Nonlinear Vibration Analysis of a Fluid-Loaded Plate in Magnetic Field

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Summary

In the present study, we perform the non-linear vibration analysis of an elastic plate subjected to weak fluid loading in an inclined magnetic field. The structural nonlinearity, fluid nonlinearity, and the effects of magnetic field are all incorporated in the formulations to derive the governing equation of the plate. The method of multiple scales is adopted to determine the eigenvalues and mode shapes of the linear vibration, and then the amplitude of the nonlinear vibration response of the plate is calculated. Based on the assumptions of ordering and formulations of multiple scales, it can be concluded that the non-linear dynamics in weak fluid loading conditions is totally dominated and controlled only by the structural non-linearity and linear effect of the magnetic field. Both thick and thin plates are investigated; the contributions due to the structural nonlinearity and acoustic linear radiation damping are of the same order for a rather thick plate. For a thin plate, the structural nonlinearity completely controls the behavior of the plate, which implies that in this case the effect of fluid loading is considerably negligible. In general, it can be concluded that both the effects of magnetic field and structural nonlinearity play important roles only on the first few modes of the plate.

Introduction

Recently the nonlinear dynamics of a magneto-elastic plate under fluid loading has drawn many researchers' attention both in classical technical applications (for example, vibrations of naval structures in fluids) and in advanced applications (for example, vibrations of magneto-electrically driven micro-electromechanical systems). In addition to the practical importance, the nonlinear vibration analysis of a magneto-elastic plate in fluid is of fundamental interest in the theory of nonlinear dynamical systems. Some researchers [1-3] have already tackled the problem of nonlinear vibrations of acoustically loaded elastic structures. The purpose of their investigations was to search for the possibility of energy exchange between vibrations at different frequencies and different modes that are uncoupled in a linear theory. Dowell [1] demonstrated that these interaction effects are generated by structural nonlinearities, while an acoustic part of the problem may be considered as a linear one in weak fluid excitation. Furthermore, it was assumed that resonant frequencies of an elastic structure are not influenced by the added mass of a

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surrounding acoustic medium. Abrahams [2] and Engineer and Abrahams [3] performed the analysis of non-linear vibrations of a baffled plate and a cylindrical shell in several excitation states by using the theory of non-linear structural-acoustic coupling. Recently, Sorokin [4] has suggested a model of heavy loading of a non-linear elastic structure by a dense and weakly compressible fluid, and he has investigated several non-linear effects produced by the fluid's non-linearity. However, sometimes the structures are subjected to the effects of a magnetic field, therefore it is necessary to reformulate the nonlinear vibration analysis of a fluid plate subjected to the effects of a magnetic field. The following researchers made significant contributions in the vibration analysis of structures in a magnetic field. Shin, Wu and Chen [5] have studied the transient vibrations of a simply supported beam with axial loads and transverse magnetic fields. Recently Liu and Chang [6] performed the vibration analysis of a magneto-elastic beam with general boundary conditions subjected to axial load. Hasanyan et al. [7] used the method of multiple scales to investigate the nonlinear vibration and instability of perfectly conductive plates in an inclined magnetic field. The purpose of the present study is to adopt the nonlinear structural-acoustic coupling formulation to the case of weak fluid loading of a structure subjected to an inclined magnetic field. Weak excitation conditions are taken into account with calculated resonant frequencies of linear vibrations and the effects of the fluid, structural nonlinearities, and magnetic field are compared and discussed.

Formulation of Problem

Consider an isotropic fluid-loaded plate, located in an external stationary inclined magnetic field $B(B_{01}, 0, B_{03})$. The governing equation of motion of the plate can be expressed as follows: (Dowell [1] and Hasanyan et al. [7])

$$D_0 \frac{\partial^4 w}{\partial x^4} + D_1 \frac{\partial^2 w}{\partial t^2} + D_2 \frac{\partial^2 w}{\partial x^2} + D_3 \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} + D_4 \frac{\partial^2 w}{\partial x^2} \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx + p - f = 0 \tag{1}$$

where

$$D_0 = \frac{Eh^3}{12(1-v^2)}, \; D_1 = \rho_p^h, \; D_2 = -\left[\frac{h}{4\pi} \left(B_{03}^2 + \frac{2B_{01}^2}{kh} \right) \right], \; D_3 = -\frac{3B_{01}B_{03}}{2k\pi}, \; D_4 = -\frac{Eh}{2L}$$

Here E, ρ_p, v and h are Young's modulus, density, the Poisson ratio, and thickness of the plate, respectively, and w is the lateral displacement, f is a driving load, p is a contact acoustic pressure and k is the wave number. The third and fourth terms of equation (1) are the linear and nonlinear effects due to the inclined magnetic field. The fifth term of equation (1) represents the nonlinear stretching effect due to the immobile edges of the plate in the axial direction. The boundary conditions are imposed at $x = 0$ and L, L being the length of a plate. The purpose of the

present study is to thoroughly perform the nonlinear vibration analysis of a plate under weak fluid loading in an inclined magnetic field.

