

## Fluid-Structure Interaction in the 3D Circle of Willis

Z. Chen<sup>1</sup>, Q. Yuan<sup>1</sup> and G. Y. Zhu<sup>1</sup>, Y. Zhao<sup>2</sup>

### Summary

The objective of this research was to provide a better insight into the blood flow dynamics in the Circle of Willis (CoW). A three-dimensional geometry model is developed by the magnetic resonance data and computer-aided design software. A numerical method is proposed to solve the fluid-structure interaction (FSI) in the CoW. The CoW was simulated under both steady-state and unsteady flow conditions, and the distribution of the blood flow rates and deformation of the arteries were thus obtained. The large deformation of the structure altered the flow field significantly while the fluid pressure affected the deformation of the structure strongly. The greatest displacement was occurred at Posterior Communicating Arteries (PCoAs). A methodology of cerebral hemodynamic modeling is proposed as a potential diagnostic tool for the future clinic application.

### Introduction

The Circle of Willis (CoW, Fig.1) is a ring-like arterial structure located at the base of the brain and responds for the distribution of oxygenated blood. It is composed of a single anterior communicating artery, paired anterior cerebral arteries, internal carotid, posterior communicating and posterior cerebral arteries.

There has been a significant body of research performed on blood flow in the CoW [1-2], treating the cerebral vasculature as a 1D structure and assuming the blood as Poiseuille fluid. However, this approach cannot describe the effects of the complex arterial geometry, in particular the effects of blood vessel junctions. Subsequent 2D models of the CoW [3-4] have improved the definition of the geometry; for obtaining more realistic hemodynamic results, 3D models must be considered. Studies have been performed on 3D models of the CoW [5-6] generated from magnetic resonance imaging (MRI) data; however, those 3D models neglected the effects of the fluid-structure interaction (FSI) between the vessel wall and blood. In bioengineering applications, problems of flow interacting with elastic solid are very common. Therefore, it is very important to investigate the fluid-structure interaction in 3D models of the CoW.

In this study, a 3D FSI model of the CoW was simulated under both steady and unsteady flow to investigate the blood flow rates and the deformation of the arteries, etc.

---

<sup>1</sup>School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an 710049, China

<sup>2</sup>School of Mechanical & Aerospace Engineering, Nanyang Technological University, 639798, Singapore

### Numerical Modeling

Based on the data from clinic, a simplified 3D geometry model of CoW (Fig. 1) was generated by SOLIDWORKS. All the geometry data were come from Hillen and Hoogstraten's study[2], and the space structure was based on the study of S. Moore[6]. The detailed geometric parameters are shown in Table 1.

