THIRD HARMONIC LIFT FORCES FROM PHASE VARIATION IN FORCED CROSS-FLOW AND IN-LINE CYLINDER MOTIONS

J.M. Dahl, F.S. Hover, and M.S. Triantafyllou
Massachusetts Institute of Technology, Cambridge, MA, USA

ABSTRACT

Forced cross-flow and in-line motion of a circular cylinder in a free stream is performed to observe the wake and forces exerted on the cylinder. The phase between in-line and cross-flow motions is varied while holding amplitudes and reduced velocity constant in order to show the effect of phase variation on the wake and observed lift force. A phase of 0 degrees, representative of observed free vibration motions of the cylinder, displays a '2P' vortex shedding pattern and the relative motion of the vortices in the wake, with respect to the cylinder, results in dominant third harmonic forces in lift. If the phase between motions is changed to 135 degrees, the wake shifts to a '2S' shedding pattern, and higher harmonic forces become negligible compared with forces at the fundamental frequency of vortex shedding.

1. INTRODUCTION

Long, slender ocean structures are particularly susceptible to fatigue damage caused by the vortex-induced vibration of these structures. The recent reviews of Sarpkaya (2004) and Williamson and Govardhan (2004) explain the fundamental problem of vortex-induced vibration and review a number of fundamental studies associated with the problem.

In long, string-like structures, a high number of modes makes excitation of the structure inevitable from excitation forces dues to vortex shedding. Vortex shedding behind bluff bodies in a free stream of fluid results in oscillatory forcing of the body in both the direction of lift and the direction of drag. The alternate shedding of vortices in the wake of the body leads to a forcing frequency in lift (cross-flow) near the frequency of vortex shedding, while forcing frequencies in the drag (in-line) direction occur with twice the frequency of vortex shedding, since all vortices shed downstream of the cylinder. Long, string-like structures exhibit structural characteristics such that the natural frequency in-line with the free stream may be twice the natural frequency in the cross-flow direction. With a structural condition such that the ratio of in-line to cross-flow natural frequency is two, the structure can undergo dual resonance, where large excitations occur both in-line and transverse to the incoming flow.

Recent studies by Dahl et al (2007) have shown that dual resonance of an elastically mounted, rigid cylinder in a free stream results in large amplitude, dominant third harmonic forces in lift, larger in magnitude than forces at the fundamental frequency of vortex shedding. The inclusion of large magnitude third harmonic forces in a simple fatigue life example showed that fatigue life may be reduced by orders of magnitude due to the presence of these third harmonic forces. Vandiver et al (2006) observed large amplitude third harmonic components of stress in large scale experiments with a high-mode number flexible cylinder.

Third harmonic forces were also observed in the elastically mounted, rigid cylinder experiments of Jauvtis and Williamson (2004). Flow visualization in these experiments showed the formation of a '2T' mode of vortex shedding, where triplets of vortices were shown to form over one half cycle of cylinder motion, with very large cross-flow excitation of the cylinder. Vortex triplets in the wake were shown to account for third harmonic forces based on the force impulse calculated from circulation in the wake. The magnitudes of third harmonic forces in these experiments were not large in comparison with forces at the Strouhal frequency, consistent with observations by Dahl et al (2007) for an in-line to cross-flow natural frequency ratio of one.

The present study expands on the observations from previous experiments to explain the presence of dominant third harmonic lift forces and analyze the wake associated with these forces. A rigid cylinder is forced to move in combined cross-flow and in-line motion while lift and drag forces on the body are measured. This paper presents a subset of a large set of experiments where the in-line amplitude of motion, cross-flow amplitude of motion, phase between in-line and cross-flow motion, and reduced velocity are varied to determine hydrodynamic force coefficients associated with specific motions of the cylinder. This study describes motions of the cylinder with fixed amplitudes and reduced velocity, varying only the phase between in-line and cross-flow motions. Quantitative flow visualization of the wake behind the cylinder combined with a simple potential flow representation of the wake is used to explain the presence of dominant third harmonic forces in lift.

2. EXPERIMENTAL METHOD

Experiments were performed in a small towing tank, 2.4 m long, 0.75 m wide, and 0.7 m deep. Two linear motors, mounted perpendicular to one another, were fixed to a moving carriage that could be towed along the length of the tank at constant speed. The test cylinder, circular in cross section, with a diameter of 38.1 mm, was cantilevered from the linear motors allowing motion in the in-line and cross-flow directions. A six-axis strain gage force sensor was mounted between the linear motor cantilever and the test cylinder in order to measure fluid forces exerted on the cylinder. An extensive six-axis calibration of the force sensor was performed to determine the cross-coupling effects between sensor directions. Digital particle image velocimetry was performed to visualize the wake of the cylinder under forced motions.

