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FE-Simulation of Nonlinear Vibration and Stability Control of Smart Structures

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Summary

In the present work the significance of considering geometrical nonlinearity in the application of piezoelectric materials in order to improve the dynamic and stability characteristics of thin structures is shown. For the numerical investigations a finite shell element has been used which employs strain displacement relations based on the first-order shear deformation moderate rotation theory. An ANS formulation has been used to overcome membrane and shear locking. Two numerical examples are shown. The first example deals with increasing the critical buckling load by incorporating piezoelectric layers into the structure. In the second example the nonlinear vibrations of a cantilevered beam are damped with a simple velocity proportional control.

Introduction

In modern structural engineering weight reduction is a top priority. The amount of required raw materials is reduced, the costs are lowered and weight reduction may be beneficial in aesthetic point of view. A negative side effect is the stability problem which occurs in both static as well as dynamic manner. One promising solution seems to be integrating piezoelectric materials into these structures.

Governing Equations

The mechanical equilibrium equations, the equation for charge for a body B_0 can be written in the weak formulation by introducing the potential functional

$$\Pi(u,\varphi) = \int_{B_0} \left[H - b\cdot u + \frac{1}{2}\dot{u}\cdot\dot{u}\rho \right] dV - \int_{\partial_t B_0} t\cdot u\, dA + \int_{\partial_q B_0} q\,\varphi\, dA \qquad (1)$$

where H is the electric enthalpy density $H = \sigma : \varepsilon - D\cdot E$, σ and ε denote the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor, respectively, $\boldsymbol{D}$ and $\boldsymbol{E}$ denote the dielectric displacement vector and the electric field vector, respectively, $\boldsymbol{b}$ is the vector of the body force densities, $\boldsymbol{u}$ is the vector of displacements, which in terms of a first order shear deformation theory can be expressed as $u = \overset{0}{v} + \Theta^3 \overset{1}{v}$. Here, the translation of the reference surface is denoted by $\overset{0}{v}$, the rotation of the reference surface normal by $\overset{1}{v}$ and Θ^3 denotes the thickness coordinate. Further ρ denotes the mass density, t the prescribed externally applied traction vector on the surface $\partial_t B_0$, q is the prescribed externally applied charge density on the surface $\partial_q B_0$, φ is the electric potential and finally $\dot{()}$ denotes the

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derivative with respect to time. The electromechanical equilibrium is found when $\delta\Pi = 0$.

Assuming small strains, small rotations around the reference surface normal and moderate rotations of the reference surface normal, the Green-Lagrange strains can be expressed as [1]

$$\varepsilon_{\alpha\beta} = \varepsilon^0_{\alpha\beta} + \Theta^3\varepsilon^1_{\alpha\beta} + \left(\Theta^3\right)^2\varepsilon^2_{\alpha\beta}, \quad \varepsilon_{\alpha 3} = \varepsilon^0_{\alpha 3} + \Theta^3\varepsilon^1_{\alpha 3}, \quad \varepsilon_{33} = 0, \tag{2}$$

where the individual components, which are geometrically nonlinear, are further described in [1].

The irrotational Lagrangean electric field is described by the negative gradient of the electric potential along the undeformed shell parameters

$$E = -\mathrm{GRAD}\varphi. \tag{3}$$

For thin piezoelectric layers it is sufficient to assume a linear function for the electric potential. Not only are the degrees of freedom reduced this way, moreover the kinematic assumption of equal potential on the electrode pairs can be more easily applied without introducing additional Lagrangean multipliers. Due to this assumption the electric field only exists in transverse direction and it can be expressed as

$$E_3 = -\frac{\varphi}{h_p}, \tag{4}$$

where h_p is the thickness between an electrode pair. The electric potential φ therefore becomes a degree of freedom per electrode pair denoting the difference of the electric potential between the electrodes.

The constitutive behaviour of the considered electromechanical structures is described by the converse and direct piezoelectric effect

$$\begin{aligned} \sigma^{ij} &= c^{ijkl}\varepsilon_{kl} - e^{ijk}E_k \\ D^i &= e^{ikl}\varepsilon_{kl} + \delta^{ik}E_k, \end{aligned} \tag{5}$$

where c denotes the elasticity tensor at constant electric field, e the piezoelectric coupling tensor and δ the dielectric permittivity tensor at constant strain. The components of the constitutive tensors can be found e.g. in [2].

