Analysis of Interface Crack Problems in Dissimilar Anisotropic Media using the Boundary Element Method

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Summary

This paper presents an application of the boundary element method (BEM) in problems of linear elastic fracture mechanics. The stress intensity factor (SIF) for an interfacial crack in a dissimilar anisotropic bimaterial is evaluated. Analysis of a plate under in-plane mixed mode is carried out and the complex SIF is computed. The sub-region technique is used to model each different sub-domain, represented by each anisotropic material. On the interface of sub-regions conditions of tractions equilibrium and displacements continuity are imposed, except in the corresponding crack region. The singular behaviour presented by the stress fields near the crack-tip is modelled by traction singular quarter point elements. Numerical examples of problems with in-plane loading are presented and compared with those established in the literature.

Introduction

Structures composed of bonded layers of dissimilar materials are becoming more and more common in a variety of applications. In many such structures failures originating from production processes, overload or aging of the structure can be observed, with serious implications on the structural reliability of bimaterial systems. Due to geometric and material discontinuities, cracks are commonly observed near the interface, thus the SIF of an interfacial crack can be used to evaluate the critical condition of these bimaterials. General loading in the interface of materials with different anisotropic properties, results in a coupled complex SIF, or $K = K_I + iK_{II}$ [1].

The BEM is known as an appropriate numeric technique for the analysis of high stress gradients in solid mechanics such as stress intensification at the crack tip. Some works using BEM on analysis of interfacial cracks in isotropic bimaterial components were presented by Tan and Gao [2, 3], Yuuki and Xu [4] and recently by de Paiva et al. [5]. The application of the method in analysis of interfacial cracks in anisotropic bimaterial components has been presented by Tan et al. [6] and Pan and Amadei [7].

In this work a boundary element computer code was implemented using continuous quadratic elements. It was also implemented the technique of sub-regions for modeling of sub-domains of laminated bimaterial. Quarter-point elements, were used to describe the displacements and stress fields near the crack tip.

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Boundary element analysis for anisotropic elasticity

The boundary integral equation for linear anisotropic elasticity is derived in the usual manner [8, 9]. Following Sollero et al. [10] we have:

$$c_{ij}(z')u_j(z')+\int T_{ij}(z',z)u_j(z)d\Gamma(z)=\int U_{ij}(z',z)t_j(z)d\Gamma(z), \quad (1)$$

where i, j = 1, 2; c_{ij} is given by $\delta_{ij}/2$ for a smooth boundary; z' and z are the source and field points on the boundary Γ; u_i and t_j are displacements and tractions computed on the boundary Γ and T_{ij} and U_{ij} are the fundamental solution for tractions and displacements given by:

$$T_{ij}(z',z)=2\,\mathrm{Re}\left[\frac{Q_{j1}(\mu_1 n_1-n_2)A_{i1}}{z_1-z_1'}+\frac{Q_{j2}(\mu_2 n_1-n_2)A_{i2}}{z_2-z_2'}\right], \quad (2)$$

$$U_{ij}(z',z)=2\,\mathrm{Re}\left[P_{j1}A_{i1}\ln(z_1-z_1')+P_{j2}A_{i2}\ln(z_2-z_2')\right], \quad (3)$$

where n_k are components of the normal vector, and P_{jk}, Q_{jk}, A_{ik} are complex coefficients, μ_k are the roots of the characteristic equation for an anisotropic material, with k = 1, 2. In real problems Eq. (1) is discretized and transformed into the following linear system of algebraic equations

$$\mathbf{H\,u}=\mathbf{G\,t} \quad (4)$$

where $\mathbf{H}$ and $\mathbf{G}$ matrices contain the numerically integrated kernels of Eq. (1), including the fundamental solutions and vectors $\mathbf{u}$ and $\mathbf{t}$ contain the boundary values, even known or not. After algebraic manipulations the unknowns are isolated in $\mathbf{f}$ vector and the system of Eq. (4) can be reduced to

$$\mathbf{A\,x}=\mathbf{f} \quad (5)$$

where unknown values are obtained by solving the linear system of equations.

