\phi^2 \Lambda \overline{\nu}_f}[\mathbf{G}]\left\{\omega v_n + \phi^2 e_{ij} n_i g_j F\, n_j + \phi^2 \Lambda f_j\, n_j + \phi^2 \frac{c_F}{\sqrt{K}} e_{ij} v_n v\right\} - \\
& \frac{1}{\phi^2 \Lambda \overline{\nu}_f}[\mathbf{B}]\left[\frac{\phi^2 \nu_f}{K} + \phi^2 \frac{c_F}{\sqrt{K}} v\right]\{\omega\} + \\
& \frac{1}{\phi^2 \Lambda \overline{\nu}_f}[\mathbf{D}_{\mathrm{j}}]\left\{\begin{aligned} & \omega v_j - \phi^2 e_{ij} g_j F - \phi^2 \Lambda f_j - \\ & \phi^2 \Lambda \tilde{\nu}_f \frac{\partial \omega}{\partial x_j} - \phi^2 \frac{c_F}{\sqrt{K}} e_{ij} v_j v \end{aligned}\right\} + \frac{1}{\phi^2 \Lambda \overline{\nu}_f} \beta [\mathbf{B}]\{\omega\}_{F-1}
\end{aligned}
\qquad (9)
$$

Results and discussion

The extended numerical algorithm, which includes the additional Forcheimer term, was tested on the problem of natural convection in a tall porous cavity heated from the side. The governing parameters for the present problem are: the porosity ϕ, the modified Rayleigh number $Ra^* = g\beta_T KH\Delta T/\gamma\, a_p$, the Darcy number $Da = (1/\phi)(K/H^2)$, the aspect ratio $A = H/D$, and the ratio of the volumetric heat capacity of the solid and fluid phase σ. Here D, H and ΔT are the width of the cavity, the height of the cavity and the temperature difference between hot and cold walls, respectively. In order to illustrate the typical numerical results the parameters are $A = 5$, $\sigma = 1$, $\Delta T = 1$, $\phi = 0.8$ and coefficient c_F is equal to 0.2. A nonuniform computational mesh of 30×40 subdomains was used with the ratio between the longest and the shortest element equal to $r_x = 30$ and $r_y = 20$. Time step is $\Delta t = 0.001$, while the convergence criterion is determined to be $\varepsilon = 5 \times 10^{-6}$. It can be argued that the convective, Forcheimer and Brinkman terms are negligible for this case but the investigation that follows focus on the Forcheimer inertial effects only. A numerical model based on the presented theoretical work and chosen parameters is, at this moment, in the phase of evaluation and testing, therefore the simple test case is presented here for $Ra^* = 100$. To demonstrate the effect of the additional Forcheimer term on the heat transfer across the cavity, the overall Nusselt number representing the total heat transfer across the cavity, $Nu = -\int_0^1 (\partial T/\partial x)_{x=0} dy$, calculated by using the Brinkman model [7] is compared with the Nusselt number gotten by using the above described numerical procedure based on the Forcheimer model. Table 1. shows that the difference in the heat transfer rate is minimal. For example for $Da = 10^{-1}$ the difference is less than 1 % and for smaller Darcy numbers the difference is never greater than 4 %. Higher distinction is expected with an increase in modified Rayleigh numbers. Table 1. further shows that Forcheimer model predicts lover total heat transfer across the cavity (Nusselt numbers) than the Brinkman model. These results are in agreement with the conclusions reported by Lauriat & Prasad [4] for vertical porous cavity and also with the conclusions reported by Lage [5] for horizontal porous layer.

Table 1.: Overall Nusselt number for $A = 5$, $Ra^* = 100$ for Forcheimer and Brinkman model

Da	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Forcheimer model	5.299	7.033	8.834	9.609
Brinkman model [7]	5.312	7.254	9.134	9.953

The effect of Forcheimer additional term is illustrated also in Fig. 1 where the vertical velocity profiles at the horizontal midplane are presented for both Brinkman and Forcheimer model for two different Darcy numbers, $Da = 10^{-2}$ and $Da = 10^{-4}$. For a small Darcy number the vertical velocity distribution shows its largest gradient near the vertical walls. But when Da is increased the maximum velocity reduces and the velocity peaks move away from the walls. It can be seen that the difference between both models is minimal.

