Stability of flow and kinetic energy dissipation in 2D annular shear flows of inelastic hard disk assemblies

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Abstract. We have used simulations of inelastic hard disks in two-dimensional shear flows to investigate the stability conditions of kinetic energy upon shearing through different circumstances. We study the characteristics of instability via the signals of kinetic energy dissipation and the statistics associated with this quantity. Our results reveal how the flow characteristics of hard-disk assembly can be modelled by a dynamical model for turbulence named as the GOY shell model. Our results clearly show that the behaviour of rapidly sheared hard-disk assemblies is similar to a fluid in highly chaotic state, so called turbulent state. The current results assist us in finding proper modelling ways in simulation of suspension flows, colloids and magnetic fluids.

1. Introduction

Hard-disk model has been employed in modeling of many complex materials such as granular matter [1,2] and metallic alloys [3]. Interesting statistical match is observed between hard disk model and turbulent flows [2]. In this context, the stabilities of flow and kinetic energy dissipation need serious investigations to clarify how a hard-disk fluid dissipates injected energy under different circumstances.

In this paper, we present some of our recent results associated with the behavior of the signals of kinetic energy dissipation for individual tracer inside a hard-disk fluid. In the following section, the geometry of simulated system and the corresponding model is described. It is followed by the section of results and discussions where the stabilities of flow and kinetic energy dissipation are demonstrated and discussed.

2. Geometry of simulations

The simulation setup has 2D annular Couette geometry with over 8000 hard-disks inside and around the system (rotating walls). Internal disks are subjected to continuous shear via the rotation of surrounding cylinders in opposite directions. The surfaces of cylinders are made of hard-disks with the same size disks identical to the flow particles [2].

The material of hard-disks is introduced by the coefficient of restitution, which characterizes the degree of the inelasticity of particles. The coefficient of restitution is defined [4] as a function of the relative impact velocity between a pair of particles; that is $e(V_n) = 1 - (1 - e_0)(V_n / V_0)^{0.2}$. Here, V_n represents the absolute value of relative impact velocity, e_0 is a reference value for the coefficient of restitution and V_0 is an adjustable parameter representing the reference impact velocity.

Hard-disk simulations are based on the event-driven algorithm in which the dynamics of system proceeds by successive binary collisions. The total momentum of colliding smooth particles conserves in the normal direction to the particle surface at contact point, while the momentum of each particle conserves in the tangential direction. The fourth equation for calculating the post-collisional velocities comes from the definition of the coefficient of restitution.

The collision table [5] is constructed after finding the minimum value (if exists) of possible collision times between any particle *i*, and the rest of particles. This is also called the characteristic equation for collision time. A collision can also occur between an inner disk and a wall disk. The characteristic equation for collision time is solved numerically. After each collision, new collision partners of the last collided pair are determined. For the rest of particles in the system the collision partners do not change.

3. Results and Discussions

Our results are organized in two subsections that focus on the dynamics of kinetic energy dissipation and the properties of hard-disk flow from our simulations.

3.1. Kinetic energy dissipation. As a result of inelastic collisions between hard-disks some amount of kinetic energy is dissipated after each collision. The time series for the energy dissipation through successive collisions of a sample tracer is displayed in Fig. 1a. In this figure, the kinetic energy dissipation is simply normalized by its mean value. The hard-disk system has a packing density of 0.43 subjected to a shear rate of about 40 per unit time scale. Singular bursts in dissipation signal can be observed that are one to two orders of magnitude greater than the mean value. Large bursts of energy dissipation stabilize the flow of kinetic energy into the system. The kinetic energy is injected to the system via collisions between internal hard-disks and the rotating walls (covered by hard-disks similar to internal hard-disks). Therefore, we need a deep insight into the nature of this important quantity. Figure 1b demonstrates the probability density function (PDF) of normalized kinetic energy dissipation for the collisions of entire particles in the system. There are three simulations with different values of e_0, which represent highly, moderately and slowly dissipative systems. The shear rate, packing density and other conditions are identical between the three systems. A general trend is observed in the PDF of energy dissipation that at least two distinct slopes can be detected as following.

