Ninth International Conference on CFD in the Minerals and Process Industries
CSIRO, Melbourne, Australia
10-12 December 2012

COMPARISON OF SOLUTION ALGORITHM FOR FLOW AROUND A SQUARE CYLINDER

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ABSTRACT

Numerical accuracy, numerical stability and calculation time are all important factors in the computational fluid dynamics. In this study, we compare two solution algorithms, the Simplified Marker and Cell (SMAC) method in the MAC-type methods and the Semi-Implicit Method for Pressure-Linked Equation (SIMPLE) algorithm in the SIMPLE-type algorithms, with respect to flow around a square cylinder in constant density and unsteady-state calculations using a staggered grid to investigate the numerical accuracy, the numerical stability and the computational time. For the flow around a square cylinder, the SMAC and SIMPLE solutions are in excellent agreement at the Strouhal number, drag and lift coefficients. However, SMAC is more unstable than SIMLE with a large Courant number. The computational time of the SMAC is shorter than that of the SIMPLE with a small Courant number.

NOMENCLATURE

A area
back back face
bottom bottom face
C_L Lift coefficient
C_D Drag coefficient
Cr Courant number
D characteristic length
Dn diffusion number,
e cell surface index
f frequency of vortex shedding
front front face
in inlet
n cell surface index
p pressure
Re Reynolds number
RC rectangular
St Strouhal number
s cell surface index
t elapsed time
top top face
$\mathbf{u}$ velocity
u velocity
v velocity
x coordinate
y coordinate
w cell surface index

α under relaxation factor
Δt time step
Δx grid width
Δy grid width
ε convergence criterion
μ viscosity
ρ density
τ non-dimensional computational speed

INTRODUCTION

Solution algorithms of pressure-velocity coupling are widely used for incompressible fluid flow calculations. The solutions expend the major part of time of the Computational Fluid Dynamics (CFD), because iterative calculations are required. In the CFD, numerical accuracy, numerical stability and calculation time are all important factors. As the calculation time influences the calculation cost directly, the calculation speed is the most important factor. But at the same time, it is necessary that the numerical accuracy and numerical stability are high. Thus, in order to perform better CFD, it is required to balance these factors. In particular, we focus on coupling schemes to decrease the calculation cost.

The solution algorithms solve for continuity and momentum equations, for which Marker and Cell (MAC) methods and Semi-Implicit Method for Pressure-Linked Equation (SIMPLE) type algorithms have been widely applied. Both common parts are solving the continuity equation in implicitly, and the most significant difference between the methods is the treatment of momentum equations; the former is solved in explicitly, and the latter is in implicitly.

Generally, MAC-type methods are used for unsteady-state calculations and SIMPLE-type algorithms are used for steady-state or pseudo-unsteady state calculations. However, as SIMPLE-type algorithms are widely used for combustion flow dynamics, SIMPLE-type algorithms are needed to correspond with unsteady-state calculations. Thus, although SIMPLE-type algorithms (SIMPLE, SIMPLE-Consistent (SIMPLEC), SIMPLE-Revised (SIMPLER), and Pressure Implicit with Splitting of Operators (PISO)) are compared with each other in steady-state or pseudo-unsteady-state [Van Doormaal and Raithby (1984) , Issa et al. (1986)], unsteady-state calculations have been only a few reported [Barton (1998)]. The comparison of MAC-type (MAC, Simplified MAC (SMAC), and Highly Simplified MAC (HSMAC)) methods and SIMPLE-type algorithms in unsteady-state fluid flow calculations have been only a few reported [Kim and Benson (1992)].

In Kim and Benson's study, the SMAC method is compared numerically with the PISO method. According to their numerical results, for a larger time step, the SMAC method is more strongly convergent and yields more accurate results than the PISO scheme, and it is more computationally efficient. They consider the steady-state calculation time and unsteady-state calculation time together. There is little consideration of different in the unsteady-state solutions.

The restriction of a time step (i.e., Courant number) directly influences the calculation cost. Therefore, many researchers consider calculation cost only in relation to the Courant number. However, calculation cost is the actual time a calculation requires; therefore, to evaluate calculation cost effectively, the real calculation time must be considered.

This study is intended to report comparison of the SMAC method and the SIMPLE algorithm. These methods have been compared for transient flow calculations in constant density using a staggered grid to investigate numerical accuracy, numerical stability, and calculation time. In the Courant number for time step restriction, we discuss about from 0.01 to 2.0 in order to investigate characteristic of each other. The treatment of discretized schemes, boundary conditions, time step restrictions, etc. with the SMAC method and SIMPLE is as equal as possible.

