LINEAR STABILITY APPROACH TO PREDICT LOCK-IN AND TIME SHARING IN VORTEX-INDUCED VIBRATIONS

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ABSTRACT

In evaluating the service life of marine structures such as risers, one must assess their dynamic response to vortices excitation (vortex-induced vibrations or VIV). At lock-in, i.e. when the frequency of the shedding of vortices synchronizes with the natural frequency of the structure, the amplitude of vibration of the structure is of the order of its diameter. Sustained vibration of this amplitude causes material fatigue which leads with time to failures. Predicting the flow velocity range for which lock-in occurs is thus of importance. It is also important in cases of flexible structures to predict which mode of vibration will lock-in with the wake. The interest here is in predicting lock-in for rigid and flexible structures like tensioned beams using a linear wake oscillator model: here, lock-in is interpreted as a linear instability caused by the merging of the two natural frequencies of a dynamic system that includes two coupled oscillators, namely the wake and the structure. This instability is also referred as couple mode flutter. It is found that the linear wake oscillator model predicts lock-in range for both elastically supported rigid cylinder and for flexible structures subjected to uniform flows. In particular, the linear approach is able to predict transition from one lock-in mode to another for flexible structures (mode-switching). It is also capable of predicting when more than one mode can lock-in with the wake for a fixed velocity (time sharing).

1. INTRODUCTION

The wake behind a fixed cylinder in a uniform cross flow is unstable past a certain critical Reynolds number. This instability takes the form of periodic shedding of vortices that are convected away from the bluff body. The frequency of vortex shedding f_f follows the Strouhal law which states that the shedding frequency is proportional to the ratio of the flow velocity over the diameter of the cylinder ($f_f=S_T U/D$ where S_T is the Strouhal number). This alternating vortex shedding produces periodic forces on the structure. The later, if flexible, undergoes what is commonly called vortex-induced vibration (VIV). The amplitude of vibration of the bluff body can attain important values (typically of the order of the diameter of its cross section) for flow velocity included in the lock-in range or region. This lock-in range is defined as the velocity range where the shedding frequency deviates from the Strouhal law and seems to "lock-in" with the structure natural frequency. The high amplitudes of vibrations reached by the structure in this region are a major concern in regards of fatigue life for offshore structures such as risers. This partly explains the large amount of literature on VIV (for comprehensive recent reviews see Williamson and Govardhan, 2004; Gabbai and Benaroya, 2005).

Here, the interested is in seeing if a stability analysis of a linearized wake-oscillator model can give insights on the physics related to lock-in for a rigid cylinders supported elastically and for flexible structures like tensioned beams. For this, a linearized version of Facchinetti's wake oscillator model (Facchinetti et al., 2004a; Facchinetti et al., 2004b; Mathelin and de Langre, 2005) is used following de Langre (2006). This non-linear wake-oscillator model was recently validated for long structures in non-uniform flows against DNS computations and experimental results by Violette et al. (2007a). Comparisons showed good agreements with both the experimental results and the DNS computations.

The first section of this paper explains the methodology used in the linear stability approach. In the second section, the reader can find details on the linear model predictions for lock-in range of a rigid cylinder subjected to a uniform flow. In the third section, the capability of the linear model to predict lock-in ranges of the different modes of a flexible structure is checked. Information on how this linear wake oscillator model can predict what will be called later “time sharing” is given in Section four.

