

Determination of the Size Effect Law with One-Size Concrete Specimens of Different Notch Depths

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Summary

The most popular expression that considers the size effect showed in concrete structures is the size effect law of Bazant. This law depends on two coefficients G_f and c_f . For the experimental determination of these coefficients is necessary obtaining the ultimate loads, in three points bending tests, of notched beams with different sizes. In this work, a method to determine the coefficients about the results on identical beams varying the deep of the notch, is presented. The necessary values of loads are obtained by a numerical analysis with FEM that considers non linear behaviour of the concrete material. Next the proposed method is applied to experimental results of concrete specimens with different relative notch depth.

Introduction

The size effect in concrete structures can be considered by the size effect law of Bazant [1,2]. This law is defined by two coefficients, G_f and c_f , which are considered as material's properties.

The experimental determination of these coefficients can be done by a procedure of the international association RILEM, which propose a three-point-bending tests of geometrically similar single edge notched bend (SENB) specimens of concrete with three different sizes.

To avoid the manufacturing of specimens of different sizes, a method is proposed in this work that allows the determination of these coefficients by means of the ultimate loads obtained in specimens of same size, but different notch depths.

A numerical and experimental application of the proposed method has been done in this work. The numerical studies have been realized with a FEM commercial program (ABAQUS) that allows obtains the peak load and softening curve of concrete specimens. For the experimental analysis, notched specimens of same size with different notch depth have been manufactured and tested.

Proposed Method

The proposed method is based in the relation given by the size effect law of Bazant, between the nominal stress at failure and the geometry of specimens tested. The size effect law is:

$$\sigma_N = \frac{P_u}{BD} = \sqrt{\frac{EG_f}{g'(\alpha)c_f + g(\alpha)D}} \quad (1)$$

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Where P_u is the ultimate load, B the thickness of the specimen, D a characteristic dimension (the depth is usually considered), E the elastic modulus of concrete, $g(\alpha)$ and $g'(\alpha) = dg(\alpha)/d\alpha$ functions of the specimen's geometry and the relative notch depth ($\alpha = a/D$) and G_f and c_f asymptotic constants of concrete. These constants are defined as the specific fracture energy and the length of the fracture process zone for specimens of infinite size [1,2]. The coefficient $g(\alpha)$ is related with the parameter $Y(\alpha)$ used for the determination of K_{IC} [2].

From size effect (1) and with the obtained (numerical or experimental) values of σ_N , a relation between G_f , c_f and $y(\alpha) = 1/\sigma_N^2$ can be determined. The relation is obtained as follows:

$$G_f(\alpha) = \frac{g'(\alpha)c_f(\alpha) + Dg(\alpha)}{E_c y(\alpha)} \quad (2)$$

The plot of the relation given by equation (2) in a $G_f - c_f$ axes for each relative notch depth, are straight lines that must be crossed in a single point. It will be able using this point to predict the G_f and c_f coefficients as properties of the material.

Numerical Studies with MEF

The determination of the ultimate loads in three-point-bending is necessary for the theoretic development of the proposed method. In the following paragraphs a numerical study about the quasibrittle materials models for concrete in commercial FEM software (ABAQUS) is presented.

One of the available models for concrete in ABAQUS is the Concrete Smearred Cracking model, which considers elasto-plastic compression behavior with flow rule and isotropic hardening, and cracking in tension by means of the softening response.

The ABAQUS modeled notched specimens have dimensions: $B = 150\text{mm}$, $D = 150\text{mm}$, $L = 600\text{mm}$, $S = 450\text{mm}$ and relative notch depth of $\alpha = 0.033 - 0.1 - 0.167 - 0.233 - 0.3 - 0.367 - 0.433$ and 0.5 . The modeled concrete have compressive strength $f_c = 25 \text{ MPa}$, tensile strength $f_{ct} = 3.16 \text{ MPa}$, elastic modulus $E_c = 27264 \text{ MPa}$ and fracture energy $G_F = 75 \text{ N/m}$.

In figure 1, the $G_f - c_f$ relationships for the numerical results are showed.

