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A STUDY OF CFD MODELLING ON VARIATION OF SOLID FRACTION IN A BATCH FLUIDIZED BED

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ABSTRACT

Two dimensional Gas-Solid batch fluidized bed is simulated in transient conditions using Eulerian-Eulerian two fluid model. The simulated results are compared with the experimental observations. The study is conducted for different sizes of the Geldart group D solid particles, different bed heights and different air velocities in the turbulent regime of fluidization. Three different models such as Gidaspow, Syamlal-O'Brien and Wen and Yu drag models have been tested to model the drag at the phase interaction for Geldart group D solid particles. At the initial stage of fluidization (where a fountain like phenomena was observed), the bed behaves like a spouting bed and later on enters into the turbulent regime. The bed height variation in the initial and the turbulent regime of fluidization is predicted using the three drag models. These predictions are compared with the experimentally observed bed height variation for different particle sizes and air velocities. It is observed that Syamlal-O'Brien drag model predictions were in good accordance with the experimental results for different particle sizes both in the initial stage and turbulent regime of fluidization.

NOMENCLATURE

α	Volume fraction
ρ	Density [kg/m^3]
υ	Velocity [m/s]
∇	The Dell operator [1/m]
P	Pressure [Pa]
$\overline{\overline{\tau}}$	The stress-strain tensor [Pa]
$\vec{g}$	The gravitational acceleration [m/s^2]
K_{gs}	Gas/ solid momentum exchange coefficient, dimensionless
μ	Viscosity [kg/m-s]
λ	Bulk viscosity [kg/m-s]
$\overline{\overline{I}}$	Stress tensor, dimensionless
g_0	The general radial distribution function
e_{ss}	Restitution coefficient for solid phase
Θ	Granular temperature [m^2/s^2]
d_s	Diameter of solid phase [m]
I_{2D}	The second invariant of the deviatoric stress tensor
ϕ_i	Angle of internal friction[^{o}]
ϕ	Shape factor used in the Ergun equation
β	The coefficient of thermal expansion
q	Diffusion of fluctuating energy (kg/s^3)
γ	Dissipation of fluctuating energy (kg/m-s)
J	Granular energy transfer (kg/m-s^3)
P1,2,3,4	Samples of different particle size.
dp1, dp2, dp3, dp4	Diameter of the particles of size P1, P2, P3, P4 respectively

Subscripts:

g	Gas phase
s	Solid phase
q	gas/solid phase
mf	Minimum fluidization velocity
c	Column
p	Particle

INTRODUCTION

Gas-Solid fluidized beds are widely used in chemical, petrochemical, pharmaceutical, paint and food industries. There are several applications of gas- solid fluidized beds like coal gasification, synthesis reactions, metallurgical operations, physical operations, cracking of hydrocarbons, combustion and incineration, biomass conversion and drying of solid particles, etc. In a Gas-Solid fluidized bed we find extremely high surface area of contact between gas and solid particles, high relative velocities of gas and solid particles, very high particle-particle and particle-wall collisions and higher heat transfer rate.

By increasing the inlet gas velocity beyond minimum fluidization velocity the bubbling fluidization region appears and sometimes smooth fluidization occurs in special conditions with very fine particles. In the Present study experimental and simulation work has been carried out using Geldart group D sand particles, varying parameters such as inlet air velocity, diameter of the particle and initial bed height of the solids.

EXPERIMENTAL SETUP

Experiments were performed in a cylindrical column of 83 mm inner diameter with 400 mm height. A circular perforated plate is used as a distributor. The sand particles of size ranging from 1.2 mm to 3.075 mm with density 2500 Kg/m^3 have been used. The solids were fluidized with air introduced from air compressor. Different static bed heights (50, 37.5 and 25 mm) were used by varying the quantity of sand in the column. Solid volume fraction in the bed was calculated to be 0.55. Schematic diagram of experimental setup is shown in Fig. 1.