2.1. Test Matrix

The test cylinder is towed at a constant velocity, U, and forced with sinusoidal motions in the cross-flow (y) and in-line (x) directions. The non-dimensional cross-flow and in-line motions are defined in equations 1 and 2, where ω_f is the forced cross-flow frequency, A_y is the cross-flow amplitude of motion, A_x is the in-line amplitude of motion, D is the cylinder diameter, and θ is the phase between in-line and cross-flow motions. These equations are equivalent to the representation of two degree of freedom motions given by Jeon and Gharib (2001) and Jauvtis and Williamson (2004).

$$\frac{y}{D} = \frac{A_y}{D} sin(\omega_f t) \quad (1)$$

$$\frac{x}{D} = \frac{A_x}{D} sin(2\omega_f t + \theta) \quad (2)$$

Jeon and Gharib (2001) found that the phase between in-line and transverse motion, θ, could delay the onset of particular vortex shedding patterns in the wake of the cylinder for varied phases between -45 degrees and 45 degrees; values of $A_y/D = 0.5$ and $A_x/D = 0.1$ were use in these experiments. Recent experiments have shown that for low mass ratio cylinder motions in a dense fluid, the free excitation of an elastically mounted rigid cylinder may have much larger amplitude motions with $A_y/D \approx 1$ and $A_x/D \approx 0.35$ for the same range of phase angles (Dahl et al, 2006). In the free vibration experiments of Dahl et al (2007), figure eight motions of the cylinder with $\theta = 0$ were shown to be associated with large amplitude third harmonic forces. The motion with dominant third harmonic forces in Dahl et al (2007) had amplitudes $A_y/D = 0.91$ and $A_x/D = 0.31$, with reduced velocity $V_r = 6.4$. These motions are representative of the motion parameters when the cylinder undergoes resonance in the in-line and cross-flow directions, hence the present study holds A_y/D, A_x/D, and V_r fixed with the values above while varying the phase, θ, between -180 degrees and 180 degrees in increments of 45 degrees to show the effect of phase variation on the cylinder wake and observed hydrodynamic forces. Fig. 1 shows the forced orbital motion of the cylinder with varied θ in the reference frame fixed to the carriage.

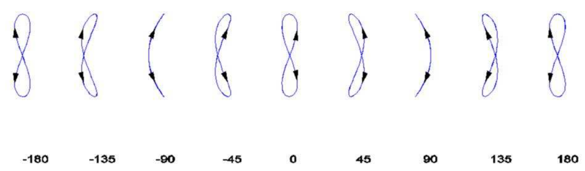

Figure 1: Cylinder orbit shape for various phases between in-line and transverse motions (θ) in degrees. Fluid flow is left to right.

3. WAKE VISUALIZATION

The phase between in-line and cross-flow motion plays an important role in determining the formation of the cylinder wake and forces exerted on the cylinder. Fig. 2 shows the wake behind the cylinder for $\theta = 0$ degrees. Two points in the cycle are illustrated, and one can see that the wake is similar to a '2P' shedding pattern, according to the nomenclature of Williamson and Roshko (1988). At this particular reduced velocity, the paired vortices of the '2P' shedding pattern is not distinct, as one vortex in the pair is very well formed, with strong, large magnitude contours of vorticity, while the other vortex in the pair is fairly weak in magnitude.