It is observed from figure 1, that there isn't a common point that allows obtain the G_f and c_f material properties. Therefore, it should be derived that these coefficients cannot be considered properties of concrete.

With $y(\alpha)$ and its derivative $y'(\alpha)$ an expression of G_f and c_f for each α can be determined. If this expressions are plot (figure 2) the variation of these coefficients respect to the relative notch depth can be observed.

However, $\alpha = 0.033$ curve cross the other curves in a similar point, which can be used to obtain a mean coefficients $G_{fm} = 33.73 \text{ N/m}$ and $c_{fm} = 10.79 \text{ mm}$. With

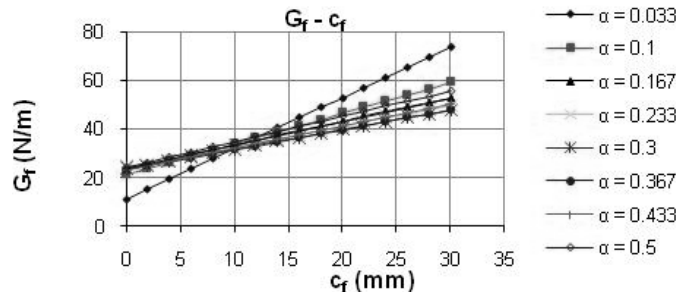


Figure 1: G_f and c_f relationships for numerical results

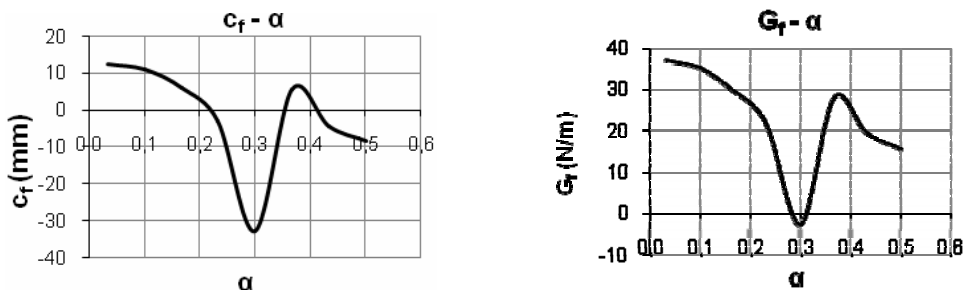


Figure 2: Variation of G_f and c_f with respect to α

these mean values of the coefficients and using equation (1), the nominal tension σ_N for other relative notch depths can be obtained. Figure 3 shows the $\sigma_N - \alpha$ curves from equation (1) with the G_f and c_f coefficients obtained with the proposed method, and the numerical results.

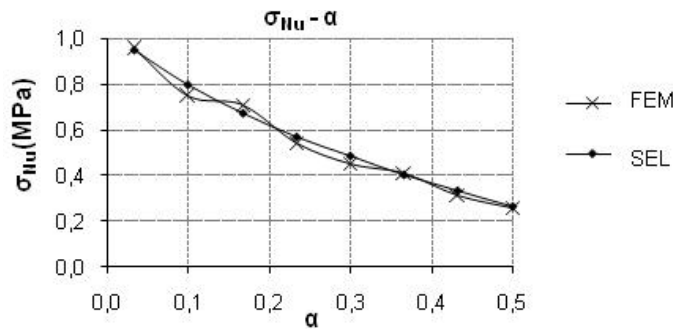


Figure 3: $\sigma_N - \alpha$ curves with numerical results

Experimental Studies

Several concrete notched specimens have been made up, of same size and identical relative notch depths than numerical models. A three-point-bending test was applied to these specimens obtaining the maximum loads for each of them. Figure 4 shows a picture of the notched specimens tested and the load and support condition

for three-point-bending tests.