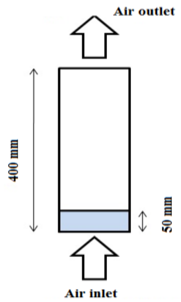

Fig. 1 Experimental Set up

CFD MODEL

A transient two dimensional Eulerian – Eulerian two fluid model has been used to model the Gas-Solid batch fluidized bed. The bed height variation has been simulated using the model for different operating air velocities and particle sizes.

3.1 Governing equations: The governing equations for the system include the conservation of mass and momentum. Equations for solid and gas phases are based on the Eulerian–Eulerian model. Continuity equation for q^{th} phase including volume fraction of each phase without mass transfer between phases is given as (Eq. 1)

$$\frac{\partial}{\partial t}\left(\alpha_q \rho_q\right) + \nabla \cdot \left(\alpha_q \rho_q \upsilon_q\right) = 0 \qquad (1)$$

α_q, ρ_q and υ_q are the volume fraction, density and the velocity of q^{th} phase respectively. Gas phase and solid phase momentum equations are given by Eqs 2 and 4

Gas phase: (Eq. 2)

$$\frac{\partial}{\partial t}\left(\alpha_g \rho_g v_g\right) + \nabla \cdot \left(\alpha_g \rho_g \upsilon_g \upsilon_g\right) = -\alpha_g \nabla p + \nabla \overline{\overline{\tau}}_g + \alpha_g \rho_g \vec{g} - K_{gs}\left(\upsilon_g - \upsilon_s\right) \qquad (2)$$

Gas phase stress – strain tensor $\overline{\overline{\tau}}_g$ is expressed as

$$\overline{\overline{\tau}}_g = \alpha_g \mu_g \left(\nabla \upsilon_g + \nabla \upsilon_g^T\right) + \alpha_g \left(\lambda_g - \left(\frac{2}{3}\mu_g\right)\right) \nabla \cdot \upsilon_g \overline{\overline{I}} \qquad (3)$$

Where $\overline{\overline{I}}$ is unity tensor (dimensionless)

Solid phase: (Eq. 4)

$$\frac{\partial}{\partial t}\left(\alpha_s \rho_s v_s\right) + \nabla \cdot \left(\alpha_s \rho_s \upsilon_s \upsilon_s\right) = -\alpha_s \nabla p - \nabla P_s + \nabla . \overline{\overline{\tau}}_s + \alpha_g \rho_g \vec{g} + K_{gs}\left(\upsilon_g - \upsilon_s\right) \qquad (4)$$

Solid phase stress – strain tensor $\overline{\overline{\tau}}_s$ is expressed as

$$\overline{\overline{\tau}}_s = \alpha_s \mu_s \left(\nabla \upsilon_s + \nabla \upsilon_s^T\right) + \alpha_s \left(\lambda_s - \frac{2}{3}\mu_s\right) \nabla \cdot \upsilon_s \overline{\overline{I}} \qquad (5)$$

Drag Models

The following three different drag models have been used to model the drag at the gas-solid phase interaction. Simulations were performed with the three models and the predicted bed height variation was compared with the experimental results.

1. Gidaspow drag model (Gidaspow, 1994):

Gidaspow drag model for drag function between particle and gas phase is given by eqn (6) and (7) for different ranges of volume fraction of gas, α_g. For $\alpha_g \leq 0.8$ eqn(6) has been used and for $\alpha_g > 0.8$ eqn (7) has been used.

$$K_{gs} = 150\frac{\mu_g\left(1-\alpha_g\right)^2}{\alpha_g\left(d_s\phi\right)^2} + 1.75\frac{\rho_g\left|\overrightarrow{\upsilon_g} - \overrightarrow{\upsilon_s}\right|\left(1-\alpha_g\right)}{d_s\phi} \qquad (6)$$

$$K_{gs} = \frac{3\rho_g \alpha_g\left(1-\alpha_g\right)}{4d_p} C_D \left|\overrightarrow{\upsilon_s} - \overrightarrow{\upsilon_g}\right| \alpha_g^{-2.65} \qquad (7)$$

$$C_D = \frac{24}{\alpha_g \mathrm{Re}_s}\left[1 + 0.15\left(\alpha_g \mathrm{Re}_s\right)^{0.687}\right] \qquad (8)$$