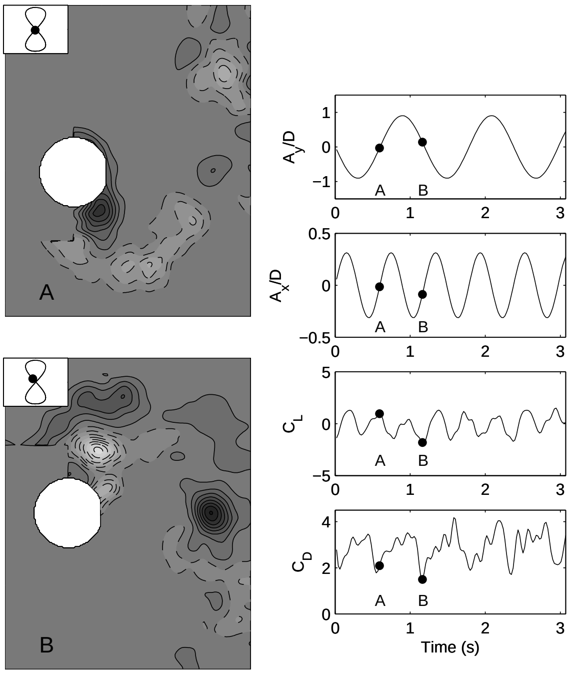

Figure 2: Cylinder wake and time traces for $\theta = 0$ degrees, $A_y/D = 0.91$, $A_x/D = 0.31$, $V_r = 6.4$. Solid lines - negative vorticity. Dotted lines - positive vorticity. Non-dimensional vorticity contours show, $\omega D/U = \pm 1, \pm 3, \pm 5, \ldots$.

As observed by Jauvtis and Williamson (2004), the '2P' shedding pattern is closely related to the '2T' pattern where additional acceleration of the cylinder causes a third vortex to form over one half cycle of cross-flow motion. The wake from Fig, 2 could resemble a '2T' mode as well, where the smaller magnitude vorticity is actually the combination of two like-signed vortices, however this cannot be distinguished from the visualization. The important feature of this wake is the large magnitude vortices from each vortex pair that move around the cylinder, in close proximity, before shedding. These vortices contribute to a large portion of the force exerted on the cylinder. The instantaneous lift coefficient, as denoted by C_L, is primarily a third harmonic force, with the dominant frequency component at three times the frequency of cross-flow motion.

At more negative values of phase between inline and transverse motion, the wake resembles a similar formation of vortices, however the phasing of vortex shedding changes. At values of θ near 90 and 135, the vortex shedding pattern changes. Fig. 3 shows the wake behind the cylinder with $\theta = 135$ degrees. In this case, the motion of the cylinder is opposite to that in Fig. 2 and there is a slight downstream curvature to the figure eight shape. This phase of motion results in the formation of a clear '2S' shedding pattern, as illustrated by two distinct opposite signed vortices that shed over one cycle of cross-flow motion. In the '2S' shedding mode, a vortex sheds on the same side of the cylinder in which it forms, and doesn't move around the cylinder as in the '2P' or '2T' shedding mode. In this case, the lift force is primarily composed of first harmonic forces with negligible higher harmonic components.

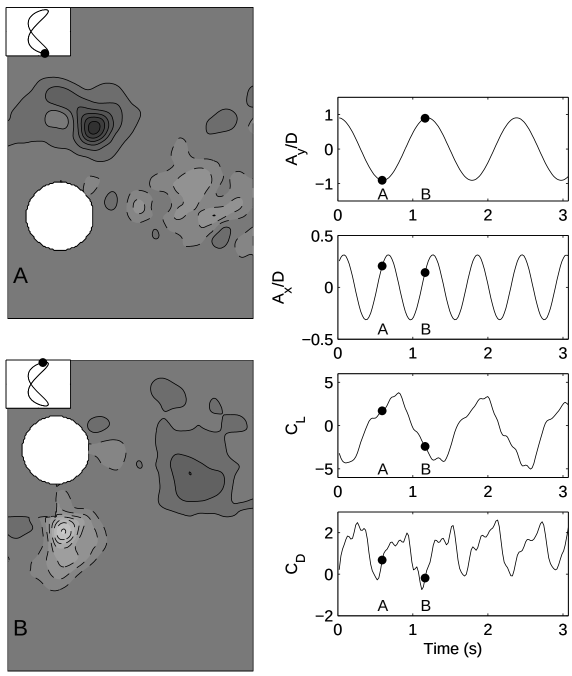

Figure 3: Cylinder wake and time traces for $\theta = 135$ degrees, $A_y/D = 0.91$, $A_x/D = 0.31$, $V_r = 6.4$. Solid lines - negative vorticity. Dotted lines - positive vorticity. Non-dimensional vorticity contours show, $\omega D/U = \pm 1, \pm 3, \pm 5, \ldots$.

At a phase angle of $\theta = 180$ degrees, a wake formation with two large magnitude, co-rotating vortices forms (Fig. 4). At this phase angle, lift forces are very large in magnitude. It is important to note that this phase of motion is completely opposite in phase to the motion observed in free vibrations (Fig. 2). This particular combination of amplitudes, reduced velocity, and phase will not likely occur for the free vibration of an elastically mounted, rigid cylinder, since this motion requires a large input of external power.

