A three-invariant hardening cap plasticity for computational modeling of the powder compaction process

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Abstract

In this paper, a three-invariant cap plasticity is developed for description of powder behavior under cold compaction process. The constitutive elasto-plastic matrix and its components are derived as the nonlinear functions of powder relative density. It is shown how the proposed model could generate the elliptical yield surface, double-surface cap plasticity, and the irregular hexagonal pyramid of the Mohr-Coulomb and cone-cap yield surface, as special cases. The single-cap plasticity is performed within the framework of large finite element deformation, in order to predict the non-uniform relative density distribution during powder die pressing. Finally, the numerical schemes are examined for efficiency in the modeling of an automotive component.

Keywords: Powder compaction; Three-invariant plasticity; Isotropic hardening; Cap model

1. Introduction

The physical and mechanical properties of powder metallurgy components are closely related to their final density. Minimizing the density gradients is an important consideration when high and consistent mechanical performance is required. The final density is determined by the press-and-sinter process parameters and by the material characteristics and filling conditions. Understanding and quantifying the main factors that influence the fill density could be a platform for ‘tailoring’ the initial density in the die. The analysis of powder pressing requires accurate material models of the various powder mixes that are used. Thus, an efficient and reliable plasticity model will play an important role in powder compaction simulation.

The cone-cap model based on a density-dependent Drucker-Prager yield surface and a non-centered ellipse was developed by Aydin et al. [1], Khoei and Lewis [2], Brandt and Nilsson [3] and Gu et al. [4]. A double-surface plasticity model was developed by Lewis and Khoei [5] for the non-linear behavior of powder materials in the concept of the generalized plasticity formulation for the description of cyclic loading. This model is based on the combination of a convex yield surface consisting of a failure envelope, such as a Mohr-Coulomb yield surface and a hardening elliptical cap. Recently, Khoei and Bakhshiani [6] developed a density-dependent endochronic theory based on coupling between deviatoric and hydrostatic behavior in finite strain plasticity to simulate the compaction process of powder material. In this paper, a generalized three-invariant single-cap plasticity is developed for 3D simulation of powder forming processes.

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2. The three-invariant cap plasticity

The single cone-cap plasticity can be written using the invariants of stress and deviatoric stress tensors in the following manner [7]

$$F(\sigma,\eta) = \psi J_{3D}^{2/3} + J_{2D} + \left(\frac{f_d}{f_h}\right)^2 J_1^2 - f_d^2 = 0 \tag{1}$$

where J_1 is the first invariant of stress tensor and J_{2D} and J_{3D} are the second and third invariants of deviatoric stress tensors. f_h is a positive increasing function of relative density, which controls the intersection of J_1 axis with yield surface function at the maximum value of compression. f_d is a function of the first invariant of stress tensor and relative density. The following

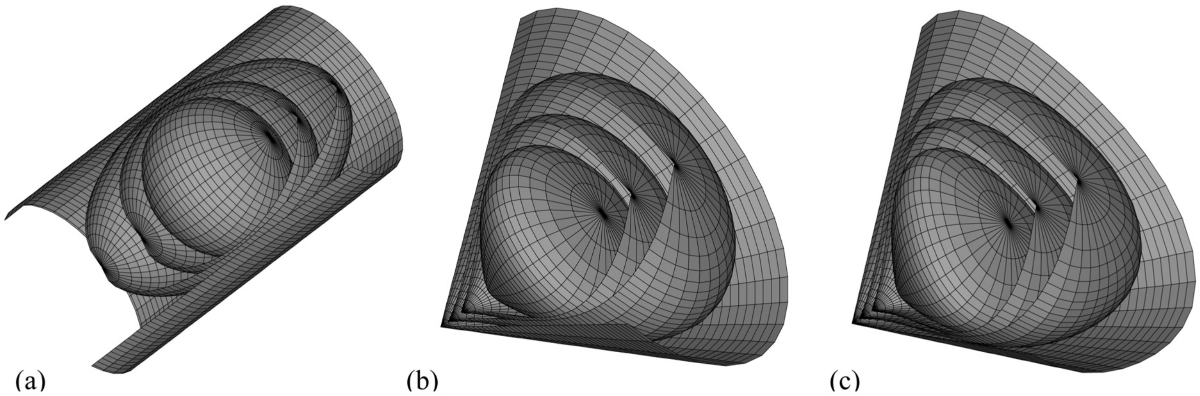

Fig. 1. Trace of 3D single-cap plasticity in principal stress space for different values of relative density: (a) the elliptical yield function, (b) the cone-cap yield function, (c) the irregular hexagonal pyramid of the Mohr-Coulomb and cone-cap yield function.

polynomial functions are proposed for the dependence of f_h and f_d on relative density

$$f_h = \left(\frac{1}{1-\eta^b}\right)(a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3) \tag{2}$$

$$f_d = (\alpha - \beta J_1) + (c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4) \tag{3}$$

where η is the relative density and ψ, b, a_0, a_1, a_2, a_3, α, β, c_1, c_2, c c_4 are parameters of the powder. In order to demonstrate different aspects of the yield surface of Eq. (1), we investigate the effects of f_h and f_d on its behavior. Consider that the first two terms of Eq. (1) are zero, it results in $(f_d/f_h)\, J_1^2 = f^2$d, which generally yields to three roots, the points of intersection of yield surface with J_1 -axis, i.e. $J_1 = \pm f_h$ and one more from $f_d = 0.0$, which has been defined in Eq. (3) as a linear function of J_1.