2. Syamlal-O'Brien drag model (Syamlal et al., 1989):
The drag model of Syamlal-O'Brien is a correlation between the drag of a sphere and multiparticle system. The correlation is given below.

$$K_{gs} = \frac{3\alpha_g \alpha_s \rho_s}{4 d_s \upsilon_r^2} C_D \left|\overrightarrow{\upsilon_s} - \overrightarrow{\upsilon_g}\right| \qquad (9)$$

$$C_D = \left[0.63 + \frac{4.8}{\sqrt{\frac{Re}{\upsilon_r}}}\right]^2 \qquad (10)$$

$$\upsilon_r = \frac{1}{2}\left[A - 0.06\mathrm{Re}\right] + \frac{1}{2}\left[\sqrt{\left(0.06\mathrm{Re}\right)^2 + 0.12\mathrm{Re}\left(2B - A\right) + A^2}\right] \qquad (11)$$

$$A = \alpha_g^{4.14} \quad B = \begin{cases} 0.8\alpha_g^{1.28} & \alpha_g \leq 0.85 \\ \alpha_g^{2.65} & \alpha_g > 0.85 \end{cases} \qquad (12)$$

Reynolds number can be calculated from

$$\mathrm{Re} = \frac{\rho_g d_s \left|\overrightarrow{\upsilon_s} - \overrightarrow{\upsilon_g}\right|}{\mu_g} \qquad (13)$$

3. Wen and Yu drag model (Wen and Yu, 1966): Wen and Yu drag model is used for dilute phase regime (Mahdi Hamzehei., (2011)). The multi-particle drag function is shown in Eq.14 and the single particle drag factor is shown in Eq. 15.

$$K_{sg} = \frac{3\rho_s \alpha_g (1-\alpha_g)}{4d_p} C_D \left|\overrightarrow{\upsilon_s} - \overrightarrow{\upsilon_g}\right| \alpha_g^{-2.65} \quad (14)$$

$$C_D = \frac{24}{\alpha_g \mathrm{Re}_s}\left[1 + 0.15(\alpha_g \mathrm{Re}_s)^{0.687}\right] \quad (15)$$

The Θ_s granular temperature is an estimate of the particle fluctuation in the granular flow and it can be expressed as

$$\Theta_s = \frac{1}{3} v^2 \quad (16)$$

The solid phase transport equation for the granular temperature can be written as

$$\frac{3}{2}\left(\frac{\partial\left(\alpha_s \rho_s \Theta_s\right)}{\partial t} + \nabla.\left(\alpha_s \rho_s v_s \ \Theta_s\right)\right) = \left(-P_s \bar{\bar{I}} + \tau s: \nabla vs - \nabla.q - \gamma - J \quad (17)\right.$$

The above equation can be stated as:
The net change of fluctuating energy = the generation of fluctuating energy due to the local acceleration of the particles+ the diffusion of the fluctuating energy+ the dissipation of the fluctuating energy due to inelastic particle – particle collisions+ the exchange of the fluctuating energy between gas and solid phase.

For solids, pressure (Lun et al., 1984) can be expressed as sum of two terms, a kinetic term and a collisions term

$$P_s = \alpha_s \rho_s \Theta_s + 2\, g_0\, \alpha_s^2\, \rho_s \Theta_s \ (1+e) \quad (18)$$

The general radial distribution function g_0 is a function that modifies the probability of collisions between particles.

$$g_0 = \left[1 - \left(\frac{\alpha_s}{\alpha_{s,max}}\right)^{1/3}\right]^{-1} \quad (19)$$

The granular bulk viscosity is the resistance that the granular particles have for compression or expansion. The model is developed from the kinetic theory of granular flow given by Lun et al. (1984).

$$\lambda_s = \frac{4}{3}\alpha_s \rho_s d_s g_0 (1+e_{ss})\sqrt{\frac{\Theta_s}{\pi}} \quad (20)$$

