

## An Experimental Method for Kinematic Measurement of a Four-Bar Mechanism by Digital Video Analysis

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### Summary

Measuring the position, speed and acceleration of the moving elements of a mechanism usually involves external instrumentation that has to be attached to it, which is not always feasible. The method discussed here provides an alternative to measure these kinematics characteristics without modifying the components. To test the method in a controlled environment a four-bar mechanism was build. It was designed to be as thin as possible and to allow arbitrary changes in its lengths. Three commercial programs and a specially designed software (KIMA®) are used to record, analyze, measure, compute and verify the kinematics of the mechanism.

### Introduction

Measuring the displacement and velocities in a four-bar mechanism (as the rear bike suspension shown in figure 1) is very important to fine adjust the performance of any kind of system. We propose that these measurements can be done by recording a digital video of the system under controlled circumstances (light, distance and alignment to the mechanism) and a later image analysis.

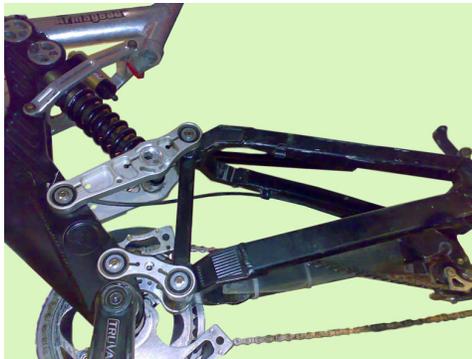


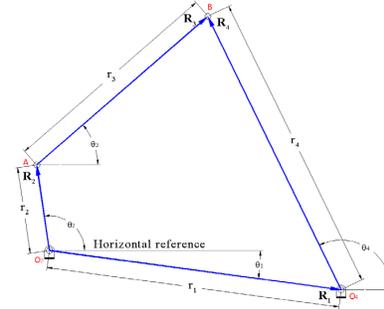
Figure 1: A four-bar linkage working as a rear bike suspension system

Figure 2 is an equivalent diagram [1] for the mechanism shown in figure 1 with four position vectors superimposed. The closed loop vector equation is (1), where vector  $\mathbf{R}_1$  must be fixed to the reference coordinate system. We will assume that vector  $\mathbf{R}_2$  is the input linkage and the kinematic conditions for  $\mathbf{R}_3$  and  $\mathbf{R}_4$  must be determined. The method proposed here is also valid in any other combination of

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input-output links.



$$\mathbf{R}_2 + \mathbf{R}_3 = \mathbf{R}_1 + \mathbf{R}_4 \quad (1)$$

The angular position for  $\mathbf{R}_3$  ( $\theta_3$ ) and  $\mathbf{R}_4$  ( $\theta_4$ ) will be solved by (2) and (3).

$$\theta_4 = 2a \tan \left( \frac{-B \pm \sqrt{A^2 + B^2 - C^2}}{C - A} \right) \quad (2)$$

$$\theta_3 = 2a \tan \left( \frac{-E \pm \sqrt{D^2 + E^2 - F^2}}{F - D} \right) \quad (3)$$

Where constants  $A$  to  $F$  has been defined [1] in terms of links lengths and angular positions.

The angular velocities for links 3 and 4 will be:

$$\omega_3 = \frac{\omega_2 r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin(\theta_3 - \theta_4)} \quad (4)$$

$$\omega_4 = \frac{\omega_2 r_2 \sin(\theta_2 - \theta_3)}{r_4 \sin(\theta_4 - \theta_3)} \quad (5)$$

Finally, angular accelerations will be defined as follows.

$$\alpha_3 = \frac{\omega_4^2 r_4 - \omega_2^2 r_2 \cos(\theta_2 - \theta_4) - \alpha_2 r_2 \sin(\theta_2 - \theta_4) - \omega_3^2 r_3 \cos(\theta_3 - \theta_4)}{r_3 \sin(\theta_3 - \theta_4)} \quad (6)$$

$$\alpha_4 = \frac{\omega_3^2 r_3 + \omega_2^2 r_2 \cos(\theta_2 - \theta_3) + \alpha_2 r_2 \sin(\theta_2 - \theta_3) - \omega_4^2 r_4 \cos(\theta_4 - \theta_3)}{r_4 \sin(\theta_4 - \theta_3)} \quad (7)$$

Equations (2) to (7) were implemented and solved by KIMA® [2].