If the third condition (i.e. $f_d = 0.0$) does not lead to any value for J_1, for example $\beta = 0.0$, the yield surface of Eq. (1) yields to two roots for J_1, the positive and negative values of f_h, which results in an elliptical shape in principal stress space, as illustrated in Fig. 1(a). This yield surface is very similar to the elliptical yield functions developed by authors for porous metal and sintered powder based on an extension of von-Mises's concept [8]. If $f_d = 0.0$ leads to the value of J_1 between $-f_h$ and $+f_h$, the cone-cap yield surface will be produced from Eq. (1) based on the intersection points of $J_1 = -f_h$ and the value obtained from $f_d = 0.0$, as demonstrated in Fig. 1(b). This yield surface is very similar to the double-surface cap models, i.e. a combination of Mohr-Coulomb or Drucker-Prager and elliptical surfaces, which has been extensively used by authors to demonstrate the behavior of powder and granular materials [1,2,3,4,5]. In Fig. 1(c), the effect of parameter ψ on the shape of yield surface Eq. (1) is shown in principal stress space for the value of $\psi = 0.1$. As can be observed from the figure, the value of ψ greater than zero causes triangularity of deviatoric trace along the hydrostatic axis. This yield surface is similar to the irregular hexagonal pyramid of the Mohr-Coulomb and cone-cap yield surface employed by researchers for description of soil and geomaterial behavior.

3. Numerical simulation results

In order to illustrate the applicability of the proposed cap plasticity model, the powder behavior during the compaction of an automotive component is analyzed numerically. The constitutive equations of single-cap plasticity along with the large finite element deformation have been implemented in a nonlinear finite element code to evaluate the capability of the model in simulating powder compaction process. A series of experimental tests performed by Doremus et al. [9] are driven to determine and calibrate the parameters of the model. The parameters of function f_h are evaluated using the confining pressure test, where the values of J_{2D} and J_{3D} in Eq. (1) are zero. Considering an initial value for parameter b in Eq. (2), parameters a_0, a_1, a_2 and a_3 can be estimated using a least square method. The parameters of function f_d are estimated performing least square method on the data obtained by the true-triaxial tests [9].

An axisymmetric automotive part which is compacted from iron powder with a mechanical press and a multi-platen die set is simulated numerically. The numerical simulation of this component is performed using a 3D finite element analysis, as illustrated by an axisymmetric representation in Fig. 2. On the virtue of symmetry, the automotive part is analyzed for one quarter of component. The simulation has been performed using the remaining pressing distance of lower inner punch of 9.75 mm and lower outer punch of 31.10 mm. In Fig.