The granular phase is defined by property models for the interactions with other particles and fluid phase. Granular collision, kinetic and frictional viscosities contribute to total viscosity.

$$\mu_s = \mu_{s,col} + \mu_{s,kin} + \mu_{s,fr} \quad (21)$$

Collision viscosity is given by Eq. 22

$$\mu_{s,col} = \frac{4}{5}\alpha_s \rho_s g_{0,ss}(1+e_{ss})\sqrt{\frac{\Theta_s}{\pi}} \quad (22)$$

Kinetic viscosity given by Gidaspow et al. (1992) is used in the present study

$$\mu_{s,kin} = \frac{10\rho_s d_s \sqrt{\Theta_s \pi}}{96\alpha_s (1+e_{ss}) g_{0,ss}}\left[1 + \frac{4}{5} g_{0,ss}\alpha_s (1+e_{ss})\right]^2 \quad (23)$$

For the frictional viscosity, Schaeffer equation (Schaeffer, 1987) is used and is shown below. The constant ϕ_i is the angle of internal friction, 30° (Benzarti etal, 2012 and Gryczka etal, 2009).

$$\mu_{s,fr} = \frac{P_{s,fr}\sin\phi_i}{2\sqrt{I_{2D}}} \quad (24)$$

Minimum fluidization velocity is calculated using Wen and Yu model (Kunii and Levenspiel., 1991)

$$\frac{d_p v_{mf} \rho_g}{\mu} = \left[(28.7^2) + 0.0494\left(\frac{d_p^3 \rho_g(\rho_s - \rho_g) g}{\mu^2}\right)\right]^{0.5} - 28.7 \quad (25)$$

Solution methodology

Two dimensional (2D) simulation of the gas-solid fluidized bed under transient conditions was performed and the results are described in this section.

The 2D computational domain was discretized into uniform rectangular fine mesh (the grid independency is given below). A fixed time step of 0.00001 has been chosen for the transient simulations. The stability and accuracy of the solution was ensured by maintaining courant number in the range of 0.2-0.4. Number of iterations in each time step is used as 30. The number of iterations was found to be adequate to achieve convergence for all time steps. Table 1 shows the values of model parameters that were used in the simulation. The discretized governing equations were solved by the finite volume approach. The second order upwind discretization scheme is used for discretizing all conservation equations except for volume fraction and turbulent dissipation formulation. For volume fraction and turbulent dissipation, first order discretizing scheme is used. The phase coupled PC-SIMPLE algorithm is used for pressure velocity coupling. The no-slip boundary condition is used for solid phase (Seyyed Hossein Hosseini etal, 2010)) and specified share of zero for x and y components were used for gas phase. The boundary conditions used are the specified inlet velocity at the bottom and pressure outlet with zero gauge pressure at the outlet.

Selecting Mesh

The size of the computational domain is 0.4 m X 0.083 m. Grid independent study has been carried out with the different mesh sizes of A, B, and C containing, 2736, 3724 and 5280 grid cells. Contours of solid volume fraction in the bed for the three meshes at 0.5 s were presented in Fig. 2. The variation in the solid volume fraction is very small between the two cases B and C. The bed height is also calculated for the three cases and the variation is observed to be less than 1%. Since these variations are very small, it is assumed that they will not affect the results to great extent. Hence in the interest of reducing the computational time, a reasonably good fine mesh with mesh size B has been chosen for further studies.

RESULTS AND DISCUSSION

The present study is carried out with Geldart group D particles of sand of diameter 3.075, 2.58, 1.7 and 1.2 mm with density of 2500 kg/m^3 at different air velocities and different bed heights. The minimum fluidization velocity of particle ranges from 0.58 to 1.31 m/s. The volume fraction of the solids in the column is varied as 0.125, 0.094, 0.0625 with bed height of 50, 37.5, 25 mm respectively and the amount of solids used is 0.4, 0.3 and 0.2 Kg and voidage of the bed is 0.55. The range of superficial gas velocity used is 2.13 to 2.98 m/s. A flat measuring scale has been used to find fluidized bed height experimentally. For analysing the simulated results, at the centre of the fluidized bed a line has been taken for predicting the fluidized bed height in simulation.