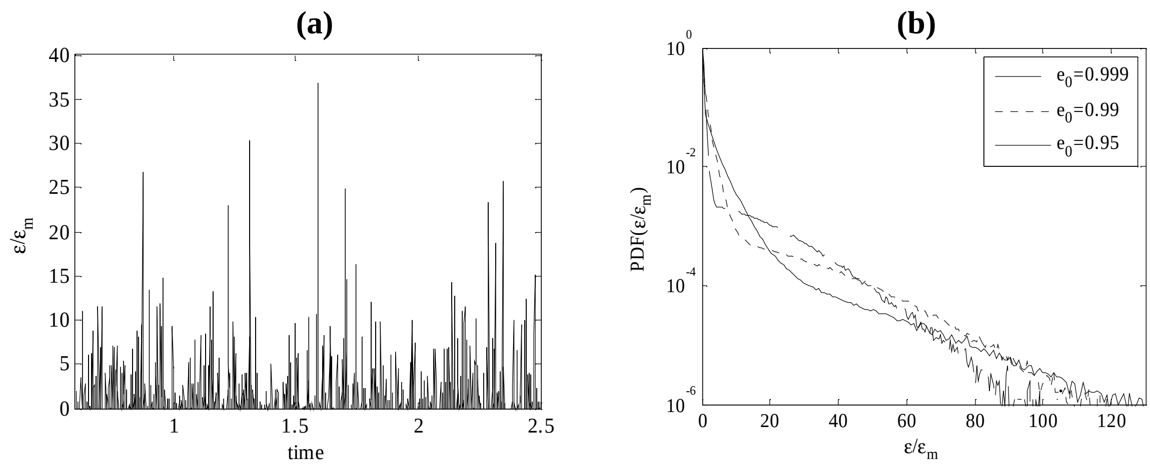

Figure 1. (a) Variation of normalized energy dissipation through successive collisions of a tracer in time for a typical simulation of hard-disks shear flow. (b) Probability density function (PDF) of normalized energy dissipation through the collisions of entire particles in the system. Solid, dotted and dash lines represent simulations with highly-, moderately- and slowly-dissipating systems.

The sharper slope corresponds to small fluctuations of normalized energy dissipation while the gentler slope at the tail of PDF stands for bursts of energy dissipation. Figure 1b shows that the inflection point of the dissipation PDF significantly changes with the reference value of restitution coefficient, e_0. Larger bursts are more probable as restitution coefficient approaches to 1.

3.2. Description via turbulent models. The intermittency of kinetic energy dissipation is the most remarkable characteristic of turbulent flows. The transfer of kinetic energy in turbulent flows starts from the largest eddies toward smaller ones where it is dissipated to heat. Similarly, we can see the kinetic energy is injected to hard-disk mixture via shearing in which the kinetic energy of individual colliding particles to walls increases. In equilibrium conditions and steady state, the injected kinetic energy is balanced with the dissipated energy associated with inelastic collisions of particles. However, on-off intermittency [6,7] is the common feature of turbulent shear flows and hard-disk shear flows. Since spatial discretization of Navier-Stokes equations is limited to rather low Reynolds numbers, we may use a dynamical system approach in order to solve turbulence problem to reveal its intermittent features in high Reynolds numbers. We have employed [2] the Gledzer-Ohkitani-Yamada (GOY) shell model [8] to reproduce the statistics of energy dissipation in shear flow of hard-disks. The GOY model forms a set of ordinary differential equations representing Navier-Stokes equations in wave-vector space,

$$\frac{du_n}{dt} = -\mu k_n^2 u_n + f_n + i k_n \left(u_{n+1}^* u_{n+2}^* - u_{n-1}^* u_{n+1}^* / 4 - u_{n-2}^* u_{n-1}^* / 8 \right), \qquad \textbf{(1)}$$

in which u_n is the velocity scale (a complex variable) corresponding to the mean energy of the n-th shell. It can be interpreted as the velocity increment on an eddy of length scale k_n^{-1}, which identifies the n-th shell. Note that the convective term is modeled as a term containing interactions between neighboring shells, i.e. (n+1,n+2), (n-1,n+1) and (n-2,n-1). f_n is the external forcing term, which may be only considered in the first two shells. μ is the kinematic viscosity and t is time. The superscript * shows the complex conjugate of the variable. The dissipation rate of kinetic energy for all N shells can be calculated as,

$$\varepsilon(t) = \mu \sum_{n=1}^{N} k_n^2 \left| u_n(t) \right|^2 . \qquad \textbf{(2)}$$