2. METHODOLOGY FOR THE LINEAR STABILITY APPROACH

In a recent paper, de Langre (2006) proposed a linear version of the Facchinetti et al. (2004a) VIV prediction model for elastically supported rigid cylinders. The original non linear coupled cylinder/wake dynamic system is formulated in a dimensionless form as

$$\begin{aligned}\frac{\partial^2 y}{\partial t^2}+\left(2\xi+\frac{\gamma}{\mu}(S_T Ur)\right)\frac{\partial y}{\partial t}+y=M\left(S_T Ur\right)^2 q\\ \frac{\partial^2 q}{\partial t^2}+\varepsilon\left(S_T Ur\right)(q^2-1)\frac{\partial q}{\partial t}+\left(S_T Ur\right)^2 q=A\frac{\partial^2 y}{\partial t^2}\end{aligned}\quad, (1)$$

where y is the dimensionless cylinder displacement in the cross flow direction and the wake variable q is the fluctuating lift coefficient $q = 2C_L(t)/C_{L0}$ felt by the later. The term C_{L0} is the fluctuating lift coefficient for a fixed cylinder. The dimensional time and length are respectively $T = t/\Omega_S$ and $Y = yD$. The parameter Ω_S is the natural pulsation of the cylinder in stagnant water and D is its diameter. In (1) $Ur=2\pi U/(\Omega_S D)$ is the reduced velocity, $\mu=(m_{cyl}+m_{fluid})/\rho D^2$ is the mass ratio (m_{cyl} being the linear mass of the cylinder, m_{fluid} the linear mass of the displaced fluid and ρ the fluid density) and $\gamma=C_D/(4\pi S_T)$ is the stall parameter (C_D being the mean sectional drag coefficient). The coupling parameter $M=C_{L0}/(16\pi^2 St^2\mu)$ and the choice of $A=12$ and $\varepsilon=0.3$ are described in details in Facchinetti et al. (2004a). If the q^2 in Eq. (1) is neglected the equation becomes linear; a further simplification is to remove all linear damping terms in the system of equations (1). By modal analysis of this simplified coupled linear system, one can compute the natural frequency of the system and the growth rate as a function of the reduced velocity Ur. This is showed on Figure 1.

It can be seen that for low Ur, the system possess two natural frequencies, one for the solid mode and one for the wake mode. These two frequencies coincide for a certain range of reduced velocities before diverging again at a high Ur. In the range of coinciding frequencies, there is a positive growth rate which means that the system is unstable. This instability is referred to as coupled-mode flutter instability. Also plotted on Figure 1 (left) is the frequency prediction of the non linear system (obtained by Fourier analysis of the cylinder displacement with time calculated by numerical time integration of Eq. 1). One can observe that the lock-in range and the linear instability range coincide very well. Lock-in can be interpreted as a coupled-mode flutter instability between two coupled linear oscillators (see also Tamura and Matsui, 1980).

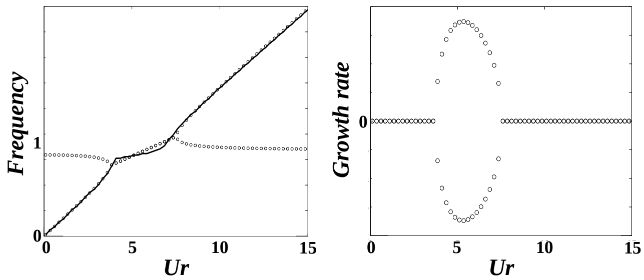


Figure 1: *Linear prediction for oscillations frequency (left figure) and growth rate (right figure) as a function of the reduced velocity (dots). Also shown on the figure is the oscillations frequency computed with the non-linear model as a function of Ur (solid line).*

3. LOCK-IN RANGE EVOLUTION WITH THE SCRUTON NUMBER.

Chen (1987) reports in his book experimental results on the lock-in range evolution with the structural parameters of an elastically supported rigid cylinder in uniform flow (namely its damping ξ linear mass m_{cyl}). This is shown in Figure 2a (lock-in range is referred as synchronization range in his book). It can be seen on this figure that beyond a certain Scruton number (around 32), there is no lock-in observed. The Scruton number is defined as

$$\delta=\frac{2\pi\xi m_{cyl}}{\rho D^2}\,. \qquad (2)$$

For the linear wake oscillator model, the growth rate evolution with the reduced velocity is plotted for the Scruton numbers of interest. The growth rate is calculated as in the previous section, i.e. without the damping terms. On the same figure, the damping felt by the cylinder expressed as