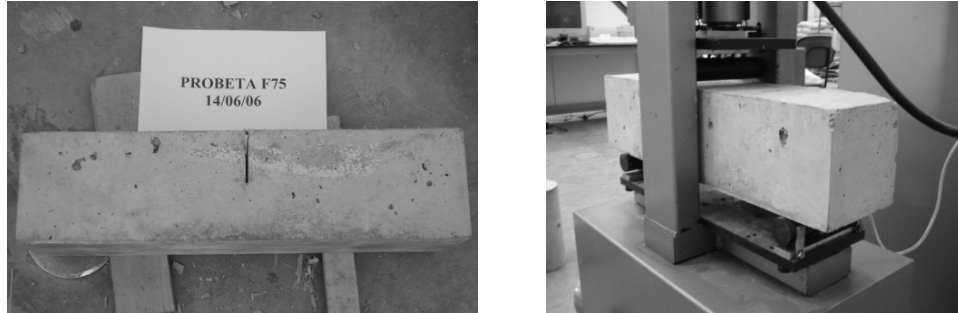


Figure 4: Notched specimen and three-point-bending test

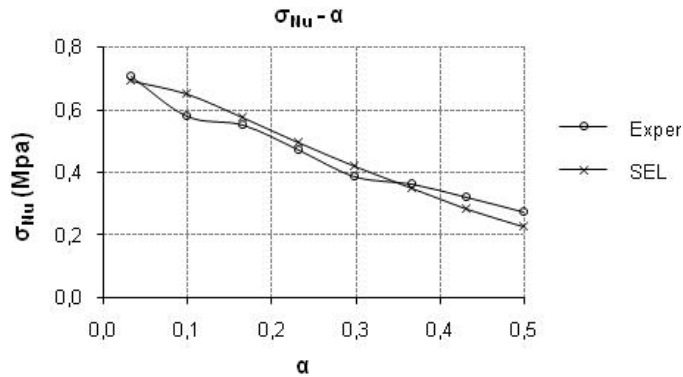


Figure 5: Experimental and proposed method $\sigma_N - \alpha$ curves

The concrete has a compressive strength of $f_c = 31.9$ MPa, tensile strength $f_{ct} = 1.94$ MPa, elastic modulus $E_c = 23466$ MPa and maximum aggregate size of 12 mm.

Table 1 shows the mean values of maximum loads and nominal stress obtained for each relative notch depth.

Table 1: Mean experimental maximum loads and nominal stress

α	P_u (N)	σ_{Nu} (MPa)	α	P_u (N)	σ_{Nu} (MPa)
0.033	15815	0.703	0.300	8635	0.384
0.100	13015	0.578	0.367	8075	0.359
0.167	12395	0.551	0.433	7165	0.318
0.233	10595	0.471	0.500	6135	0.273

Applying the proposed method to the experimental results, a common point for all $G_f - c_f$ relations cannot be obtained again, as in numerical results. So it confirms that G_f and c_f coefficients cannot be considered as material properties. Similar conclusions has been determinate for other authors (Karihaloo et al.) [5,6]. These coefficients depend on the depth of the notch and geometry of the specimens.

Experimental mean values of the coefficients can be derived from the cross point of all curves with the corresponding one to $\alpha = 0.033$. The mean values of the coefficients so obtained are $G_{fm} = 47.78\text{N/m}$ and $c_{fm} = 32.27\text{mm}$.

Using these experimental mean values into equation (1), it can be obtained the nominal stresses σ_N for other relative notched depths. In figure 5, $\sigma_N - \alpha$ curves for the experimental results and by size effect law with the coefficients obtained are showed.

Conclusions

1. A method to determine the G_f and c_f coefficients of the size effect law of Bazant, from results on concrete specimens of same size and different relative notch depths is presented.
2. To obtain the G_f and c_f coefficients with the proposed method is necessary to obtain the ultimate load of relative notch depths lower than 0.1.
3. Mean values of the coefficients of the size effect law are obtained with numerical and experimental results. With these values the determination of the ultimate loads for other relative notch depths can be obtained.
4. Variations in the values of the G_f and c_f coefficients relative to the notch depth are showed. That concludes these coefficients should not be considered properties of the material.
5. The obtained results are not directly applicable to obtain the nominal stresses corresponding to specimens in different sizes, as derived from preceding conclusions.

References

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