The model built for this work is shown in figure 2. It was built in aluminum 6063-T5 with a CNC mill. The length of each link can be modified by loosening a pair of Allen screws located near each mobile joint. The joint between link 1 and

link 4 can also be shifted giving an infinite combination of distances and angles. The round yellow-black position markers were placed using an 1/8 in cylindrical guide that was inserted in a center hole for each joint. The rectangular and circular position markers located in the corners were plotted on a sheet of paper and pasted on the wooden base of the model. It was required that the sheet was placed using the minimum inner cuts to avoid bad edges detection.



Figure 2: The four-bar prototype

The input link (the smaller one in figure 2) is driven by a variable speed drill battery operated (Wolf WIDEASIA Mod. 013C 12V). Between this and the input axle is located a planetary gear train with a 5:1 reduction stage. This gear was needed to increment the torque exerted by the drill to provide a smooth movement on the mechanism and to reduce the speed in order to get better pictures in the video.

### **Method**

The first step is to align horizontally and vertically the base of the prototype. Next the camera must be located at a distance between 2,5 m to 3 m from the base of the mechanism. This distance was determined by several tries looking for the best balance between a correct lighting and a reduction of perspective effect in each frame analyzed. The lens camera must be aligned with the center of the base. To achieve this, an electronic laser level (Surtek) mounted on a tripod was used behind the camera. The preparation for the shoot took almost an hour.

Three full cycles were recorded with a Sony DCR-HC96 video camera. The video was downloaded with Adobe Premier® and three frames were selected. The following criteria must be observed for this method to work properly.

1. None of the joint markers must overlap with any link.
2. All of the frames must be selected within one cycle of the driving link.
3. Each link must move in the same direction.

These three frames are post processed using Adobe Photoshop®. First they are translated to JPG format, because this is the image format that is best managed by

AutoCad®. Next a *Find Edges* filter is applied, and finally, the background noise is cancelled by adjusting the *Input Levels* value. Although there are several works [3, 4, 5] about automatic edge detection, the goal of this method is to not use these algorithms and be able to measure the kinematic values for the mobile link of a four-bar mechanism.

The camera introduces a vertical/horizontal distortion, which derives in a deformation of the round markers to make them appear as oval. It also produces different sizes for the same link depending upon its position. This will be fixed by two scale factors, one for the horizontal dimension and another for the vertical dimension,  $F_x$  and  $F_y$ , respectively. Values  $F_x$  and  $F_y$  are a relation between the real dimensions ( $dr_x, dr_y$ ) and those measured inside AutoCad ( $da_x, da_y$ ).

$$F_x = \frac{dr_x}{da_x} \times 100 \quad (8)$$

$$F_y = \frac{dr_y}{da_y} \times 100 \quad (9)$$

Once the frame's dimensions are corrected using Adobe PhotoShop, all the frames are reinserted in AutoCad and the location of the fixed joints for the first frame is determined using a circular mark. This procedure is manual and depends on the expertise. These marks are then copied to the other three frames. Now, the location of the moving joints must be determined. Again, the expertise is crucial for the correct location. At this point, each frame has a different value for lengths of link 2, 3 and 4 (link 1 must be the same for all frames). The average is computed for each link and the mobile joints are relocated with these new values.