The velocity potential function Φ can be formulated by a linear wave equation as follows:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} - \frac{1}{c_f^2} \frac{\partial^2 \Phi}{\partial t^2} = 0 \qquad (2)$$

Here c_f is the velocity of sound in a fluid.

The above equation is assumed to be valid in an acoustic domain with the pressure defined by the Bernoulli relation in the following expression:

$$p = -\rho_f \left[\frac{\partial \Phi}{\partial t} + \frac{1}{2} \left(\frac{\partial \Phi}{\partial z} \right)^2 + \frac{1}{2} \left(\frac{\partial \Phi}{\partial x} \right)^2 \right]. \qquad (3)$$

In addition, it is definitely essential to present the continuity condition to adopt the full Bernoulli relation for the acoustic pressure on the structure; that is, the following equation is presented to consider the deformation of the plate [6]:

$$\frac{\partial \Phi}{\partial z} = \frac{\partial w}{\partial t} + \frac{\partial w}{\partial x} \frac{\partial \Phi}{\partial x}, z = 0. \qquad (4)$$

Equations (1-4) constitute a non-linear formulation of the problem of dynamics for a fluid-loaded plate in a magnetic field. In the present study, the method of multiple scales to the time variable is applied to investigate the non-linear fluid-structure interaction in a magnetic field.

Numerical Results

For simplicity, the boundary conditions of the plate are assumed as simply supported at both ends, and the following parameter values are adopted:

$$L = 1.0\text{m}, \quad h = 0.01\text{m}, \quad \rho_p = 0.04\text{Kg/m}, \quad \rho_f/\rho_p = 0.128,$$

$$c_f/c_p = 0.308, \quad v = 0.3, \quad R = D_2/D_0 = 8.0/\text{m}^2, \quad S = D_3/D_0 = 1.0/\text{m}^2$$

In order to produce a very large amplitude of displacement of a rather thick plate, first we choose h/L=0.1 and $f_0/E = 0.20 \times 10^{-3}$, and we denote $A = \frac{r}{L}$ as a normalized amplitude. For simplicity, we assume that the driving load is distributed in the kth mode shape, i.e., $F(x) = W_k(x)$. Without the effect of magnetic field (R=0), the first resonant eigenvalue is computed as $\lambda_1 = 2.501$, and the corresponding first natural frequency is $\omega_{01} = 29.93$ rad/sec; while if we consider the effect of magnetic field (R=8), the first resonant eigenvalue is calculated as $\lambda_1 = 1.602$, and the corresponding first natural frequency is $\omega_{01} = 12.28$ rad/sec.
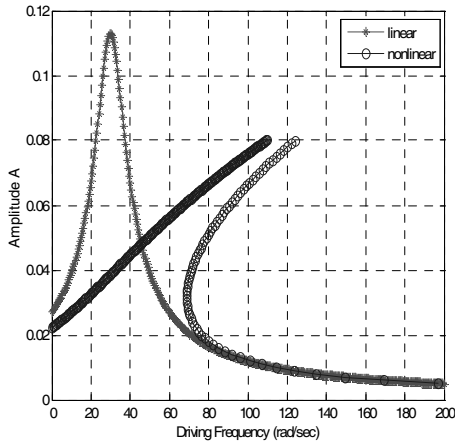
Figure 1: Frequency response near the first natural frequency of plate with h/L=0.1 and R=0.
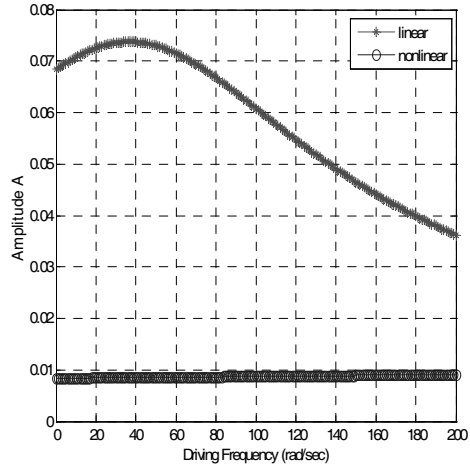