NUMERICAL METHODS

Governing equations

The governing equation of fluid flow is composed the continuity and momentum equation as followed. In this study, the property of density and viscosity is constant.
Continuity equation:

$$\nabla \cdot (\rho \mathbf{u}) = 0. \tag{1}$$

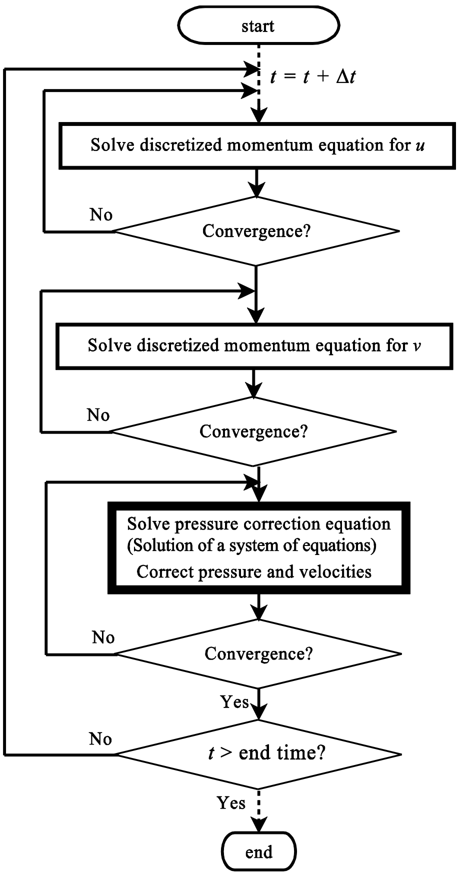
Momentum equations:

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla p + \nabla \cdot (\mu \nabla \mathbf{u}). \tag{2}$$

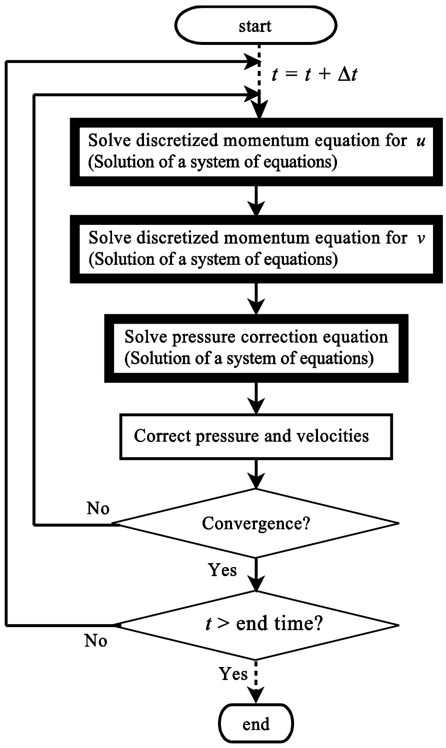
Solution algorithms of pressure-velocity coupling

We compare two coupling schemes: the Simplified Marker and Cell (SMAC) method and Semi-Implicit Method for Pressure-Linked Equation (SIMPLE) algorithm.

Those coupling schemes are analysis for incompressible fluids, and the continuity equation and momentum equations are solved.



(a) SMAC



(b) SIMPLE

Figure 1: Flowchart.

Figure 1 presents flowcharts for the coupling schemes. In this study, we use an implicit method for the unsteady term of the momentum equations. Although discretised momentum equations in SMAC are solved using a matrix solver, it is a few to solve the equations. Thus, in the figure of SMAC and SIMPLE, the thick-bordered boxes are a part which requires calculation cost.

Numerical schemes

Table 1 shows a summary of the numerical schemes used. The governing equations are discretized via the finite volume method.

The 2nd-order Crank-Nicolson method is applied to the discretization scheme of unsteady term with SMAC and SIMPLE in order to equate the temporal accuracy.

Since the spatial schemes become the equal for the SMAC method and SIMPLE, it is also necessary to equalize the temporal schemes. If an explicit method (e.g., Euler-explicit, Adams-Bashforth, or Runge-Kutta method) is used for SIMPLE, the calculation technique infringes upon the principles of SIMPLE. We use the implicit method (i.e., Crank-Nicolson method) in the discretization of temporal term for the momentum equation.