$$\zeta(Ur) = 2\xi + \frac{\gamma}{\mu}(StUr)\,, \qquad (3)$$

is also plotted. The lock-in region is defined as the range of reduced velocities where the growth rate is above the damping value (shown on Figure 3). The limits of the lock-in region as a function of the Scruton number predicted by the linear wake oscillator model appear on Figure 2b for St=0.18, C_{L0}=0.37, C_D=2 and ξ=0.002 (Scruton number is varied by changing the linear mass of the cylinder m_{cyl}).

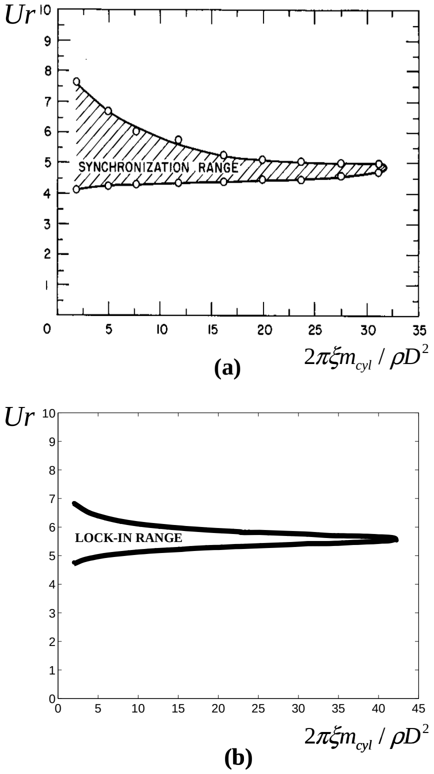


Figure 2: *Evolution of the lock-in range for an elastically supported cylinder in unform flow with the Scruton number: (a)Chen (1987), (b) linear wake oscillator model prediction.*

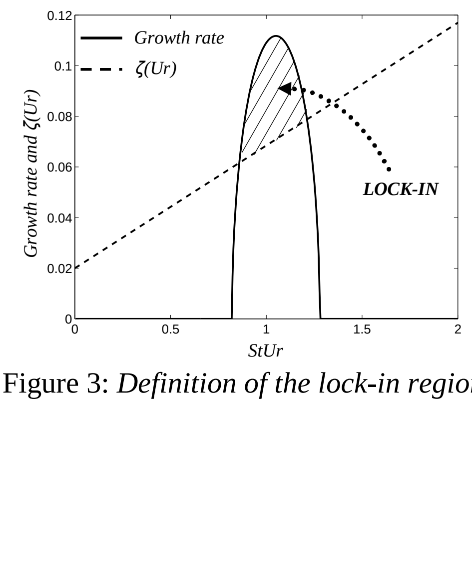


Figure 3: *Definition of the lock-in region*

By comparing Fig. 2a and Fig. 2b one can conclude that the influence of the structural parameters on the lock-in region reported by Chen (i.e. widening of the lock-in region when decreasing the Scruton number and disappearance of lock-in past a critical value) is qualitatively reproduced by the linear wake oscillator model.

4. LOCK-IN RANGES FOR FLEXIBLE STRUCTURES: "MODE SWITCHING".

The interest here is to predict the lock-in regions for the different modes of a flexible structure subjected to a uniform flow. Predicting which mode of the structure will lock-in with the wake is of practical importance since the amplitude of the bending stress that it experiences is a function of curvature, and thus of the mode number. The frequency of vibration is also very important in fatigue analysis.