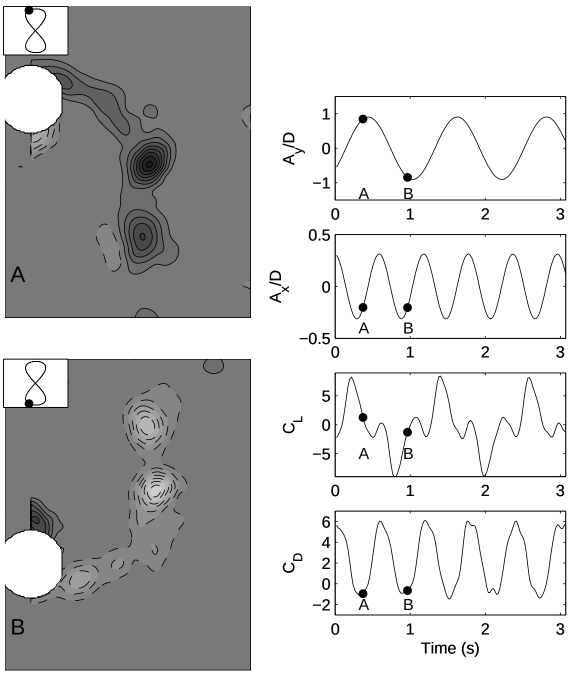

Figure 4: Cylinder wake and time traces for $\theta = 180$ degrees, $A_y/D = 0.91$, $A_x/D = 0.31$, $V_r = 6.4$. Solid lines - negative vorticity. Dotted lines - positive vorticity. Non-dimensional vorticity contours show, $\omega D/U = \pm 1, \pm 3, \pm 5, \ldots$.

4. LIFT FORCES

Jauvtis and Williamson (2004) showed that third harmonic forces in lift exist due to the presence of vortex triplets in the wake of a cylinder oscillating with combined in-line and cross-flow motion where the cross-flow motion of the cylinder was very large in magnitude ($A_y/D \approx 1.5$). Using quantitative flow visualization of the wake, the impulse force on the cylinder was calculated from the wake velocity field, illustrating the presence of third harmonic forces. We take a similar, but simplified, approach to reconstructing forces from the wake by considering a potential flow wake.

The potential for the cylinder in this flow is a combination of a time dependent free stream combined with vortices present in the wake. Using the Blausius and circle theorems from Milne-Thompson (1960), we define the lift force exerted on the cylinder as in equation 3, where ρ is the fluid density, D is the cylinder diameter, $\dot{U}$ is the free stream acceleration, Γ is the vortex circulation, u_n and v_n are the relative in-line and cross-flow velocities of vortex n with respect to the cylinder, and x_n and y_n are the relative in-line and cross-flow position of vortex n with respect to the cylinder. The reference frame is fixed to the center of the cylinder such that U accounts for in-line and cross-flow motion of the cylinder. The force is summed over n, the number of vortices present in the wake.

$$F_y = \rho\pi\frac{D^2}{4}\dot{U} + \sum_n \rho\Gamma_n\left(\frac{D}{2}\right)^2\left(\frac{u_n(x_n^2 - y_n^2) + 2v_n x_n y_n}{(x_n^2 + y_n^2)^2}\right) \tag{3}$$

The first portion of equation 3 is the ideal added mass force, while the second portion of the equation describes forces due to the presence of vortices. This representation allows one to split the hydrodynamic forces into an added mass component and a vortex component, as suggested in Jauvtis and Williamson (2004). The lift force on the cylinder is a function of free stream acceleration, vortex strength, relative motion of vortices with respect to the cylinder, and proximity of vortices to the cylinder.

In the previous section, it was shown that for a phase angle of $\theta = 0$ degrees, the lift force exerted on the body was primarily composed of a third harmonic force while for $\theta = 135$ degrees, the lift force was primarily composed of a first harmonic force. These two wakes are dominated by two large magnitude vortices that shed over one cycle of cross-flow motion, although the phasing of the shedding is different in each case. The magnitude of vorticity associated with these vortices is at least one order of magnitude larger than other vorticity in the field. Considering only these large magnitude vortices in the wake of the cylinder, we can compute the location, relative velocities, and strength of these vortices from the flow visualizations in order to calculate the derived lift force from potential flow.