In SIMPLE, an under-relaxation factor (α) is needed for the momentum and pressure equations. We compared α = 0.3, 0.5 and 0.7, relative to calculation time. The calculation at α = 0.5 was determined to be the fastest. This result was computed using the sum of calculation time where the Courant number was 0.1, 0.5, 1.0 and 1.5. Hence, the under-relaxation for following SIMPLE calculation was used α = 0.5.

Table 1: Numerical schemes.

Discretization method	Finite volume
Discretization scheme	
Convection term	3rd-order upwind difference
Diffusion term	2nd-order central difference
Unsteady term for SMAC	2nd-order Crank-Nicolson
Unsteady term for SIMPLE	2nd-order Crank-Nicolson
Coupling scheme	SMAC, SIMPLE
Matrix solver	Residual cutting method

Convergence criterion

A residual error of the continuity equation with the SMAC method and SIMPLE is evaluated using

$$\frac{\sum|u_\mathrm{e}A_{\mathrm{n-s}}-u_\mathrm{w}A_{\mathrm{n-s}}+v_\mathrm{n}A_{\mathrm{e-w}}-v_\mathrm{s}A_{\mathrm{e-w}}|}{u_\mathrm{ch}D_\mathrm{ch}^2}=\epsilon, \tag{3}$$

where, ε is the convergence criterion. The convergence criterion is $\varepsilon = 1.0 \times 10^{-6}$ for rigid conditions. In general, the error is evaluated by the numerator of the above equation. However, we evaluate by very rigid conditions because the difference in the system size might be ignored.

Stability condition for time step

The time step Δt is given to satisfy the following stability condition:

$$\Delta t = \min\left[\frac{Cr}{\max\left(\frac{u}{\Delta x},\frac{v}{\Delta y}\right)}, \frac{Dn}{\nu\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)}\right], \tag{4}$$

where, *Cr* is the Courant number and *Dn* is the diffusion number. *Cr* is varied and *Dn* is set to 0.3. In any case, the diffusion number did not exceed 0.3.

Calculation time

In this study, we compare calculation times to determine calculation costs. We measure the time from the start of the calculation to the end, which is defined as CPU time. Then, to compare the SMAC method and SIMPLE, we introduce non-dimensional computational speed ($1/\tau$), which is defined as follows:

$$\frac{1}{\tau}=\frac{\text{Standard CPU time}}{\text{CPU time}}, \tag{4}$$

where the standard CPU time for the normalization is the computational time of SIMPLE and *Cr* = 0.01. The larger the value, the more quickly the calculation is completed.

Calculation environment

All the calculations are carried out using workstations with a single Intel(R) Core i7-980X Extreme 3.33 GHz CPU.

Numerical conditions

The analytical object is two-dimensional flow in a duct with an inserted square cylinder. **Figure 2** shows the computational configuration. The width of the square cylinder is $D = 4$ mm. The computational domain is $40D \times 11D$ in the *x*- and *y*-coordinate directions, respectively.

The inlet boundary is located at $5D$ upstream of the cylinder; the outlet boundary is located at $34D$ downstream of the cylinder; and the side boundaries are located at $5D$ away from the cylinder. A uniform flow is prescribed at the inlet boundary, and the zero-gradient boundary condition is used at the outlet boundary. The free-slip condition is given to the side boundaries, and the no-slip condition is given to boundaries of the cylinder.

The flow domain is discretized by uniform 200×55, 400×110, and 600×165 grid points in the *x*- and *y*-coordinate directions, respectively. The coarse and fine meshes were used solely to study the dependence of the mesh size on numerical results. All numerical results presented were obtained using the medium mesh.

Fluid properties are assumed to be $\rho = 1$ kg/m^3, and $\mu = 1 \times 10^{-5}$ Pa·s. The Reynolds number, based on a side of the cylinder and inlet velocity, is $Re_\mathrm{RC} = \rho \times u_\mathrm{in} \times D/\mu = 190$. The calculation is advanced to 1.3 s. The characteristic velocity is $u_\mathrm{ch} = u_\mathrm{in}$ and the characteristic length is $D_\mathrm{ch} = 40D$ in the convergence criterion.

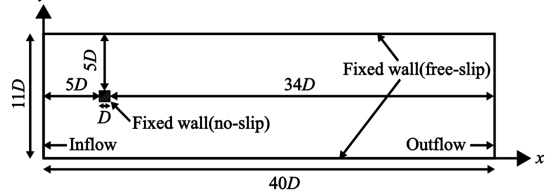


Figure 2: Flow around a square cylinder configuration.