Finite Element Formulation

Due to geometrical nonlinearity in the present formulation the variation of the potential in the unknown configuration is approximated by the first-order Taylor series, which leads to the construction of the tangential stiffness matrix. After approximating the fields by form functions, assembly of the elemental matrices and vectors and neglecting structural damping, the following set of equations need to be solved

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{u\varphi} \\ K_{u\varphi}^T & K_{\varphi\varphi} \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta \varphi \end{Bmatrix} = \begin{Bmatrix} F_e \\ Q_e \end{Bmatrix} - \begin{Bmatrix} F_i \\ Q_i \end{Bmatrix}, \tag{6}$$

where $\boldsymbol{M}$ denotes the lumped mass matrix, $\boldsymbol{F}_e$ and $\boldsymbol{Q}_e$ are the externally applied forces and charges respectively, and $\boldsymbol{F}_i$ and $\boldsymbol{Q}_i$ denote the in-balance forces and charges respectively.

A nine-node element has been deployed using the ANS-formulation of Park and Stanley [3], which was previously extensively tested by Kreja et al [4].

Numerical Examples

The first numerical example is taken from Sabir and Lock [5]. It deals with a symmetrically hinged cylindrical roof as depicted in Fig. 1. The roof is loaded with a uniform pressure field. In order to increase the critical buckling load the structure is covered with piezoelectric patches on the bottom and the top surface made of PZT G1195. It is attempted to increase the critical load by applying electric potentials to the piezoelectric patches. One quarter of the roof is discretised by a [4×4] mesh of 9 noded ANS elements. Each element constitutes one electrode pair on the bottom and one on the top. In case the electric potential and the direction of the macroscopic polarisation of the piezoelectric patches is constant throughout the structure, the observed effect is minimal. In order to increase the intended effect part of the patches were polarized in opposite direction. The optimal configuration was obtained as shown in Fig. 1. The critical pressure (see Fig. 2), obtained at the bifurcation point, for the PZT covered roof without actuation is $5.727 \cdot 10^4$ N/m^2. If a constant voltage of 500V is applied to each electrode pair, the critical pressure increases to $5.952 \cdot 10^4$ N/m^2, which is an improvement of 3.9 %.

The second example was originally experimentally investigated by Bailey and Hubbard [6] within the range of geometrically nonlinear vibrations. Afterwards this example was numerically investigated by Lammering [7] with a slightly modified configuration. In the present work the configuration as described by Lammering [7] is taken with consideration of the tip inertia, even though this does not result into considerable differences. The beam is discretised by a [10×1] mesh of 9 noded ANS elements. One electrode pair is applied to each piezoelectric layer.

In both the linear as well as the nonlinear case a transverse tip displacement of 2cm is enforced after which the beam is released to conduct velocity proportional control in order to damp the vibrations. The tip displacement of 2cm is the one originally used by Bailey and Hubbard [6]. On both cases tip velocity proportional control is applied with a gain of $g = -62.5\,$Vs/m and $g = -125\,$Vs/m resulting into a maximum applied voltage of approximately 50V and 100V per layer, respec-