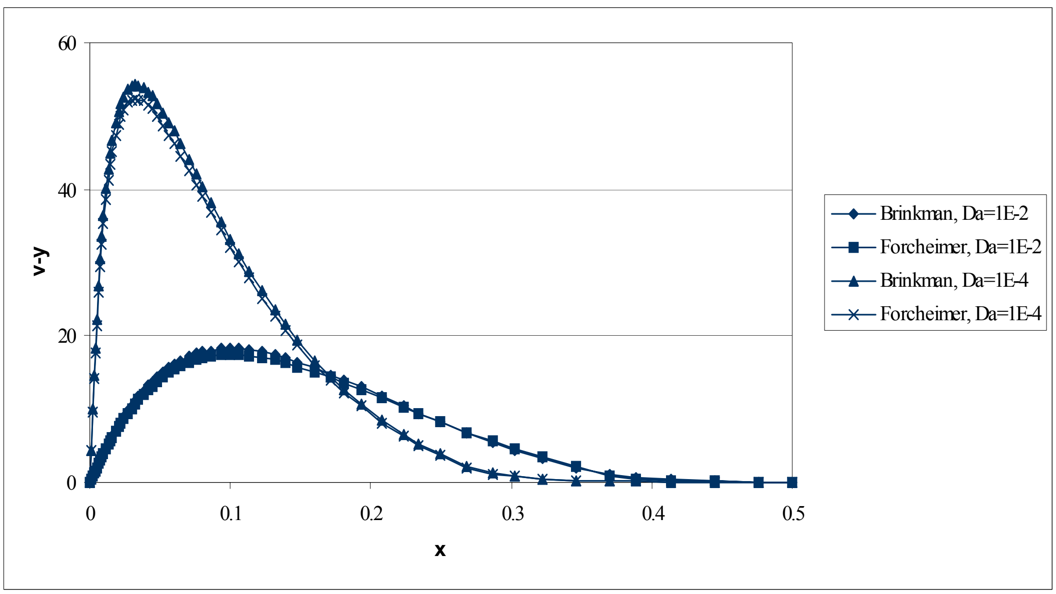

Figure 1.: Vertical velocity profiles (left half of the cavity) at the horizontal midplane for a modified Rayleigh number $Ra^* = 100$ at two various values of Darcy number

The complete analysis for different values of Ra^* and ϕ will likely serve to confirm the fact that, when using the Brinkman-Forcheimer momentum equation, the effect of the additional inertia term (Forcheimer term) is not considerable for the moderate values of the governing parameters.

Conclusion

The problem of natural convection in porous cavity heated from the side saturated with Newtonian fluid is investigated utilizing a Boundary Domain Integral Method (BDIM). The Brinkman-Forcheimer extended Darcy model is used to examine the influence of the additional inertia term. The inclusion of the Forcheimer term in the momentum equation leads to a reduction of the heat transfer rate but the results indicates that the effect on the heat transfer results is minimal. The numerical code becomes more complex and also the computation time required to achieve convergence is increased. Therefore it is possible to conclude that in the range covered by the present investigation the Forcheimer term is not really relevant to the calculation of the global heat transfer parameter, as reported in the literature in which the calculations were performed with other numerical methods.

Acknowledgements

The first author (R. Jecl) acknowledges the financial support to the research project Z2-3289 (2001) as provided by the Ministry of Education, Science and Sport of Republic of Slovenia.

Reference

1 Lauriat G., Prasad V. (1987): "Natural convection in a vertical porous cavity: a numerical study for Brinkman-extended Darcy formulation", *Journal of Heat Transfer*, Vol. 109, pp. 688-696.

2 Vasseur, P., Wang, C.H., Sen, M. (1990): "Natural convection in an inclined rectangular porous slot: Brinkman-extended Darcy model", *Journal of Heat Transfer,* Vol. 112, pp. 507-511.

3 Jecl, R., Škerget, L., Petrešin, E. (2001): "Boundary domain integral method for transport phenomena in porous media", *International Journal for Numerical Methods in Fluids,* Vol. 35, pp. 39-54.

4 Lauriat, G., Prasad, V. (1989): "Non-Darcian effects on natural convection in a vertical porous enclosure", *International Journal of Heat and Mass Transfer,* Vol. 32, pp. 2135-2148.

5 Lage, J.L. (1992): "Comparison between the Forcheimer and the convective inertia terms for Benard convection within a fluid saturated porous medium", *HTD 193, Fundamentals of Heat Transfer in Porous Media,* ASME, pp. 49-55.

6 Nield, D. A. and Bejan, A. (1999): *Convection in porous media (Second ed.)*, Springer-Verlag.

7 Jecl, R. and Škerget, L. (2003): "BEM for natural convection in a tall porous enclosures saturated with a power-law fluid". *proceedings from ICCES 2003, eds.:* Atluri, S. N., Beskos, D. E., Polyzos, D., Corfu, Greece.

8 Škerget L., Hriberšek, M., Kuhn, G. (1999): "Computational Fluid Dynamics by Boundary Domain Integral Method", *International Journal for Numerical Methods in Engineering*, Vol. 46, pp. 1291-1311.