RESULTS

The calculated Strouhal numbers ($St = fD/u_\mathrm{in}$) are compared with the measured data [Davis and Moore, (1982)] in **Figure 3**, where SIMPLE data at *Cr* = 0.01

indicates that the flow does not reach a steady-state, and if t = 2.0 s, the St value with SIMPLE at Cr = 0.01 is 0.141. The f value is calculated by the vortex shedding period. This figure shows that the present numerical results are in good agreement with experimental data. Hence, the numerical accuracy of SMAC and SIMPLE is excellent. The St values of SIMPLE are constant with an increase in the Cr value, while those of SMAC decreases. This indicates that SIMPLE is more stable than SMAC method as the Cr values increase.

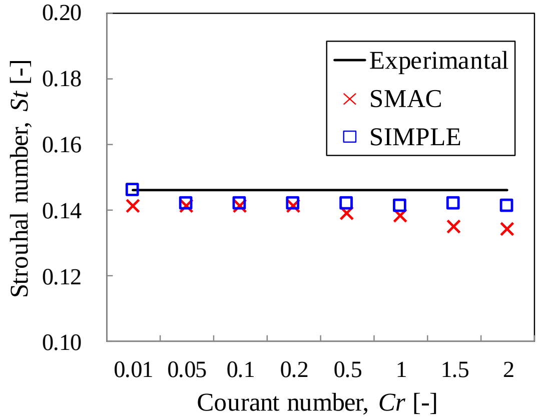


Figure 3: Strouhal number at Re = 190 and t = 1.3 s.

Figure 4 and **Figure 5** show the drag and lift coefficients at t = 1.3, where the C_D and C_L values are defined as following equation:

$$C_D = \frac{\sum(p_{front} - p_{back})\delta y}{\frac{1}{2}\rho u_{in}^2 D}, \quad (5)$$

and

$$C_L = \frac{\sum(p_{top} - p_{bottom})\delta x}{\frac{1}{2}\rho u_{in}^2 D}. \quad (6)$$

These figures show that the SMAC and SIMPLE results are in good agreement with the C_D and C_L values.

This is obtained because of the all discretization schemes used in this study are same with SMAC and SIMPLE: the convection, diffusion and unsteady term discretization schemes. Therefore, it is considered to require treating the relating schemes in the same way, when the numerical accuracy in the coupling schemes is discussed.

Figure 5 shows a significant difference between the SIMPLE and SMAC at Cr = 0.01 and t = 1.3 s. This is similar to the problem with the Strouhal number because the calculation of SIMPLE at Cr = 0.01 has not reached steady-state. The C_L value will be set to 0.00 if the calculation is continued.

Figure 4 and **Figure 5** also show that SMAC is less stable and SIMPLE is stable as the Cr value increases.

Figure 6 shows the non-dimensional computational speed, where the normalization CPU time is the computational time (216892.66 s) of SIMPLE at Cr = 0.01. This figure indicates that the calculation time of SMAC is shorter than that of SIMPLE in all case. When the results are compared at Cr = 0.1, SMAC is 1.5 times faster than SIMPLE.

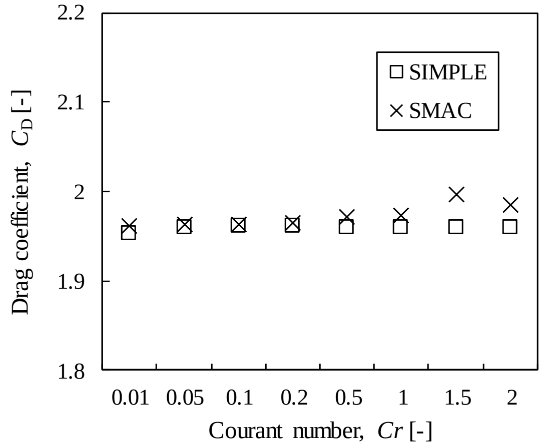


Figure 4: Drag for the flow over the square cylinder.

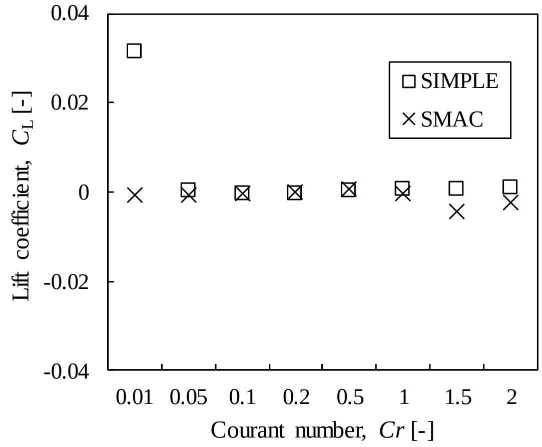


Figure 5: Lift for the flow over the square cylinder.