The links angular position is measured by AutoCAD. These values will be used to compute speed and acceleration for the mobile links. To calculate speed, the time increment is determined knowing that commercial video cameras take 29.99 fps (frames per second) which can be rounded up to 30 fps. From the Adobe Premier interface we can know the exact time that corresponds to each frame selected. Equation (10) was used to calculate (in seconds) the time between two consecutive frames. Time will be specified with two pair of numbers as  $ss: ff$ , where  $ss$  are seconds and  $ff$  are frames.

$$TI = \frac{\text{total frames in between}}{30} s \quad (10)$$

At this point all the experimental data has been measured (angular positions) or computed (angular velocities and accelerations). The software KIMA® will be used to calculate the theoretical position, speed and acceleration for links 3 and 4, based on the experimental data for link 1 and 2. These values will be compared against those obtained experimentally to determine the percentage of deviation.

**Results**

Three frames were selected, with time frames as follows: 01:20, 02:26 and 04:12. The time intervals were:  $TI_{1,2} = 1.2s$  and  $TI_{2,3} = 1.533s$ . Measured links lengths were  $r_1= 140.6$  mm,  $r_2= 61.8$  mm,  $r_3= 173.5$  mm,  $r_4= 141.6$  mm and  $\theta_1 = 0.5^\circ$ . The experimental results are shown in Table 1.

Table 1: Experimental results obtained from AutoCAD measurements

Frame 1:	$\theta_2 = -153.8870^\circ, \theta_3 = -35.9605^\circ, \theta_4 = -112,9715^\circ$
Frame 2:	$\theta_2 = -134,1030^\circ, \theta_3 = -31,7749^\circ, \theta_4 = -104,5960^\circ$ $\omega_2 = 0,2877 \text{ rad/s}, \omega_3 = 0,0609 \text{ rad/s}, \omega_4 = 0,1218 \text{ rad/s}$
Frame 3:	$\theta_2 = -108,4602^\circ, \theta_3 = -28,0318^\circ, \theta_4 = -92,6484^\circ$ $\omega_2 = 0,2919 \text{ rad/s}, \omega_3 = 0,0426 \text{ rad/s}, \omega_4 = 0,1360 \text{ rad/s}$ $\alpha_2 = 0,0027 \text{ rad/s}^2, \alpha_3 = -0,0119 \text{ rad/s}^2, \alpha_4 = 0,0092 \text{ rad/s}^2$

Table 2 shows the computed data obtained from KIMA®. Kinematical data for the driving link was taken from Table 1.

Table 2: Data computed by KIMA®.

Frame 1:	$\theta_3 = -35,9274^\circ, \theta_4 = -113,1167^\circ$
Frame 2:	$\theta_3 = -31,7588^\circ, \theta_4 = -104,7628^\circ$ $\omega_3 = 0,0525 \text{ rad/s}, \omega_4 = 0,1283 \text{ rad/s}$
Frame 3:	$\theta_3 = -28,0466^\circ, \theta_4 = -92,8491^\circ$ $\omega_3 = 0,0309 \text{ rad/s}, \omega_4 = 0,1388 \text{ rad/s}$ $\alpha_3 = -0,0151 \text{ rad/s}^2, \alpha_4 = 0,0022 \text{ rad/s}^2$

Minimum and maximum percentage deviations were as follows:

Table 3: Percentage deviation values between experimental and theoretical values

Magnitude	Deviation (%)	
	Min	Max
Position	0.0530	0.2166
Velocity	2.0771	27.4274
Acceleration	26.5823	76.0652

**Discussion**

The results were excellent for position values. We can say that speed results were good enough for the remote image analysis presented here. But our acceleration values had a big peak deviation of 76.0652%. Work has to be done to fine adjust this stage because of the small acceleration values.

Another cause for this peak could be the way we measure the link's lengths. We need to develop a device to measure the distance between adjacent joints with more precision. Although the prototype was designed and built to be as flat as possible, there is a 7 mm difference between the front faces. The measurements are made with a Vernier caliper which can not compensate this thickness.

The next step in this work will be to implement automatic edge detection algorithms and compare the results from this job with them. Also, an ultimate goal will be to develop a real time video analysis system.

### References

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