If we consider the mean energy dissipation rate in sheared hard-disk system within n_c collisions taken place in time interval δt as,

$$\overline{E} = -\frac{m_p}{4\delta t} \sum_{i=1}^{n_c} (1 - e_i^2) V_{n_i}^2 , \qquad \textbf{(3)}$$

we interestingly see that Eqs. (2) and (3) are analogous. The relative impact velocity is the only source for the dissipation of energy in present simulation. e_i is the restitution coefficient in i-th collision, which is a function of V_{n_i}, the relative impact velocity in a collision n_i between two particles with mass m_p. Equations (2) and (3) indicate that energy dissipation rate is proportional to the square of the velocity in both turbulence models and sheared hard-disk system. As a comparison, Fig. 2 is presented in which the PDF of ε is shown for both cases of simulated sheared hard-disk system and the GOY shell model of turbulence. Two simulated cases are shown where their shear rates were 3 and 6 per unit time. The forcing term in the shell model is found by trial and error so that the statistics of the energy dissipation from the model fits to those from the simulation. It is about 5×10^{13} and 10^{14} times $\mu^2 k_0^3$ for the cases of shear rates 3 and 6 per unit time, respectively. Note that the wave number of n-th shell is expressed as $k_n = k_0 2^n$, where k_0 is the smallest wave number set to 1.8. The number of shells was set to N=13 and $\mu = 10^{-7}$ (equivalent to Re ~10^7).

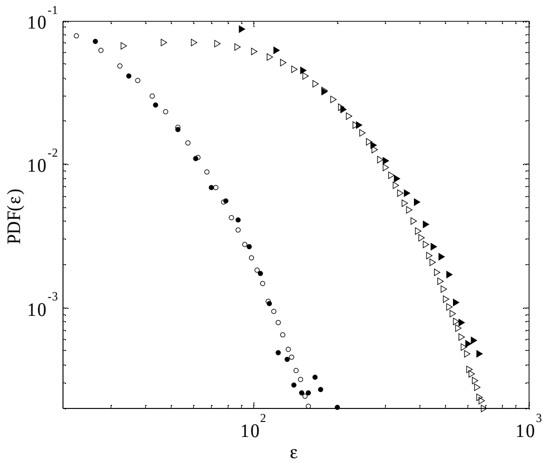

Figure 2. PDF of energy dissipation rate in hard-disk shear flow simulations and GOY shell model. Open symbols stand for simulation results and filled ones for the GOY model. Circles and triangles represent simulations with the shear rates of 3 and 6 per unit time. Note that the solid volume fraction is 0.46 and e_0=0.90.

4. Summary and Conclusions

Hard-disk assemblies are simulated in 2D annular Couette geometry subjected to rapid shear flows. Any individual particle may be assumed as a tracer which is affected by a surrounding hard-disk fluid. Due to discrete nature of this fluid the tracer suffers impulses from the particles of the fluid. In each impulse (collision), there is some amount of kinetic energy lost due to the inelastic nature of particles. The lost kinetic energy is made up in average by driving the walls of the system, namely the shearing walls. However, variation of dissipated kinetic energy with time has interesting statistics according to our results. We employed a dynamical system model, namely the GOY shell model, to reproduce the same statistics of kinetic energy dissipation as calculated in shear flow of hard-disks. More investigations are underway to elucidate such statistical match between simulations and the model of turbulence. In this context, more evidences are being collected from simulations to clarify the relevance of Eqs. (2) and (3), which are the expressions for kinetic energy dissipation in the model and the simulation.

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