Finite Element Formulation of Anisotropic Hyper-Elastic Materials Using Polyconvex Strain Energy Function

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Summary

In this presentation, a finite element formulation and implementation of anisotropic hyper-elastic material models using polyconvex strain energy functions are presented. Through comparison between the computational results using a polyconvex strain energy function and a non-polyconvex one, the validity and effectiveness of polyconvex strain energy functions are investigated. A number of constitutive models for composite rubber materials and biological tissues, which show strong anisotropic behavior, have recently been proposed using hyperelastic constitutive material. Hyperelasticity allows the formulation of large deformation, including anisotropic effects, using a structural tensor which consists of fiber-directional basis vectors. The present work is based on the invariant formulation, in which pseudoinvariants are required in addition to the isotropic 3-invariants. In order to avoid non-physical behavior which may appear in general anisotropic hyper-elastic models, the related strain-energy function must be polyconvex. There exists a number of studies on polyconvexity, and these show that polyconvexity of a strain-energy function insures the existence of the global minimum of the total elastic energy, which implies that the material is stable.

Through simple numerical examples, the present work shows that a non-polyconvex strain energy function may cause non-physical behavior whereas polyconvex one can avoid this defect. Current research also utilizes this model in order to capture the response of cloth considering interaction of yarn families, i.e. warp and weft, in deformed configuration. Present anisotropic hyper-elastic models predict the outcome of uniaxial and biaxial tensile tests on knitted strips loaded in the warp, weft and bias directions. In all cases, the model can accurately predict the experimentally observed deformations and load-extension responses.

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