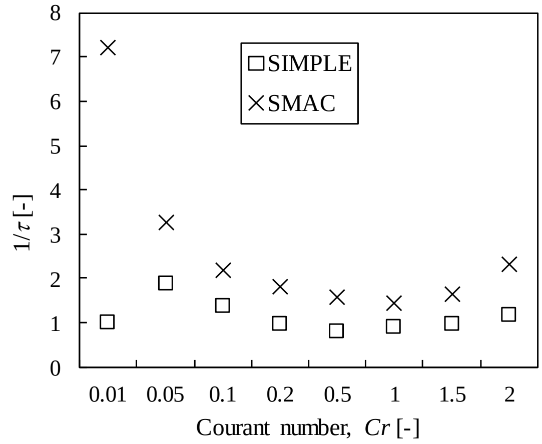


Figure 6: Computational speed at t = 1.3 s

Figure 7 shows total iteration number of the matrix solver for the momentum pressure correction equation.

In SMAC, the total iteration number increases with an increase Cr value, and decreases as the Cr value exceed 1. This is because SMAC is less stable as the Cr value increases. Although the iteration number increases within

1 time step with an increases the *Cr* value, the number of time steps decrease so that the iteration number decreases.

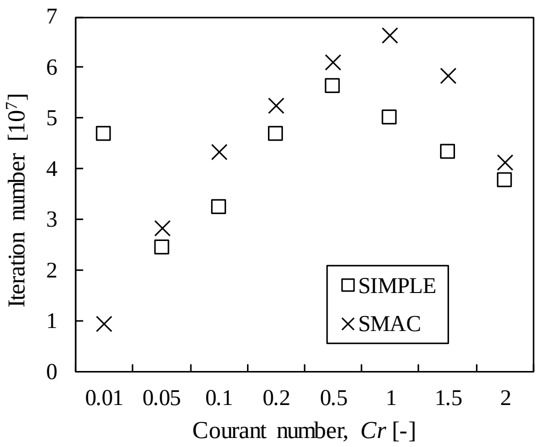


Figure 7: The number of solver is called at $t = 1.3$ s.

SIMPLE shows a tendency similar to SMAC. But, The iteration number at *Cr* = 0.01 is large. This is because the number of time steps is large at the *Cr* value is small.

When the iteration number of SMAC and SIMPLE at *Cr* = 0.01 are compared, that of SMAC is less than that of SIMPLE. This is caused for the iteration number is a few, because SMAC is calculated explicitly.

Expect the case of *Cr* = 0.01, this figure shows that the iteration number of SMAC is larger than that of SIMPLE. However, the calculation time of the SMAC is shorter than that of the SIMPLE in all cases. This is because that the pressure correction equation is only solved after the momentum equation is solved in SMAC, while the momentum equation and pressure correction equations are solved to satisfy those equations at the same time in SIMPLE. This also shows that it is difficult to determine which the calculation time completely depends on the iteration number. Therefore, not evaluation of the total iteration number but the evaluation of the actual calculation time is required.

From above mentioned, the accuracy of SMAC and SIMPLE is the same in the small Courant number. Hence, if the target system is stable and the Courant number is small, we show SMAC is useful.

CONCLUSION

In the present study, the SMAC method and SIMPLE algorithm are evaluated for unsteady-state fluid flow calculations. The following results are obtained.

- The calculated Strouhal numbers of SMAC and SIMPLE are in good agreement with experimental data. The numerical accuracy of SMAC and SIMPLE in steady-state is excellent.
- The calculated drag and lift coefficients of SMAC and SIMPLE are in good agreement with each other. The numerical accuracy of SMAC and SIMPLE is the same in the small Courant number.
- SMAC is more unstable than SIMLE in the large Courant number.
- The computing time of the SMAC method is short compared with the SIMPLE algorithm.

As a result, the SMAC method is effective in case of incompressible, constant density and unsteady-state fluid flow calculations.

ACKNOWLEDGMENTS

This work was partially supported by a grant for the Global COE Program "World Center of Education and Research for Trans-disciplinary Flow Dynamics".

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