Figs. 5 and 6 show the measured hydrodynamic lift coefficient exerted on the cylinder compared with the value computed from the potential flow simplification of the wake. Lift coefficient values do not match perfectly since the potential flow assumption describes a two dimensional flow while the actual experiment has three dimensional effects. Additionally, the phasing of vortex shedding in the flow visualization will not necessarily correspond with the phasing of forces measured along the length of the cylinder, since the correlation of vortex shedding may not be consistent along the span. This is particularly apparent in Fig. 6, where the trend and magnitude of the potential flow force is equivalent to the measured force, however the phasing is slightly different.

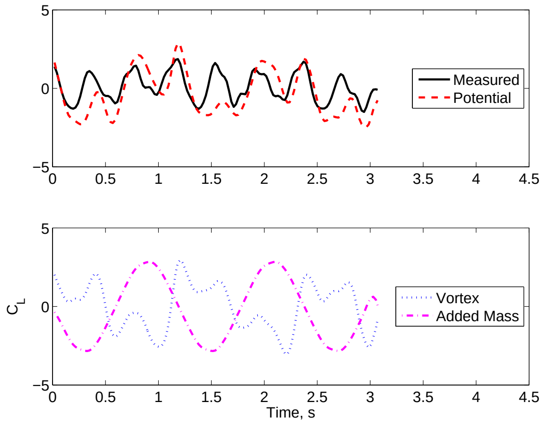

Figure 5: Potential flow representation of lift force coefficient for $\theta = 0$ degrees. Figure shows the measured force compared with calculated force from potential flow and the break-up of the potential flow force into an ideal added mass force and a vortex force.

Figs. 5 and 6 also show the calculated potential flow forces divided into an added mass force and a force due to vortex shedding. The ideal added mass force is only a function of the accelerating fluid, thus forces must occur with the same frequency as cross-flow motion. All higher harmonic components of force must exist due to vortex shedding. In Fig. 5, one can see that the vortex force has higher harmonic components in addition to a first harmonic component, while the added mass force occurs at the frequency of motion. The particular phasing between these two forces determines the total force. In this case, the first harmonic portion of vortex force nearly cancels with the total ideal added mass force, resulting in a dominant third harmonic lift force. This illustrates a condition where the effective added mass (total force in phase with acceleration) of the system is nearly zero and the lift force in phase with velocity is nearly zero.

Since we have only considered the two large magnitude vortices present in the wake in Fig. 2, we show that it is not necessary for triplets of vortices to be present in order to account for third harmonic forces. The relative motion of the cylinder with respect to these two strong vortices accounts for large amplitude third harmonic forces in lift. Third harmonic forces can be caused by both vortex triplets, as observed by Jauvtis and Williamson (2004), and by large amplitude combined in-line and cross-flow motions with '2P' vortex shedding.

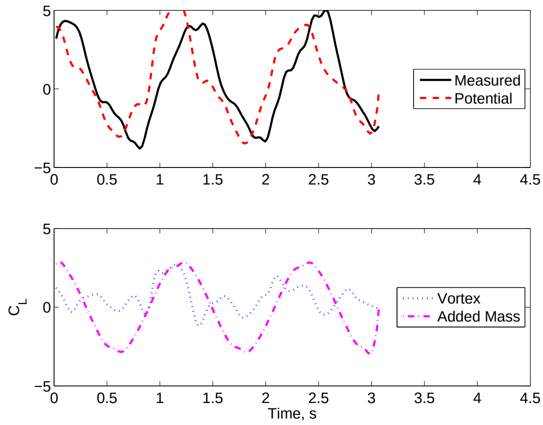

Figure 6: Potential flow representation of lift force coefficient for $\theta = 135$ degrees. Figure shows the measured force compared with calculated force from potential flow and the break-up of the potential flow force into an ideal added mass force and a vortex force.

In the case of '2S' vortex shedding, lift forces do not contain a large third harmonic force, as seen in Fig. 6. In this case, vortices shed and move directly downstream, without moving around the cylinder. This means that vortices do not stay in as close proximity to the cylinder while the cylinder moves with in-line motion. This results in an attenuation of higher harmonic force magnitudes, although the total lift coefficient magnitude is large due to first harmonic forces.

5. DISCUSSION

This paper shows a small number of visualizations associated with forced cylinder motions in combined cross-flow and in-line motion, however the phase relation between these motions is shown to have a large impact on the observed wake as well as the observed lift forces associated with the cylinder motions. In particular, the '2P' or '2T' mode of vortex shedding is shown to be associated with higher harmonic forces in lift, while the '2S' mode of shedding results in lift forces dominated by the fundamental frequency of vortex shedding.