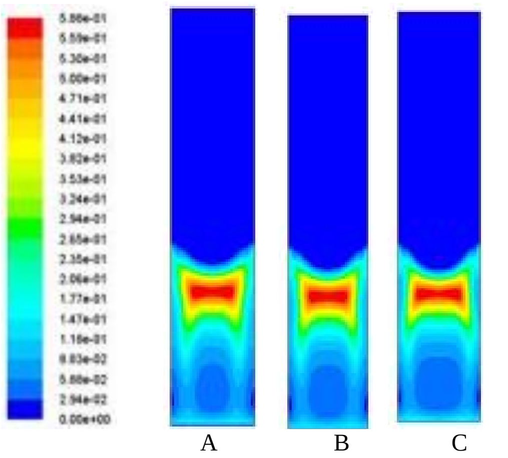

Fig. 2 Contours of Solid volume fraction at 0.5 s with three mesh sizes of A, B and C.

Various drag models such as Gidaspow (Gidaspow et al., 1994), Syamlal– O'Brien (Syamlal et al.,1989) and Wen and Yu drag models (Wen and Yu, 1966) have been used for simulation. Fig. 3 presents simulation results of three different drag models at time of 0.5 s. All other parameters were kept constant and only drag model at fluid solid interactions was changed. Fig.3, it has been observed from the results that at the initial stage of fluidization three drag models achieved same fluidized bed height with slight variation. In turbulent fluidization regime the Gidaspow and Wen and Yu drag models estimated less height as compared with Syamlal– O'Brien drag model. From the results it has been observed that Syamlal– O'Brien drag model results match closely with experimental results. Hence for further simulation the Syamlal– O'Brien drag model has been chosen in the present study.

At the initial stage of fluidization the fountain like behaviour was observed, later on the particles settled at the bottom of the bed in first few fractions of seconds and then the steady state turbulent fluidization is takes place.

The present bubbling behaviour matches the sudden inlet jet velocity bubble behaviour reported by Kunii and Levenspiel (1991). In the present study the inlet is assumed as 100% and due to this, the inlet behaves as a jet inlet at initial stage of fluidization.

Generally the jet velocity is introduced with small diameter in to the column due to which the fountain like behaviour is observed at that particular location only. In this present study the total inlet has been assumed as a jet inlet and the velocity of air used is just above the minimum fluidization range. Because of this reason the fountain like behaviour is observed at the initial stage and after few seconds the particles settle down and the normal turbulent fluidization takes place. The spout (fountain) height varies due to variation in particle size, initial amount of solids and air velocity.

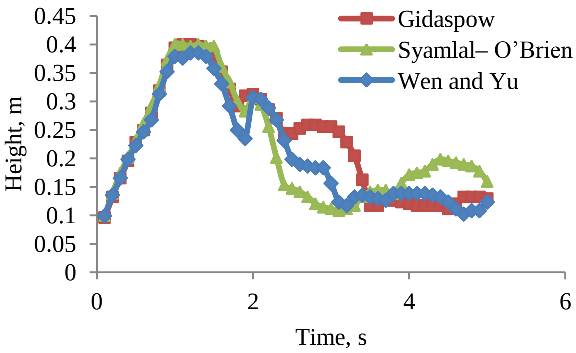

Fig. 3 At the center of fluidized bed, height measured for three different drag models.

Table1. Model parameters used in experimental and Simulation work.