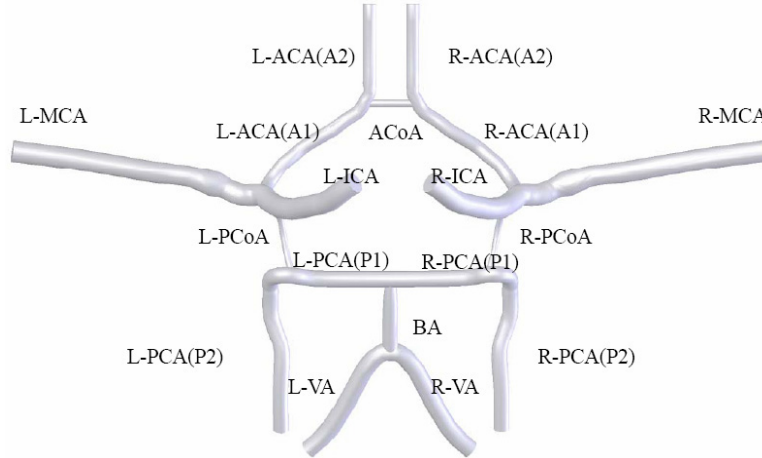


Figure 1: The 3D geometry model of CoW

Table 1: Arterial anatomical dimensions of the CoW

| ID Code | Artery                             | Diameter /mm | Length /mm |
|---------|------------------------------------|--------------|------------|
| ACA-A1  | Proximal Anterior Cerebral Artery  | 2.5          | 20         |
| ACA-A2  | Distal Anterior Cerebral Artery    | 2.5          | 50         |
| ACoA    | Anterior Communicating Artery      | 1.5          | 5          |
| BA      | Basilar Artery                     | 3.0          | 30         |
| ICA     | Internal Carotid Artery            | 4.0          | 250        |
| PCA-P1  | Proximal Posterior Cerebral Artery | 3.0          | 20         |
| PCA-P2  | Distal Posterior Cerebral Artery   | 3.0          | 70         |
| PCoA    | Posterior Communicating Artery     | 1.2          | 20         |
| VA      | Vertebral Artery                   | 3.0          | 300        |

The blood flow was considered as incompressible, viscosity, Newtonian and laminar flow. The artery wall was assumed to be hyperelastic, isotropic, incompressible and homogeneous. On all walls, no-slip boundary condition was specified. The incompressible Navier-Stokes equations with arbitrary Lagrangian-Eulerian (ALE) formulation were used as the governing equations which are suitable for problems with FSI. The governing equations are following:

$$\frac{\partial \mu}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla \cdot V = 0 \quad (1)$$

$$\rho \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = -\nabla p + \mu \nabla^2 V \quad (2)$$

$$\rho_w \frac{\partial^2 \xi}{\partial t^2} = \frac{p}{H} - \frac{Eh}{(1-\sigma^2)H} \left( \frac{\sigma}{R} \frac{\partial \zeta}{\partial x} + \frac{\xi}{R^2} \right) \quad (3)$$

$$\rho_w \frac{\partial^2 \zeta}{\partial t^2} = \frac{Eh}{(1-\sigma^2)H} \left( \frac{\sigma}{R} \frac{\partial \xi}{\partial x} + \frac{\partial^2 \zeta}{\partial x^2} \right) - \frac{\eta}{H} \left( \frac{\partial u_r}{\partial r} \right) \Big|_{r=R} - \frac{K}{H} \zeta \quad (4)$$

$$\text{here: } H = h + \frac{\rho_1 r_1 h_1}{\rho_w R}; \quad K = \rho_w H \omega_0^2$$

Where  $V$  is the velocity vector,  $\rho$  is the density,  $P$  is the pressure,  $\mu$  is the fluid viscosity.  $H$  signify for the equivalent thickness of the vascular wall,  $\rho_w$ ,  $R$  and  $h$  are respectively the density, balance radius and thickness of the vascular wall.  $E$  and  $\sigma$  are the Young's modulus and Poisson ratio of the vascular wall.  $\omega_0$  is the natural cyclic frequency of vascular which take the elastic effect of connective tissue into consideration.  $\zeta$  and  $\xi$  are the displacements in radial and Circumferential of vascular wall.

### Results

The commercial software ADINA was used to solve the full coupled FSI model. The density of blood was set to  $1056 \text{kg/m}^3$  and the viscosity of blood was set to  $0.0035 \text{Pa} \cdot \text{s}$ . The boundary conditions chosen for the present study were systemic and venous pressures at the inlets and outlets of the afferent and efferent arteries respectively. The afferent pressure at ICAs and VAs was set at  $12265.6 \text{Pa}$  ( $92 \text{mmHg}$ ); the efferent pressure at other vessels was set at  $11500 \text{Pa}$  ( $86 \text{mmHg}$ ).

Both in FSI model and rigid wall model of the CoW, the ICAs supply almost 4 times as much blood to the VAs (Fig. 2). There is almost no flow through the ACoA and only a relatively small equal amount of blood flows through the PCoAs due to the symmetry of the CoW. The direction of blood flow through the PCoAs is from anterior to posterior. The mass flow rates have much more difference between FSI model and rigid wall model of the CoW. The mass flow rates in FSI model are lower than that in rigid wall model. In FSI model, flow rate in every single segments are 6%-15% lower than that in rigid model. When considering FSI effects, the deformation of the CoW are shown in fig. 4. The biggest deformation occurred at PCoA, about 2% of the artery wall thickness.

