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A Boundary Element Formulation for Boundary Only Analysis of Thin Shallow Shells

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Summary

This paper presents a boundary element formulation for the analysis of thin shallow shells where only the boundary is discretized. Classical plate bending and plane elasticity formulations are coupled and effects of curvature are treated as body forces. The body forces are written as a sum of approximation functions multiplied by coefficients. Domain integrals that arises in the formulation are transformed into boundary integrals by the radial integration method. Two different approximation functions are compared. Results are obtained for a spherical shallow shell and the accuracy of each approximation function is assessed by comparison with results from literature.

Introduction

Considerable progress has been made in the past few years in applying the boundary element method (BEM) to the analysis of shell structures. One of the first work was due to Newton and Tottenham [5] who presented an application of the BEM to shallow shells by decomposition of the fourth order governing equation into a second order equation. Since this work, many different approaches arise in the literature as can be seen in the review by Beskos [1]. In some formulations there is no domain integration, as Lu and Huang [4] who developed a direct BEM for shallow shells involving shear deformation. However, the direct BEM involves complicated fundamental solutions. An alternative to the direct BEM is the coupling of plate bending and plane elasticity formulations, as proposed by Zhang and Atluri [8] who derived a formulation for static and dynamic analysis of classical shallow shells. The domain integrals were computed by domain discretization into cells. Dirgantara and Aliabadi [2] extended this approach to the analysis of shear deformable shallow shells. However, the discretization of the domain into cells reduces one of the main advantages of the BEM that is the boundary only discretization. Wen *et al.* [7] used the formulation proposed by Dirgantara and Aliabadi [2] and transformed their domain integrals to boundary integrals using the dual reciprocity technique.

The dual reciprocity method (DRM) and the radial integration method (RIM) are techniques used in BEM to transform domain integrals into boundary integrals. They are suitable for boundary element formulations where a complete fundamental solution is either unavailable or very complex, because in these cases one or

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more terms can remain as domain integrals in order to use a simpler fundamental solution. Thus, a large number of problems can be solved with the knowledge of a few number of fundamental solutions and additional terms as inertia or non-linear effects, can be treated as body forces and taken to the boundary. In both methods, the remaining terms are approximated through a finite series expansion involving proposed approximating functions and coefficients to be determined. This expansion is substituted in the generated domain integrals that are, subsequently, transformed into boundary integrals.

In this paper, a boundary element formulation for thin shallow shells with no domain discretization is presented. The domain integrals due to the curvature of the shells are transformed into boundary integrals using the radial integration method. Two approximation functions are used. Displacements computed using both approximation functions are in good agreement with results available in literature.

Boundary integral equations

Consider a shallow shell of an isotropic elastic material with the mid-surface being described by $z = z(x_1, x_2)$. The base-plane of the shell is defined in a domain Ω in the plane x_1, x_2 whose boundary is given by Γ.

Using the equilibrium equation of isotropic shallow shells, the reciprocity relation, and the Green theorem, Zhang and Atluri [8] derived integral equations that can be divided in terms of plane elasticity and plate bending formulations. These formulations are coupled by the domain integrals that arise in the equations. Integral equations for the plane elasticity formulation are given by:

$$c_{ij}u_j + \int_\Gamma t^*_{ik}(Q,P)u_k(P)d\Gamma(P) = \int_\Gamma u^*_{ik}(Q,P)t_k(P)d\Gamma(P) + \int_\Omega C\kappa_{kj}u_3u^*_{ik,j}(Q,P)d\Omega, \quad (1)$$

where $i,j,k = 1,2$; u_k is the displacement in directions x_1 and x_2, $t_i = N_{ij}n_j$, N_{ij} are membrane forces applied in shell; u_3 stands for the displacement in the normal direction of the shell surface; κ depends on the curvature radii R_{ij} of the shallow shell; k_{ij} are the inverse of curvarture radii. P is the field point; Q is the source point; and asterisks denote fundamental solutions. The constant c_{ij} is introduced in order to take into account the possibility that the point Q can be placed in the domain, on the boundary, or outside the domain.

The integral equation for the plate bending formulation is given by:

$$Ku_3(Q) + \int_\Gamma \left[V^*_n(Q,P)w(P) - m^*_n(Q,P)\frac{\partial w(P)}{\partial n} \right] d\Gamma(P) + \sum_{i=1}^{N_c} R^*_{c_i}(Q,P)u_{3_{ci}}(P)$$

$$
\begin{aligned}
&= \sum_{i=1}^{N_c} R_{c_i}(P) u^*_{3_{ci}}(Q,P) + \int_\Omega q_3(P) u^*_3(Q,P)\, d\Omega \\
&\quad + \int_\Gamma \left[V_n(P) u^*_3(Q,P) - m_n(P) \frac{\partial u^*_3}{\partial n}(Q,P) \right] d\Gamma(P) \\
&\quad + \int_\Gamma C\kappa n_j u_i(P) u^*_3(Q,P)\, d\Gamma(P) + \int_\Omega C \frac{\kappa_{ij}}{\rho_{ij}} u^*_3(Q,P) u_3(P)\, d\Omega \\
&\quad + \int_\Omega \left[C\kappa_{ij}(P) u^*_3(Q,P) \right]_{,j} u_i(P)\, d\Omega,
\end{aligned} \tag{2}
$$

where $\frac{\partial()}{\partial n}$ is the derivative in the direction of the outward vector $\mathbf{n}$ that is normal to the boundary Γ; m_n and V_n are, respectively, the normal bending moment and the Kirchhoff equivalent shear force on the boundary Γ; R_c is the thin-plate reaction of corners; $u^*_{3_{ci}}$ is the transverse displacement of corners; q_3 is the domain force in the x_3 direction; The constant K is introduced in order to take into account the possibility that the point Q can be placed in the domain, on the boundary, or outside the domain.