Symbol	Description	Value
ρ_s	Solids density (sand, kg/m^3)	2500
ρ_g	Air density at room temp(kg/m^3)	1.205
d_p	diameter of the particle (mm)	dp1 –3.075, dp2–2.58, dp3–1.7, dp4–1.2
ε_s	Solids Volume Fraction (In bed)	0.55
D_c	Diameter of the column(m)	0.083
H_0	Initial bed height(m)	0.05, 0.0375,0.0 25
H_c	Height of the column(m)	0.4
U_g	Superficial gas velocity	2.13, 2.56, 2.98 m/s
U_{mf}	Minimum fluidization velocity Wen and Yu (m/S) (Eq. 2.28)	P1 – 1.31 P2 – 1.14 P3– 0.82 P4– 0.58 m/s

Effect of air velocity

Effect of air velocity has been studied at three different velocities ranging from 2.13 to 2.98 m/s. The results were presented in Fig. 4. It has been found that with increase in velocity, fluidization increases and further increase in velocity leads to change in the regime of fluidization. It has been found that bubbling regime of fluidization is consistent upto five times of minimum fluidization velocity.

Effect of initial bed height

The simulations have been carried at three different bed heights ranging from 0.025 m to 0.05 m keeping the air velocity at 2.13 m/s and the particle size to be 3.057 mm. Generally increase in the fluidized bed height is anticipated with increase in the initial bed height upto a certain initial bed height. Later on, the fluidized bed height decreases with increase in the initial bed height since the inlet air velocity will not be sufficient enough to fluidize the large quantities of bed of solids and also the contact between the gas and solid decreases. Hence, it is nessary to predict the optimum initial bed height for good Gas-

Solid contact in fluidized bed. From the simulation and experimental resutls it has been observed that for all particle sizes the best initial bed height has been found to be 0.05 m among the three initial bed heights studied. The effect of initial bed height has been presented in Fig.5.

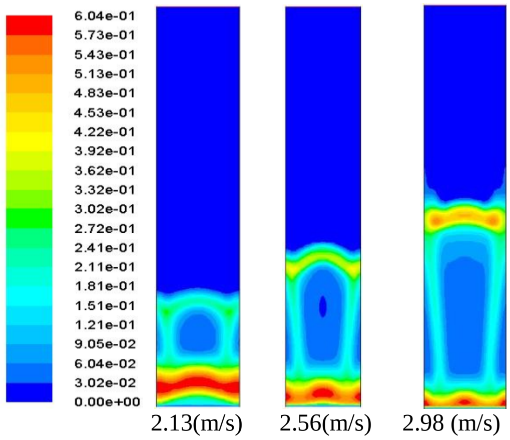

Fig. 4 Contours of solid volume fraction at different velocities with initial bed height of 50 mm after 1 s for particle diameter of 3.057mm.

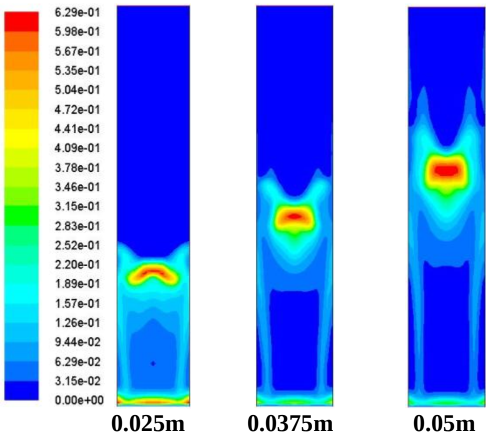

Fig .5 Contours of solid volume fraction at different initial bed heights at initial velocity of 2.13 m/s with particle diameter of 1.2 mm after 1s.

Effect of particle diameter

Experiments have been carried out for four different particle sizes ranging from 1.2 to 3.057 mm. It has been presented in fig. 6 and found from results that increase in the particle diameter the fluidized bed height is decreases. The simulated and experimental results have been compared.

Experimentally determined fluidized bed height and simulated fluidized bed height were presented in Figs.7 at various velocities and bed heights using uniformly sized particles of different sizes. For all type of particle sizes the simulated results were in good agreement with experimental values.