The pulsatile waveform is used as inlet pressure. To simulate the circulation in the CoW more accurately and observe the time profile of outlet hemodynamics, computation of blood flow in the CoW under unsteady conditions were carried out. Periodic pulse pressure was set as the boundary condition.

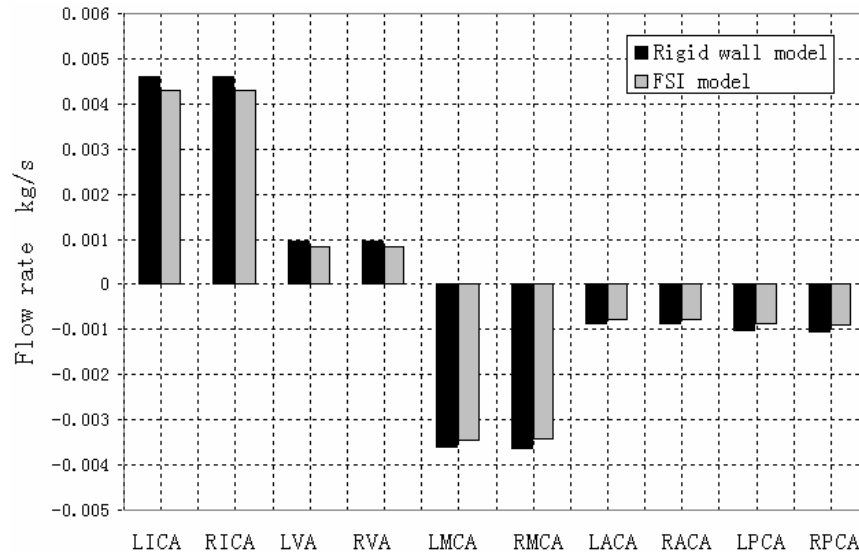


Figure 2: The comparison of blood flow rates between FSI model and rigid wall model

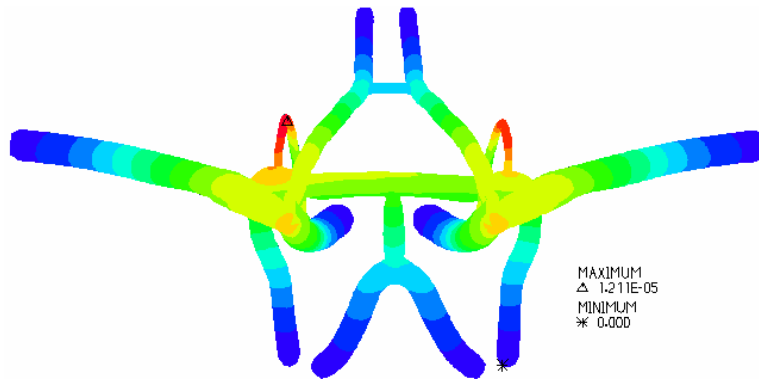
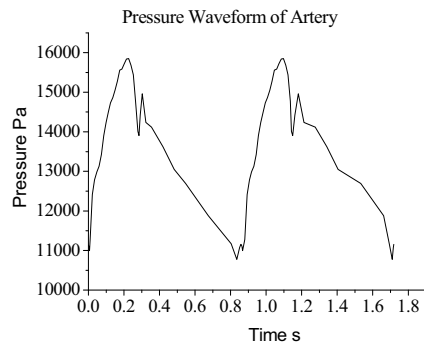
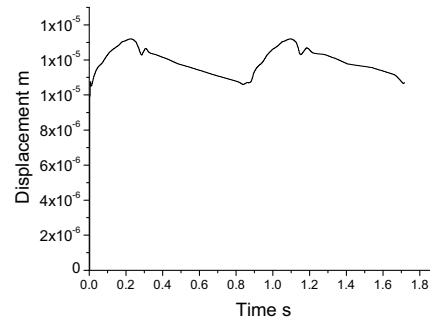