For a tensioned beam, system (1) is rewritten (less the non linearity)

$$\begin{gathered}
\frac{\partial^2 y}{\partial t^2} + \left(\frac{\gamma}{\mu}(S_T Ur)\right)\frac{\partial y}{\partial t} - \left(\frac{\Lambda}{\pi}\right)^2 \left(\frac{1}{1+\frac{1}{\chi^2}}\right)\frac{\partial^2 y}{\partial z^2} + \\
+ \left(\frac{\Lambda}{\pi}\right)^4 \left(\frac{1}{1+\chi^2}\right)\frac{\partial^4 y}{\partial z^4} = M\left(S_T Ur\right)^2 q \\
\frac{\partial^2 q}{\partial t^2} - \varepsilon\left(S_T Ur\right)\frac{\partial q}{\partial t} + \left(S_T Ur\right)^2 q = A\frac{\partial^2 y}{\partial t^2}
\end{gathered} \qquad . \quad (4)$$

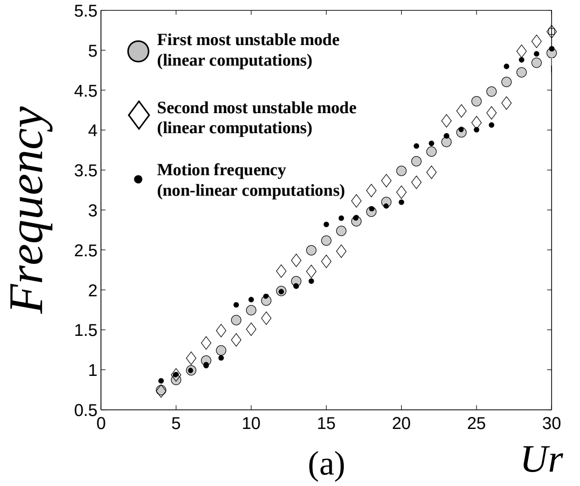
Where $\Lambda=L/D$ is the aspect ratio of the structure (L being its length) and parameter χ is a dimensionless quantity expressed as

$$\chi = \frac{T}{EI}\left(\frac{L}{\pi}\right)^2, \qquad (5)$$

where T is the tension in the structure and EI its bending stiffness. No structural damping is included in (4). It is neglected since it is considered small when compared to the fluid induced damping (this is quite standard, Violette et al., 2007a). The reduced velocity is defined with the fundamental structural frequency that is written

$$f_s = \frac{1}{2L}\sqrt{\frac{T}{m_T} + \frac{EI}{m_T}\left(\frac{\pi}{L}\right)^2}\,, \qquad (6)$$

where m_T is the sum of the linear mass of the cylinder and displaced fluid. In (4), the second and fourth derivatives with respect to the spanwise direction z are expended using finite differences method. Thus, for n computation points on the cable, we have $2n$ linear coupled oscillators (n structural and n wake oscillators). The modal analysis applied here on the system (including the linear damping term this time) is the same as in Section 2. Depending on the reduced velocity Ur studied, one finds a number of unstable modes, i.e. modes with a positive growth rate.



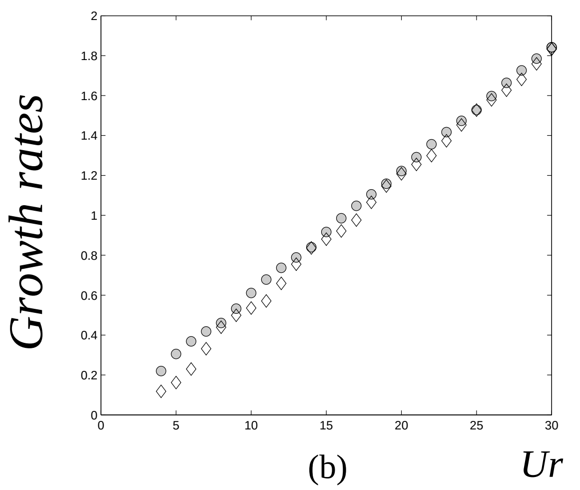


Figure 4: *(a) Frequency evolution with reduced velocity for both linear (open signs) and non-linear model (dots), (b) vibrations modes growth rates obtained by the linear model.*

For a tensioned cable (χ infinitely large) the frequencies of the two most unstable modes with respect to their growth rate as a function of Ur are plotted on Figure 4a for reduced velocity up to 30. Shown on Figure 4b are the growth rates of the two most unstable vibrations modes of the system as a function of Ur. The mass ratio used here is μ = 2 and the aspect ratio L/D is set at 10π. The phenomenological parameters used are: C_{L0}=0.37, C_D=2 and St=0.18.