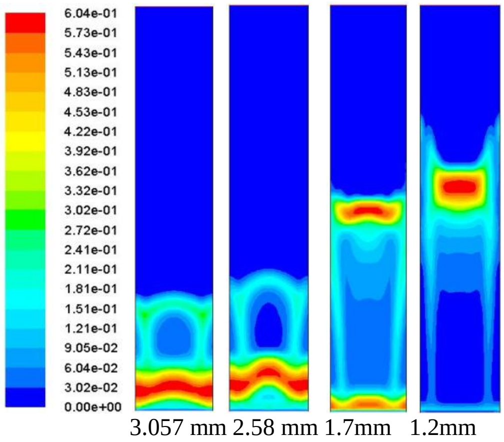

Fig. 6 Contours of solid volume fraction for different particle diameters at velocity of 2.13 m/s with initial bed height of 0.05m at time of 1.2s.

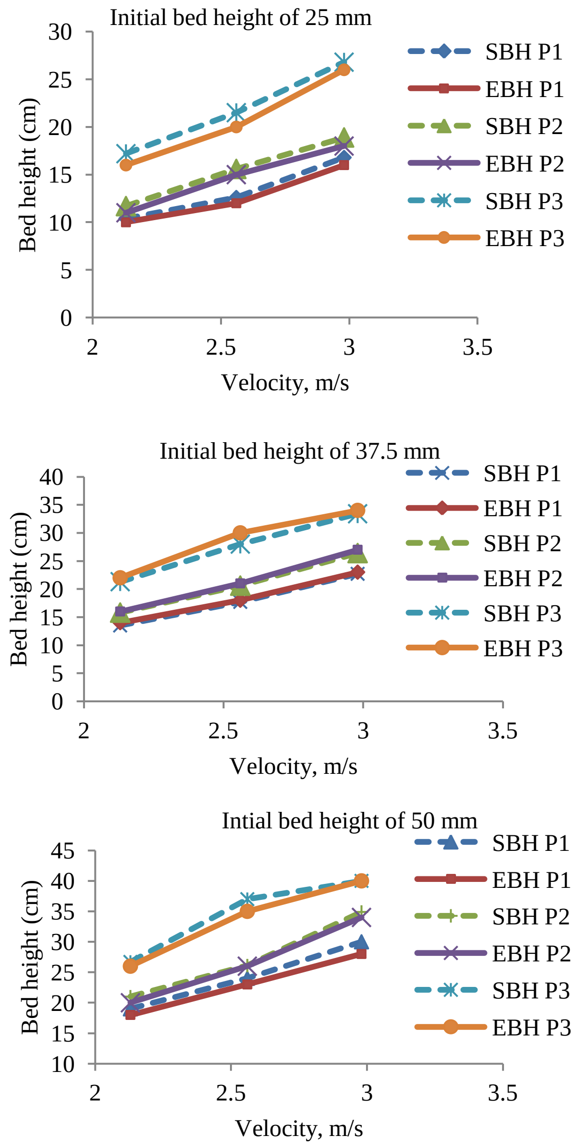

Fig. 7 (A, B and C) Fluidized bed height vs velocity at different velocities and at different initial bed heights.
SBH: Simulated bed height EBH: Experimental bed height

CONCLUSIONS

Three different drag models were used namely, Gidaspow, Syamlal-O'Brien and Wen and Yu drag model for gas-solid fluidized bed simulation. Syamlal-O'Brien model simulation results were in good agreement with experimental values. Four different sizes of particles at various velocities at different bed heights have been used and simulation as well as experimental work was performed. At the initial stage of fluidization the fluidization behaviour was shown in Fig. 5 at various velocities with initial bed height of 50 mm for different Geldart group D particles. The simulated results using Syamlal-O'Brien drag model were compared with experimental results and are found to be in good agreement.

The present simulated and experimental results were seen to be similar to the contours reported by van Wachem et al. (2001) for Geldart group D particles of size 1.545 mm using Lagrangian-Eulerian model. In the present study Eulerian-Eulerian model using, Syamlal-O'Brien drag model has been used with Geldart group D particles (sand) of size ranging from 1.2 to 3.075 mm and results are comparable with van Wachem et al. (2001) where the authors (van Wachem et al., 2001) worked with particles of relatively lower size and density (Polystyrene). This shows that Eulerian-Eulerian model using Syamlal-O'Brien drag model can be used for Geldart group D particles even for particles of higher size and density as seen in the present study.

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