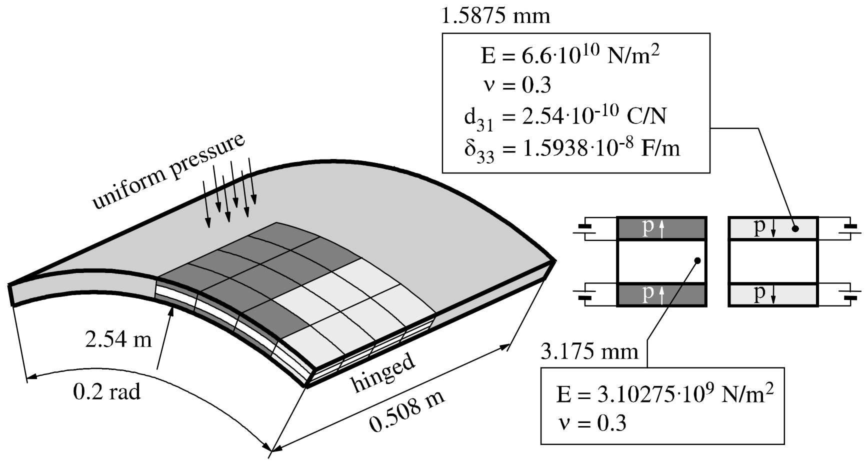

Figure 1: Hinged cylindrical roof covered with piezoelectric material

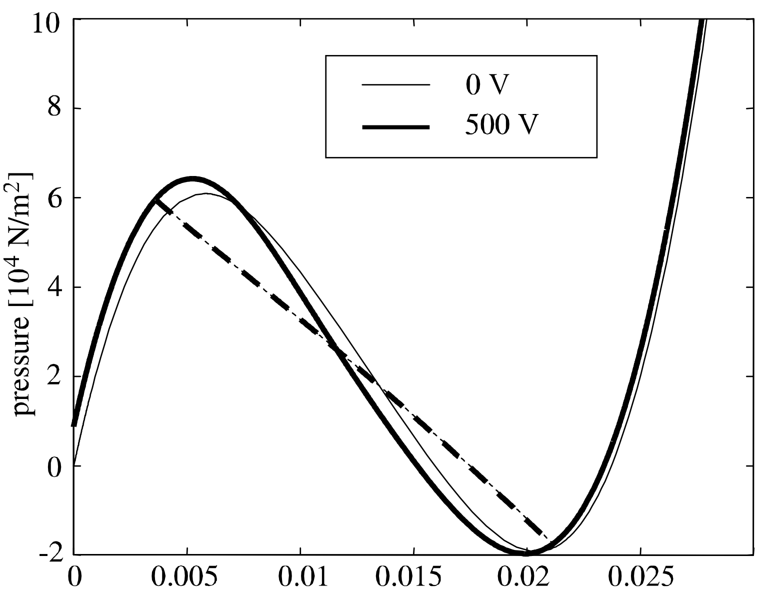

Figure 2: Symmetric and asymmetric buckling curves of the hinged roof

tively. This corresponds to the experiment of Bailey and Hubbard [6] who applied maximum voltages of 100V and 200V, respectively, by using only one piezoelectric layer. A significant difference between the linear and the nonlinear results is shown in Fig. 4.

Summary

A geometrically nonlinear finite shell element for the investigation of thin piezolaminated structures is presented. Two numerical examples are discussed in

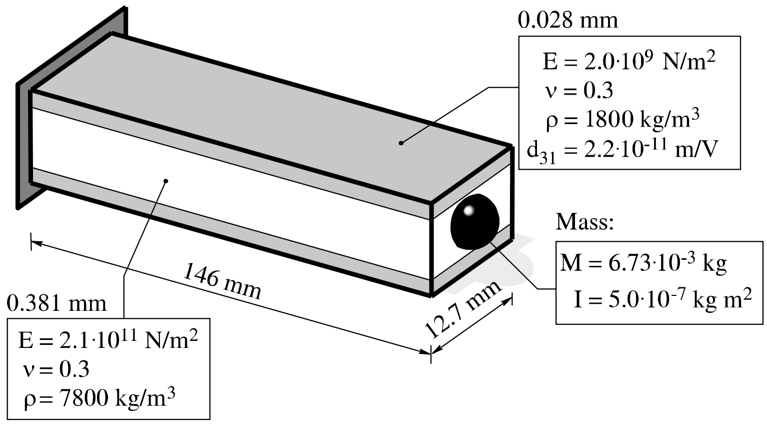

Figure 3: Piezolaminated cantilevered beam with a tip mass

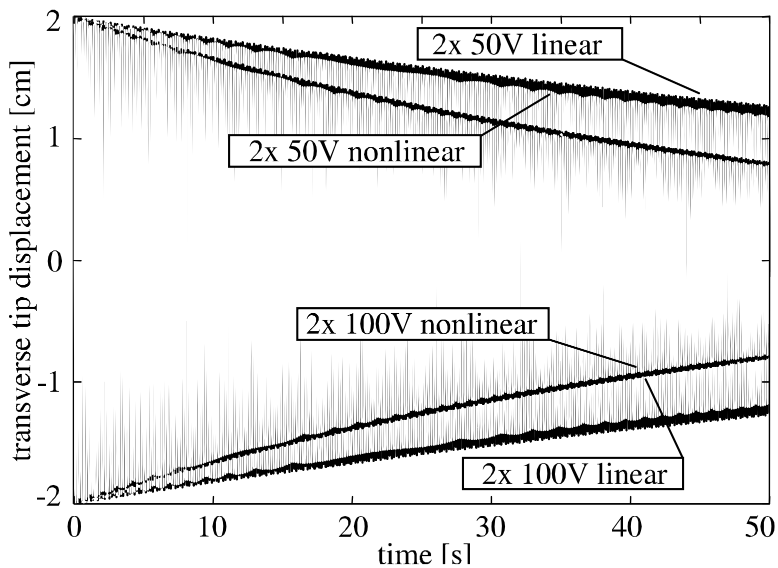

Figure 4: Velocity proportional control of the cantilevered beam

which the geometrical nonlinearity plays a profound role. One example deals with the static stability control of a hinged cylindrical roof and the other example deals with the vibration control of geometrically nonlinear vibrations of a cantilevered beam.

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