Let us consider an anisotropic bimaterial divided in two sub-regions Ω_1 and Ω_2, with boundary Γ_1 and Γ_2 separated by an interface I delimited by Γ_1^{I} and Γ_2^{I} (Fig. 1). Displacements $\mathbf{u}$ and tractions $\mathbf{t}$ in the boundary of each sub-domain must satisfy conditions of displacements continuity and tractions equilibrium. After setting up these conditions of in the interface, Eq. (4) can be rewritten in the following way

$$\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_1^I & -\mathbf{G}_1^I & 0 \\ 0 & \mathbf{H}_2^I & \mathbf{G}_2^I & \mathbf{H}_2^I \end{bmatrix} \begin{Bmatrix} \mathbf{u}_1 \\ \mathbf{u}^I \\ \mathbf{t}^I \\ \mathbf{u}_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{G}_1 & 0 \\ 0 & \mathbf{G}_2 \end{bmatrix} \begin{Bmatrix} \mathbf{t}_1 \\ \mathbf{t}_2 \end{Bmatrix} \tag{6}$$

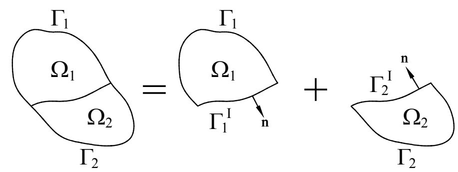

Figure 1. Domain division using sub-region.

Applying boundary conditions in the system of Eq. (6), the system shown in Eq. (5) is obtained.

The SIF can be evaluated directly using the nodal values of tractions [2], namely

$$K_0 = \frac{\sqrt{2\pi\, l}}{\cosh(\pi\zeta)} \sqrt{(t_1)^2 + (t_2)^2} \tag{7}$$

where ζ is a function of material properties, t_1 and t_2 are tractions calculated at boundary nodes of crack tip and l is the quarter-point element length, as shown in Fig. 2.

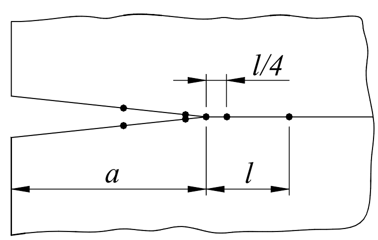

Figure 2. Crack tip element and its characteristic dimensions.

Numerical Results and Discussions

The first analysis presents a problem of a crack lying in an interface of two isotropic bonded dissimilar half-planes subjected to tension remotely applied. This problem has analytic solution presented by Rice and Sih [11]. Fig. 3 shows the physical problem considered. In the numerical analysis, a finite size plate was considered, but the width and height of the plate was taken to be 20 times the size of the crack. Plane strain conditions were assumed. Also, to ensure continuity conditions for the strain ε_{xx} along the bimaterial interface, the applied stress $(\sigma_x)_{II}^{\infty}$ was taken to be

$$(\sigma_x)_{II}^{\infty} = \frac{E_2}{E_1}(\sigma_x)_I^{\infty} + \left[\nu_2 - \frac{E_2}{E_1}\nu_1\right](\sigma_y)^{\infty} \tag{8}$$

where E_1 and E_2 are the Young moduli of materials I and II, respectively. Due to symmetry, only one half of the physical problem was modelled. The mesh employed has a total of 24 boundary elements. Four different values of E_2/E_1 ratio were considered. The ratio of the lengths of the quarter-point crack-tip elements to the crack length, l/a, was varied from 0.06 to 0.15.

Table 1 presents the normalized SIF values $K_0/\overline{K}$, where $\overline{K} = (\sigma_y)^{\infty}\sqrt{\pi a}$. Present results, evaluated with the anisotropic fundamental solutions using quasi-isotropic elastic properties, are compared with analytical results [11], using isotropic elastic properties.

Table 1. SIF for an interfacial crack in a dissimilar half-plane.