Figure 2: Frequency response near the first natural frequency of plate with h/L=0.1 and R=8.
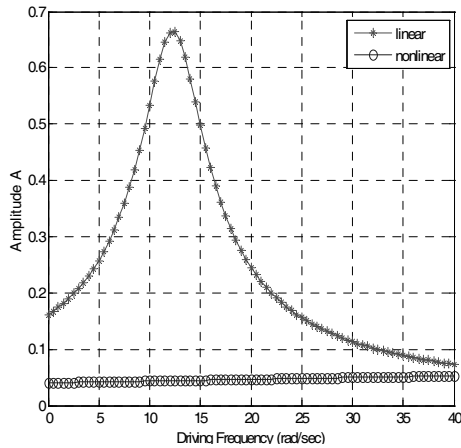

Figure 3: Frequency response near the first natural frequency of plate with h/L=0.01 and R=0.
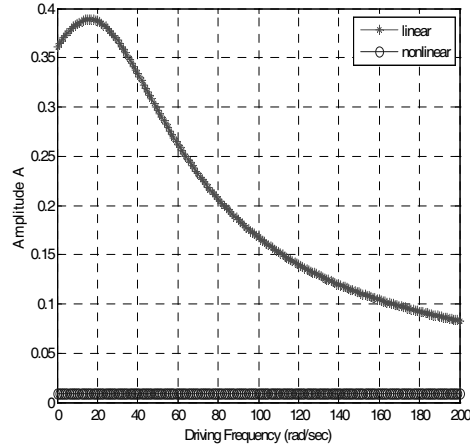

Figure 4: Frequency response near the first natural frequency of plate with h/L=0.01 and R=8.

In Figure 1, the normalized amplitude A is presented with respect to the driving frequency of the driving load without the effect of magnetic field (R=0), as seen in the figure, the amplitude based on the linear theory is larger than that of the nonlinear theory, which is fairly reasonable. In Figure 2, with the presence of the magnetic field (R=8) the amplitude from the linear theory is much larger than that

of the same case without the magnetic field as presented in Figure 1; furthermore, the amplitude of the nonlinear theory is much less than that of the linear theory. It can be inferred that the effect of magnetic field has a significant impact on the amplitude of the plate, especially when the driving frequency of the excitation load is near the first natural frequency of the plate. It can also be concluded from Figure 1 that in the case of a thick plate the contributions due to the structural nonlinearity and acoustical linear radiation damping are of the same order without the effect of magnetic field. However, as it can be seen from Figure 2, in the presence of a magnetic field the influence of structural nonlinearity is still pronounced but the resonant peak of the response has disappeared. Now we select a different set of parameters, i.e., h/L=0.01 and $f_0/E = 0.20 \times 10^{-6}$ to simulate a thin plate. In the case without the magnetic field (R=0), the first three resonant eigenvalues are computed as $\lambda_1 = 1.561$, $\lambda_2 = 4.511$, $\lambda_3 = 7.137$, and with the magnetic field (R=8) the first three resonant eigenvalues are estimated as $\lambda_1 = 1.030$, $\lambda_2 = 4.263$, $\lambda_3 = 6.971$. In Figure 3, the amplitude of the plate is plotted with respect to the driving frequency near the first natural frequency of the plate, which is computed as $\omega_{01} = 36.89$ rad/sec without the magnetic field. The structural nonlinearity completely controls the behavior of the plate, which implies that in this case the effects of fluid loading is considerably negligible as compared with the effect of structural nonlinearity. In Figure 4, the effect of magnetic field is included and the first natural frequency of the plate is calculated as $\omega_{01} = 16.05$ rad/sec. The plots show very similar trends as those in Figure 3 except that the amplitude of the linear response has increased tremendously due to the fact that the fundamental natural frequency has reduced a lot. In general, it can be concluded that the effects of magnetic field play an important role only on the first few modes, say first two modes for the case in this study. In addition, the effects of nonlinearity on the behavior of the plate is quite remarkable for the first few modes, however, it is negligible as the higher modes are concerned.

Acknowledgement

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