It can be seen in Fig. 4a that the system (structure/wake) goes through successive regimes characterized by one dominant unstable mode (mode with the most important growth rate) when increasing the reduced velocity. For example, Mode 2 dominates for Ur between 9 and 13 and Mode 3 for Ur from 14 to 19. Inside those reduced velocity ranges, the frequency seems to be varying linearly with Ur. However, the transition from one dominant mode to another, which will be called here "mode switching", is characterized by a discontinuity in the frequency which forms the stairs-like shape of the frequency as a function of Ur. Each steps of the stairs being the lock-in range of a structural mode.

For the same range of Ur, non-linear computations were performed using the full non-linear wake oscillator model. For each non-linear computation random values of q on each point of order 10^{-3} are used as initial conditions. Oscillations frequency is derived from the evolution of displacement with time using Fourier analysis for each reduced velocity. Those oscillations frequency predictions are compared on Fig. 4a with the linear model results. The same stairs-like evolution of the cable vibration frequency with Ur is predicted, i.e. that the mode switching observed with the non linear model is also characterized by a frequency discontinuity. According to Fig 4a., there seems to be a reasonable agreement between the linear model and non linear model prediction for lock-in range of the different modes of the structure.

Thus, one can suggest that the vibration mode with the highest linear growth rate will be the one locked-in with the wake (and thus observed in the structure response). Following this logic, mode switching between two adjacent modes is observed when respective linear growth rate curves cross each other.

5. MULTIPLE RESPONSES: "TIME SHARING".

When looking at Fig. 4a, one can observed some discrepancies between the modes predicted by the linear and the non-linear models. For example, at Ur=25, the linear model predicts that Mode 6 will be dominant when the dominant mode observed in the response computed numerically with the non linear model is Mode 5. However, when performing a second non linear computation using again random values on the distributed wake oscillators q, and thus different initial values than the previous calculation, Mode 6 is observed in the permanent responses.

By looking at Fig. 4b, it can be noted that the growth rates of the two most unstable modes, namely Mode 5 and 6 in this case, have the same values. This means that both unstable modes are thus equally capable of appearing in the response. Violette et al. (2007b) verified the effect of the relative linear growth rates of two modes on their occurrences in the non linear computations. They found that similar linear growth rates leaded to almost even split of the occurrences in the non linear computations. This interesting result indicates that at a single flow velocity the wake can “select” to lock-in between more than one structural modes characterized by an individual frequency.

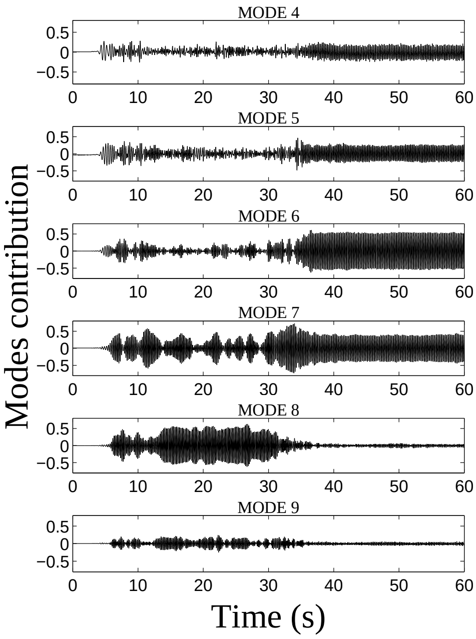


Figure 5: *Modes contribution to the response of a tension beam subjected to a uniform flow on half of it’s length (Fig.7 in Chaplin et al. 2005).*