As can be seen, domain integrals arise in the formulation owing to the curvature of the shell. In order to transform these integrals into boundary integrals, consider, as in the DRM, that a body force b is approximated over the domain Ω as a sum of M products between approximation functions f_m and unknown coefficients γ_m, that is:

$$
b(P) = \sum_{m=1}^{M} \gamma_m f_m. \tag{3}
$$

for approximation functions based on pure radial basis function, or

$$
b(P) = \sum_{m=1}^{M} \gamma_m f_m + ax + by + c \tag{4}
$$

with

$$
\sum_{m=1}^{M} \gamma_m x_m = \sum_{m=1}^{M} \gamma_m y_m = \sum_{m=1}^{M} \gamma_m = 0 \tag{5}
$$

for approximation functions based on radial basis function combined with augmentation functions.

Two approximation functions are used in this work. The first is the radial basis function that has been used extensively in the DRM given by:

$$
f_{m_1} = 1 + R, \tag{6}
$$

and the second is the well known thin plate spline:

$$
f_{m_3} = R^2 \log(R), \tag{7}
$$

used with the augmentation function given by equations (4) and (5). It has been shown in some works from literature that this approximation function can give excellent results for many different formulations (see Partridge [6] and Golberg *et al.* [3]).

Equation (3) and (4) can be written in a matrix form, considering all source points, as:

$$\mathbf{b} = \mathbf{F}\gamma \tag{8}$$

Thus, γ can be computed as:

$$\gamma = \mathbf{F}^{-1}\mathbf{b} \tag{9}$$

Body forces of integral equations (1) and (2) depend on the displacements. So, using equation (9) and following the procedure presented by Albuquerque *et al.* [9], domain integrals that come from these body forces can be transformed into boundary integrals. Then, by discretization of these boundary integrals, a matrix equation can be obtained. Finally, after applying boundary conditions, this matrix equation is transformed in an linear system that can be solved to find the unknowns of the shell problem.

Numerical results

In order to compare the accuracy of the different approximation functions, the method is applied to a shperical shallow shell under an internal pressure. The properties of the shell are as follows: thickness $t = 0.1$ m; radius of the base of the shell $r = 5$ m; $k_{11} = k_{22} = 1/R = 0.01$ m, $E = 210000$ MPa and $\nu = 0.3$. The internal pressure is $q_3 = 1$ MPa. The edge of the shell is clamped, i.e., the boundary conditions are $u_1 = u_2 = u_3 = \partial u_3/\partial n = 0$.

The transversal displacement is computed using the two radial basis functions, given by equations (6) and (7), with a mesh of 20 constant boundary elements and 17 internal points as shown in Figure 1.

Figure 2 shows the transverse displacements of the plate computed using the first and the second approximation functions, given by equations (6) and (7), respectively. Displacements are compared with results presented by Dirgantara and Aliabadi [2] for the the same problem. As it can be seen, there is a good agreement between all results.

Conclusions

This paper presented a boundary element formultaion for the analysis of thin shallow shells where domain integrals are transformed into boundary integrals by the radial integration method. As the radial integration method doesn't demand particular solutions, it is easier to implement than the dual reciprocity boundary

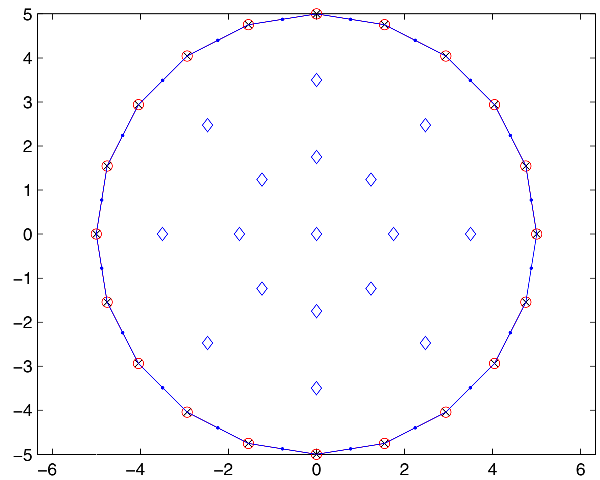

Figure 1: Mesh and internal points for the spherical shell.

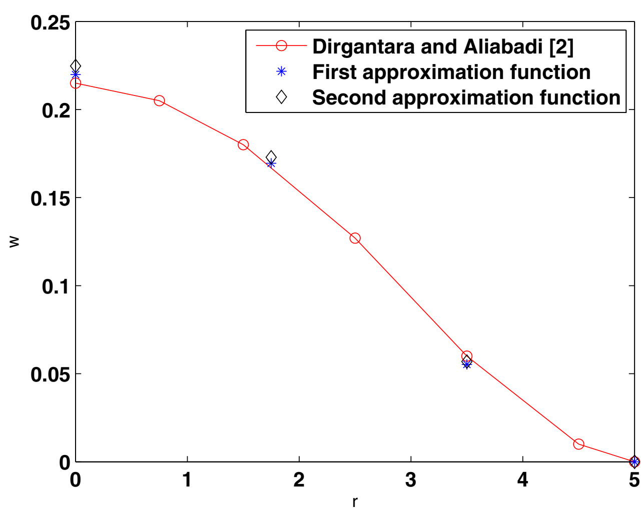

Figure 2: Transversal displacement for the spherical shell with clamped edge.

element method. Two different approximation functions are used in the radial integration method. Results obtained with both approximation functions are in good agreement with results presented in literature.

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