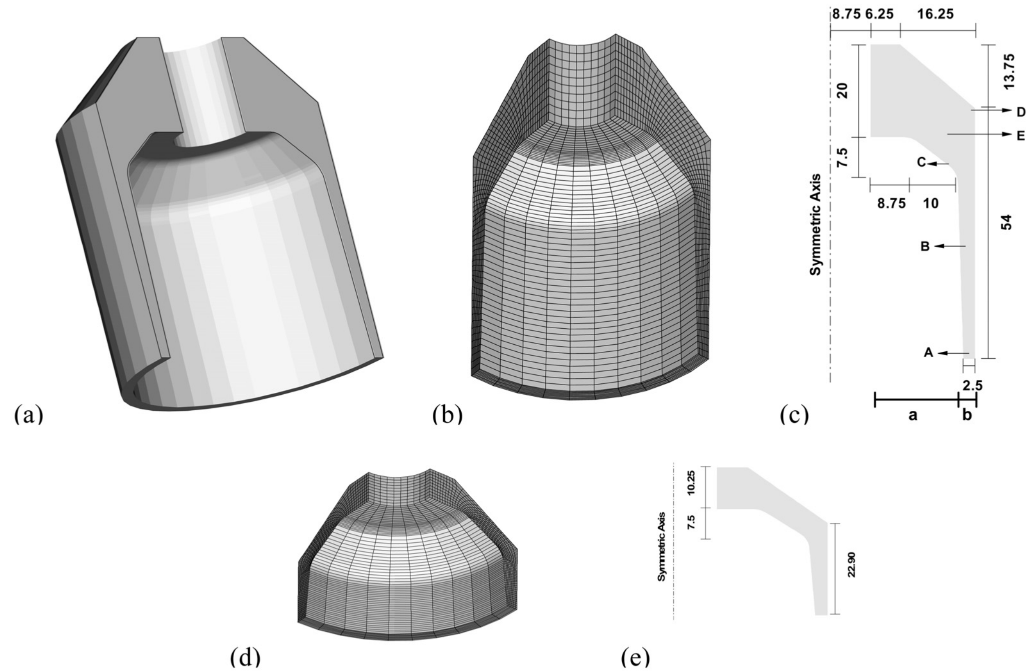

Fig. 2. An automotive component: (a) the uncompacted powder component, (b) one quarter of 3D FE mesh, (c) geometry and boundary conditions, (d) final deformed mesh, (e) geometry of compacted specimen.

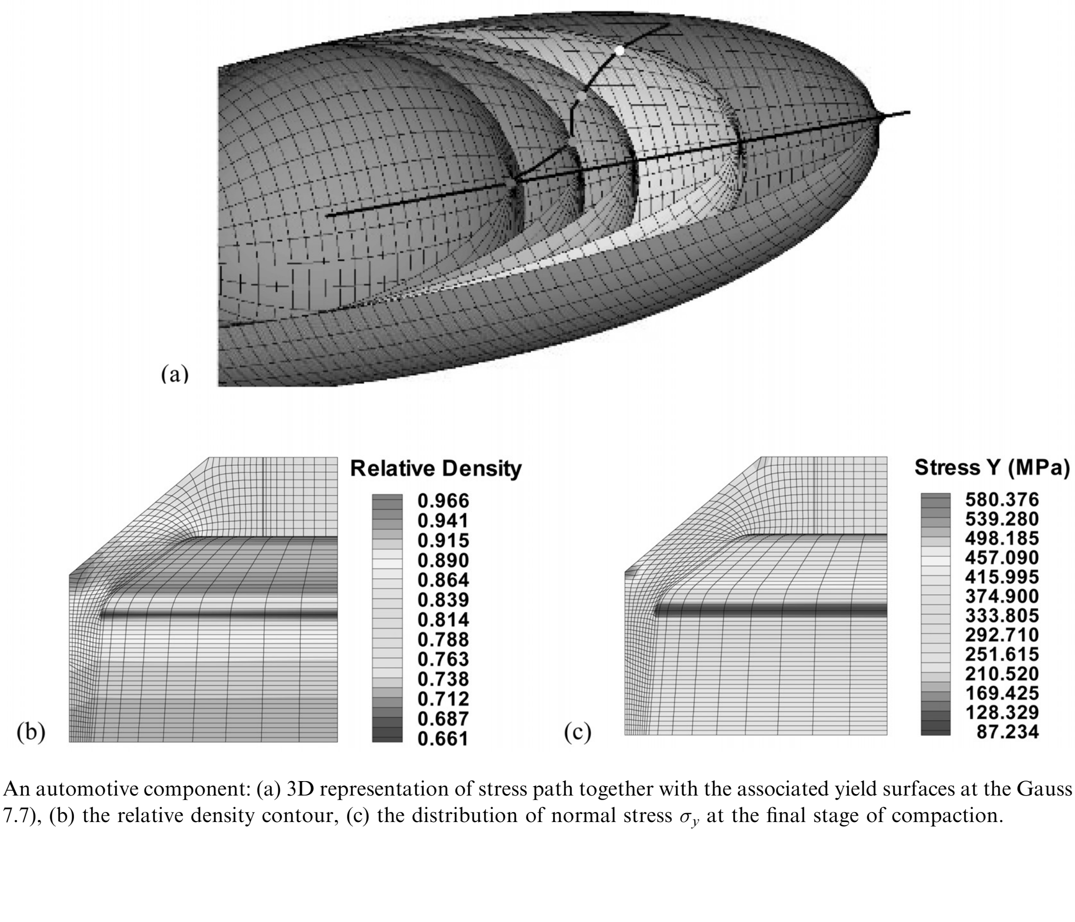

Fig. 3. An automotive component: (a) 3D representation of stress path together with the associated yield surfaces at the Gauss point C (25.6, 17.7), (b) the relative density contour, (c) the distribution of normal stress σ_y at the final stage of compaction.

3(a), the 3D representation of the expansion of yield surfaces along with the obtained stress paths are presented at a Gauss point of an automotive component indicated in Fig. 2(c). This representation clearly shows how the yield surface grows with densification and expands when the material hardens. The applicability of the single cap plasticity model to handle the compaction simulation of powder is clearly evident in Fig. 3(a). In Figs. 3(b) and 3(c), the predicted relative density and normal stress σ_y distributions are shown at the final stage of compaction.

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