Figure 3: The displacement of the CoW for FSI model

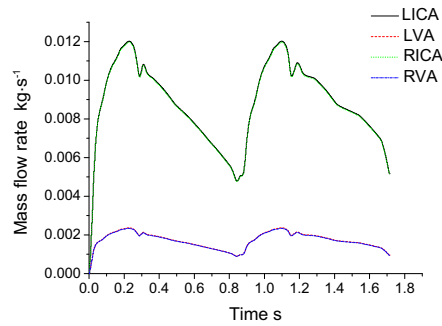
In the unsteady FSI model of the CoW, the afferent blood supply is essentially split symmetrically between the left and right afferent arteries. But in efferent arteries, the blood flow rates are not of absolute symmetry (Fig.4(d)). This phenomenon may be caused by the arteries walls displacement in the FSI model. As shown in fig 4(d), in any given point, the mass flow rates in MCA, PCA, ACA which obtained from our simulation are quite close to that from clinic measurement[7]. This result proved the accuracy of our computer model, based on this, we can get more information about the CoW and make our study closer to clinical applications. Fig. 4(b) shows the deformation of PCoAs in two cardiac cycles. The quantitative change of deformation is a periodic function of time, but the absolute deformation value is



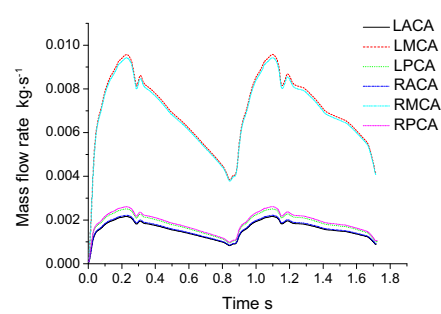
(a) Pulse wave-time profile of inlet pressure



(b) Maximum displacement scale-time profile of PCoA



(c) Flow rate-time profile of afferent blood



(d) Flow rate-time profile of efferent blood

Figure 4: Results of unsteady CFD simulation

very small.

### Conclusion

1. The FSI effect between vessel wall and blood has a significant influence on the blood flow in the CoW. The flow rates are lower in rigid wall model than that in FSI model. The greatest difference is about 15%.
2. When considering FSI effect under steady conditions, the greatest displacement occurred at PCoA.
3. Study under unsteady conditions can simulate the circulation in the CoW more accurately. A flow rate-time profile in all segments can be drawn from this study.
4. In FSI model, the inlet flow rates are no longer symmetrical as in the rigid wall model. This may be due to the FSI effect.

### References

1. Moorhead, K. T., Doran, C. V., Chase, J. G. et al. (2004): "Lumped parameter and feedback control models of the auto-regulatory response in the circle of Willis", *Computational Methods in Biomechanics and Biomedical Engineering*, Vol. 7, pp. 121-130.
2. Hillen, B. and Hoogstraten, H. W. (1986): "A Mathematic Model of the Flow in the Circle of Willis", *Journal of Biomechanics*, Vol. 19, pp. 187-194.
3. Ferrandez, A., David, T., Brown, M. D. (2002): "Numerical models of auto-regulation and blood flow in the cerebral circulation", *Computational Methods in Biomechanics and Biomedical Engineering*, Vol. 5, pp. 7-19.
4. David, T., Brown, M., Ferrandez, A. (2003): "Auto-regulation and blood flow in the cerebral circulation", *International Journal for Numerical Methods in Fluids*, Vol. 43, pp. 701-713.
5. Cebal, J. R., Castro, M. A., Soto, O. et al. (2003): "Blood-flow models of the circle of Willis from magnetic resonance data", *Journal of Engineering Mathematics*, Vol. 47, pp. 369-386.
6. Moore, S., David, T., Chase, J. G. et al. (2006): "3D models of blood flow in the cerebral vasculature", *Journal of Biomechanics*, Vol. 39, pp. 1454-1463.
7. Zhang, Z. S. (2004): *Correlative Anatomy and Clinic of the Human Cerebral Blood Vessels*, Scientific and Technical Documents Publishing House Press.