Experimental evidences of this can be found in the literature. Chaplin et al. (2005) reported that for certain test conditions, they observed high modulations with time of the modes contribution of a tension beam, or riser, with low flexural rigidity subjected to a uniform flow on approximately half of its length (the other part being in stagnant water). In their paper (Chaplin et al., 2005) mentioned that those modulations were triggered by perturbations in the system like small vibrations of the assembly. Figure 5 shows one example of this. It shows the modal contribution of Mode 4 to Mode 9 (in the cross flow direction) in the response of the riser with time. It can be seen that Mode 8 dominates the response of the structures for time between say 15 and 30 seconds. For time higher than 30 seconds, a combination of Mode 4 to 7 is observed. When looking at the frequency of the response of the riser in the two time segments of the test, one can observe a jump in the frequency. In other words, in each time segment, the cable vibrate at one single frequency, but this frequency changes around t=30s. Similar experimental results are reported by Swithenbank (2007). In this text, this phenomenon will be referred as “time sharing” (following the terminology used by Swithenbank, 2007).

Computation is made with the linear wake-oscillator model for the configuration for which the results of Fig. 5 are obtained. The parameters used are summarized in Table 1. In this table, $(L/D)_{flow}$ is the ratio of the length of the cylinder expose to the flow over its cross section diameter. Inversely, $(L/D)_{stagnant}$, is the length of the part of the structure in stagnant water over the cylinder diameter. The linear stability analysis of system (4) gives 2 modes with identical growth rate (difference being of the order of the 0.1% between them). The dimensionless frequency predicted for both modes are 7.13 and 6.67. The frequencies in Hz are recovered by multiplying by the fundamental frequency f_S (Eq. 6). This frequency is calculated by replacing the bending stiffness EI, tension T and riser length L in Eq. 6 with the values for the test given by Chaplin et al. (2005) corresponding to the results shown on Fig. 5, which are T=1073 N, EI=29.9 Nm^2 and L=13.12m. That way, one finds 5.31 Hz and 5.68 Hz respectively for both modes.

Structural Parameters

L/D	469
$(L/D)_{flow}$	211
$(L/D)_{stagnant}$	258
μ	π
χ	625.9

Phenomenological Parameters

C_{L0}	0.37
C_D	1.2
St	0.18

Table 1: Structural and phenomenological parameters used for the linear wake oscillator model.

The evolution of the riser frequency response with time can be found by wavelets analysis of the time trace of Fig. 5. For the first time segment (t=15-30s), the frequency of the riser vibration is 5.5 Hz and is 5.1 Hz for the second segment (t=33-60s). The agreement between the linear theory and the experiment is quite good for the frequency difference (which is 0.37 Hz for the linear wake oscillator model and 0.4 Hz for the experiment).

6. CONCLUSIONS

The results presented in this paper show that:

1. the main features of the evolution with the Scruton number of the lock-in range for an elastically supported rigid cylinder in uniform flow are reproduced qualitatively by a linear wake oscillator model.

2. lock-in ranges of the different modes of a flexible structure subjected to a uniform flow obtained with a full non linear wake oscillator mode (validated against DNS and experiments) can be predicted by a simple linear wake oscillator model.

3. similar linear growth rates for different modes leads to multiple possible responses of the structure to the wake. This multiple response behavior, which is referred as “time sharing” in this text, has been experimentally observed (Chaplin et al., 2005; Swithenbank, 2007). One case where the linear wake oscillator model successfully predicts experimentally observed time sharing has been presented.

Those results on time sharing suggest that the structure and the wake can be regarded as one dynamic system that possesses a number of linearly unstable modes. Thus, insights on the probability of occurrences of those modes in the fully developed response can be obtained by a linear stability analysis.

7. ACKNOLEGMENTS

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