$\frac{E_2}{E_1}$	$\frac{l}{a}$	$\left(\frac{K_0}{\overline{K}}\right)_{[11]}$	$\left(\frac{K_0}{\overline{K}}\right)_{PRESENT}$	*Error* Δ%
	0,15		0.9994	0.0581
1	0,08	1,000	0.9988	0.1212
	0,06		1.0035	0.3523
	0,15		1.0191	3.0424
5	0,08	0,989	0.9973	0.8411
	0,06		0.9813	0.7792
	0,15		1.0339	5.5019
20	0,08	0,980	1.0124	3.3100
	0,06		0.9966	1.6909
	0,15		1.0391	6.4669
100	0,08	0,976	1.0178	4.2842
	0,06		1.0020	2.6685

We can notice from Table 1 that for a refined mesh with $l/a = 0.06$ the maximum error was 2.6 %, even for the ratio of elastic properties E_2/E_1 assuming value of 100.

The second analysis presents a problem of an interfacial crack in an anisotropic bimaterial under a uniform pressure p (Fig. 4). Plane stress condition is assumed. Twenty-four quadratic boundary elements were used to discretize the bimaterial system. The normalized SIF under plane stress condition are listed in Table 2. The anisotropic elastic properties in material (I) were assumed to be those of glass/epoxy with $E_1 = 48.26\ GPa$, $E_2 = 17.24\ GPa$, $G_{12} = 6.89\ GPa$, $\nu_{12} = 0.29$. For material (II), a graphite/epoxy with $E_1 = 144.8\ GPa$, $E_2 = 11.7\ GPa$, $G_{12} = 9.66\ GPa$, $\nu_{12} = 0.21$ was selected. While the material axis E_1 in material (I) was assumed to be along the horizontal direction (i.e. $\alpha_1 = 0$), the E_1-axis in material (II) makes an angle α_2 with respect to the horizontal direction. The interfacial SIF at crack tip A was obtained by the present method, listed in Table 2 and compared to the numerical results presented by Pan and Amadei [6], showing a good approximation.

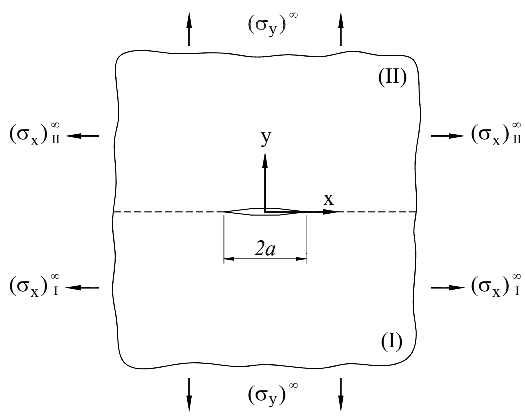

Figure 3. Physical problem of an infinite isotropic bimaterial plate with an interfacial crack subjected to remote normal stresses.

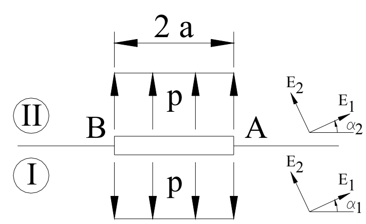

Figure 4. Physical problem of an infinite anisotropic bimaterial plate with an interfacial crack subjected to uniform pressure p.

Table 2. Normalized SIF for an interfacial crack in infinite anisotropic bimaterial.

α_2	$K_n = \sqrt{K_I + K_{II}} / \bar{K}$ [6]	$K_n = K_0 / \bar{K}_{PRESENT}$	*Error* (%)
0	1.0007	1.0198	1.9059
30	0.9974	0.9941	0.3259
45	0.9970	0.9856	1.1426
60	0.9949	0.9873	0.7575
90	1.0002	1.0109	1.0678

Conclusions

In this paper the implementation of a computer code for SIF calculation of interfacial cracks in anisotropic bimaterial structures was presented. The code is based on BEM and uses the sub-region technique. The SIF were calculated from nodal values of tractions obtained by traction singular quarter-point elements. Numerical analysis with various ratios of the material properties have been carried out on some known problems and results show a good agreement with the results presented in the literature.

Acknowledgement

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