The '2P' and '2T' modes of vortex shedding are characterized by the alternate shedding of pairs or triplets of vortices where at least one vortex crosses the cylinder wake before shedding.

For instance, in Fig. 2, frame A shows a large magnitude, negative vortex which formed along the top edge of the cylinder but has now moved below the cylinder. In this process, the vortex has remained in close proximity with the cylinder while the cylinder has made one in-line fluctuation. The total relative cylinder motion with respect to this vortex over one cycle of motion results a large third harmonic force. Similar fifth harmonic forces may exist if one considers downstream vortices shed from the previous cycle, however these vortices are not in close proximity to the cylinder; hence, they will not largely affect lift forces.

In the case of '2S' shedding, vortices do not cross the wake near the cylinder, but rather shed directly downstream of the cylinder. In this case, the vortex moves directly away from the cylinder, so the effect of relative velocity changes are attenuated by the effect of vortex proximity to the cylinder. This is clearly seen in equation 3, where the lift force is a function of relative velocities over distance from the cylinder squared.

Understanding the wake and forces associated with these motions is essential to properly predicting forces exerted on long marine structures that exhibit these types of motions. Although the free vibrations of an elastically mounted, rigid cylinder do not exhibit some of the particular motions shown in this study, the forced motions in this study may be seen locally along a long, flexible cylinder such as a marine riser.

It is important to note that the motions presented in this study as representative of free vibrations are very limited since amplitudes and reduced velocity are fixed with values where third harmonic forces are dominant. These motions are only representative of free vibrations when the inline natural frequency of the cylinder is twice the cross-flow natural frequency, a condition that differs in the experiments of Jauvtis and Williamson (2004). Additionally, changing values of reduced velocity and amplitude will alter the phasing of vortex shedding and the phasing of first and third harmonic forces exerted on the cylinder. The phasing between first and third harmonic components of force since peak amplitudes of force are dependent on this phase relationship.

6. CONCLUSION

In this paper we have studied the how the variation of phase between in-line motion and cross-flow motion of a circular cylinder in a free stream affects the lift force on the cylinder and the wake behind the cylinder. For particular fixed amplitudes of in-line motion ($A_x/D = 0.31$) and cross-flow motion ($A_y/D = 0.91$), with fixed reduced velocity ($V_r = 6.4$), we show that a phase of 0 degrees corresponds to a '2P' vortex shedding pattern with one strong vortex paired with one weak vortex. The relative motion of the strong vortex with respect to the cylinder results in large magnitude, dominant third harmonic force in lift. At a phase angle of 135 degrees, the wake displays a '2S' shedding pattern and the relative motion of vortices with respect to the cylinder results in much smaller magnitude third harmonic forces, although first harmonic forces are still large.

7. REFERENCES

Dahl, J.M., Hover, F.S., Triantafyllou, M.S., 2006, Two-degree-of-freedom vortex-induced vibrations using a force assisted apparatus. *Journal of Fluids and Structures* **22**: 807-818.

Dahl, J.M., Hover, F.S., Triantafyllou, M.S., Dong, S., Karniadakis, G.E., 2007, Resonant vibrations of bluff bodies cause multi-vortex shedding. *Physical Review Letters* **99**: 144503.

Jauvtis, N., Williamson, C.H.K., 2004, The effect of two degrees of freedom on vortex-induced vibration at low mass and damping. *Journal of Fluid Mechanics* **509**: 23-62.

Jeon, D., Gharib, M., 2001, On circular cylinders undergoing two-degree-of-freedom forced motions. *Journal of Fluids and Structures* **15**: 533-541.

Milne-Thompson, L.M., 1960, *Theoretical Hydrodynamics* The McMillan Company: New York.

Sarpkaya, T., 2004, A critical review of the intrinsic nature of vortex-induced vibrations. *Journal of Fluids and Structures* **19**: 389-447.

Vandiver, J.K., Swithenbank, S., Jaiswal, V., Jhingran, V., 2006, Fatigue damage from high mode number vortex-induced vibration. *25th International Conference on Offshore Mechanics and Arctic Engineering* June 2006, Hamburg.

Williamson, C.H.K., Roshko, A., 1988, Vortex formation in the wake of an oscillating cylinder. *Journal of Fluids and Structures* **2**: 355-381.

Williamson, C.H.K., Govardhan, R., 2004, Vortex-induced vibrations. *Annual Review of Fluid Mechanics* **36**: 413-455.